#### Wetting-phase relative permeability in multi-scale porous media

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#### Abstract

Modeling relative permeability in multi-scale rocks and fractured networks has broad applications to understanding oil production and recovery in reservoir formations. Natural porous media are typically composed of two domains; one incorporates macropores, while the other contains micropores. In the literature, numerous theoretic models have been developed based on the series-parallel tubes approach (Mualem, 1976; van Genuchten, 1980) to estimate wetting-phase relative permeability (krw) from pore size distribution or capillary pressure curve. In this study, we, however, invoke concepts from critical path analysis (CPA), a theoretical technique from statistical physics. CPA has been successfully used to model flow and transport in porous media (Hunt, 2001; Ghanbarian-Alavijeh and Hunt, 2012; Hunt et al., 2013; 2014; Ghanbarian et al., 2016; Ghanbarian and Hunt, 2017). We estimate the wetting-phase relative permeability from the measured capillary pressure curve using two methods: (1) critical path analysis (CPA), and (2) series-parallel tubes (vG-M). To evaluate these models, we use 26 experiments from the literature for which capillary pressure and wetting-phase relative permeability data were measured at 500 data point over a wide range of wetting-phase saturation (Sw). Results demonstrate that CPA estimates krw more precisely than vG-M. We show that accurate krw estimation by the CPA-based model needs precise characterization of capillary pressure curve and accurate calculation of the crossover point (Swx) separating the two domains.



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### BACKGROUND

#### **Factors affecting wetting-phase relative** permeability

- Connectivity
- Pore-throat size distribution
- Tortuosity
- Porosity
- Pore shape geometry

#### **Theoretical upscaling techniques**

- Bundle of capillary tubes approach
- Effective-medium approximation (EMA)
- Critical path analysis (CPA)
- Perturbation theory
- Volume averaging method

#### **PURPOSES AND ASSUMPTIONS**

#### **Objectives**

- ✓ To apply concepts from CPA to dual-porosity media by revisiting the Hunt et al. (2013) model.
- $\checkmark$  To evaluate  $k_{\rm rw}$  estimation from accurately characterized  $S_w$ - $P_c$  curve using a database including 26 samples.
- To compare the accuracy of the bi-modal CPA model with that of the bi-modal vG-M model.

#### Assumptions

- Pores are cylindrical.
- $\Box$   $k_{\rm rw}$  is mainly controlled by pores with sizes greater than some critical pore size.
- $\Box \phi_1$  and  $\phi_2$  are porosities corresponding to the structural and textural pores, respectively.

### **MATERIALS AND METHODS**

#### Hydraulic flow in a tube of radius R

$$g_h = \frac{\pi R^4}{8\mu l}$$

#### **Samples**

The database used in this study is from Schwen et al. (2015) and includes 26 samples. The  $S_w$ - $P_c$  and  $S_w$ - $k_{rw}$ curves, each of which includes 500 measured data points determined using the evaporation method with extended range of measurements.

#### **Critical path analysis**

Concepts of critical path analysis (CPA) from percolation theory have been successfully used to model hydraulic properties of porous media (Hunt et al., 2014; Hunt, 2001). Based on CPA, transport in a network of pores is dominated by those pores whose sizes are greater than some critical size (Fig. 1). Accordingly, pores smaller than the critical pore size make trivial contribution to the overall transport.

#### **Bimodal capillary pressure curve model**

For dual-porosity media the capillary pressure curve model is

$$S_{w} = \begin{cases} 1, \\ 1 - \frac{\beta_{1}}{\phi} \left[ 1 - \left(\frac{P_{c}}{P_{d1}}\right)^{D_{1}-3} \right], \\ S_{wx}, \\ \frac{\phi_{2}}{\phi} - \frac{\beta_{2}}{\phi} \left[ 1 - \left(\frac{P_{c}}{P_{d2}}\right)^{D_{2}-3} \right], \end{cases}$$

#### Wetting-phase relative permeability model

Hunt et al. (2013) proposed the following bi-modal model for  $k_{rw}$ :

$$k_{rw} = \begin{cases} \left[\frac{\beta_1 - \phi + \phi S_w}{\beta_1}\right]^{\frac{3}{3 - D_1}}, & S_w > \phi_2/\phi \\ \left(\frac{\beta_1 - \phi_1}{\beta_1}\right)^{\frac{3}{3 - D_1}} \left[\frac{\beta_2 - \phi_2 + \phi S_w}{\beta_2}\right]^{\frac{3}{3 - D_2}}, & S_w < \phi_2/\phi \end{cases}$$

#### **RESULTS**

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Sample	φ	<b>D</b> <sub>1</sub>	$\beta_1$	P <sub>d1</sub> (cm)	<b>D</b> <sub>2</sub>	$\beta_2$
CT_1	0.50	2.875	0.47	1.8	2.571	0.22
CT_2	0.49	2.871	0.44	1.3	2.511	0.17
CT_3	0.56	2.235	0.30	6.6	2.687	0.22
CT_4	0.56	2.511	0.31	3.9	2.637	0.22
CT_5	0.51	2.489	0.24	5.0	2.601	0.24
CT_6	0.49	2.573	0.24	4.2	2.606	0.25
CT_7	0.55	2.194	0.30	6.8	2.636	0.22
CT_8	0.55	2.535	0.30	2.6	2.631	0.23
CT_9	0.56	2.595	0.31	2.0	2.638	0.21
CT_10	0.60	2.347	0.30	6.6	2.690	0.22
CT_11	0.53	2.852	0.49	4.5	2.511	0.16
CT_12	0.54	2.935	0.58	1.1	2.620	0.25
CT_13	0.54	2.551	0.32	3.0	2.645	0.22
NT_1	0.40	2.958	0.74	16.0	2.447	0.25
NT_2	0.45	2.945	0.70	13.9	2.408	0.29
NT_3	0.41	2.928	0.34	26.2	2.561	0.30
NT_4	0.47	2.970	0.86	2.3	2.550	0.25
NT_5	0.49	2.662	0.20	8.8	2.646	0.31
NT_6	0.46	2.951	0.91	16.3	2.458	0.26
NT_7	0.44	2.951	0.61	3.8	2.515	0.24
NT_8	0.44	2.962	1.00	21.3	2.515	0.25
NT_9	0.45	2.959	1.00	9.8	2.526	0.25
NT_10	0.45	2.974	1.00	3.5	2.512	0.32
NT_11	0.41	2.968	1.00	10.2	2.518	0.23
NT_12	0.42	2.965	1.00	9.7	2.545	0.24
NT_13	0.46	1.875	0.13	55.5	2.605	0.27

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 $P_c < P_{d1}$  $P_{d1} < P_c < P_x$  $P_x < P_c < P_{d2}$  $P_c > P_{d2}$ 



Fig. 1. Two-dimensional scheme of the critical path analysis. (a) A pore network composed of six different pore sizes (i.e., 0.5, 1, 1.5, 2, 2.5 and 3) with arbitrary units) randomly distributed in the medium. (b) The same network with only the first two largest pores (2.5 and 3) in their original locations. Pores smaller than 2.5 were removed from the pore network. As can be seen, the medium does not percolate. (c) The network after adding the third largest pores with size 2 (critical pore size). The samplespanning cluster is first formed and the network starts percolating.



Fig. 2. (a) Measured capillary pressure curve, (b) derived bi-modal pore size distribution, (c) fitted bimodal capillary pressure curve model to the data at higher water saturations, and (d) fitted bimodal capillary pressure curve model to the data at lower water saturations for sample CT 1.

$P_{d2}$	$P_{\rm x}$	$\phi_1$	$\phi_2$
		0.05	0.00
1418	679	0.25	0.22
3508	2590	0.27	0.19
1105	373	0.28	0.22
1696	469	0.28	0.22
1527	373	0.22	0.24
1423	340	0.20	0.25
1564	373	0.29	0.22
1245	310	0.27	0.23
1048	296	0.27	0.21
501.1	224	0.26	0.22
2863	2360	0.29	0.18
835.6	195	0.16	0.25
1340	340	0.28	0.22
2293	1080	0.12	0.28
1730	816	0.14	0.31
904.8	539	0.07	0.32
1106	679	0.14	0.28
1412	409	0.15	0.30
1575	1420	0.18	0.28
1556	1080	0.15	0.26
2050	2050	0.16	0.28
1677	1290	0.18	0.27
1734	282	0.11	0.33
1964	1490	0.15	0.26
1614	1560	0.16	0.26
1743	428	0.12	0.27





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