Dependence of streamer propagation on background field conditions

Nikolai Lehtinen[1]

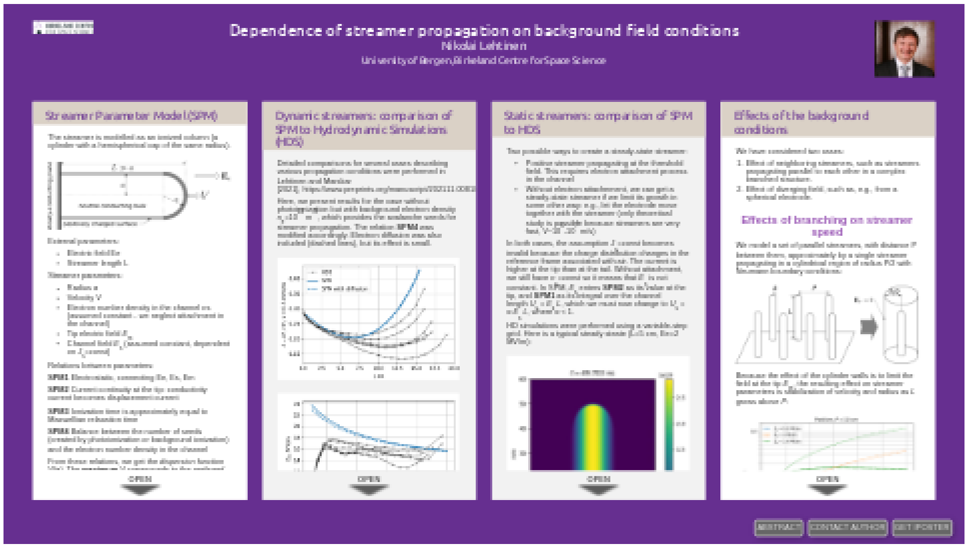
[1]University of Bergen

November 26, 2022

Abstract

Positive and negative electric streamers are column-shaped discharges which are an important stage in lightning development. The mechanism, by which the streamer acquires a certain velocity and radius of its head, had been a long-standing puzzle. In [1], we proposed a parametric streamer model (PSM), which may explain this mechanism, as well as the mechanism of the streamer threshold electric field. The PSM results for a positive isolated streamer in constant uniform external electric field are verified by comparing with hydrodynamic calculations of a 'steady-state' streamer, namely a streamer whose length is kept constant by synchronizing the position of the electrode to which it is attached with the moving streamer head. For this particular streamer configuration, we found that both velocity and radius of a streamer increase with its length, a results which was also observed in previously performed hydrodynamic calculations and experiments. Beside the velocity and radius of the streamer, PSM allows to quickly estimate all other streamer parameters, such as the maximum field at the streamer tip and the field inside the streamer channel. The relatively low, compared to, e.g., hydrodynamic or particle-in-cell (PIC) models, computational costs of PSM suggest that generalizion of PSM principles to more complicated streamer structures may be very valuable. In the present work, we generalize PSM to evaluate how different background conditions affect streamer propagation. In particular, we find that, unlike the uniform-field case described above, (1) for the non-uniform electric field such as that of a spherical electrode, the streamer velocity decreases with length; (2) for a streamer propagating parallel to other similar streamers (as in a streamer "bunch''), both velocity and radius stabilize to an approximately constant value. [1] N. G. Lehtinen, "Physics and mathematics of electric streamers," Radiophysics and Quantum Electronics, 2021, in print [Russian version: DOI: 10.52452/00213462_2021_64_01_12].

Dependence of streamer propagation on background field conditions



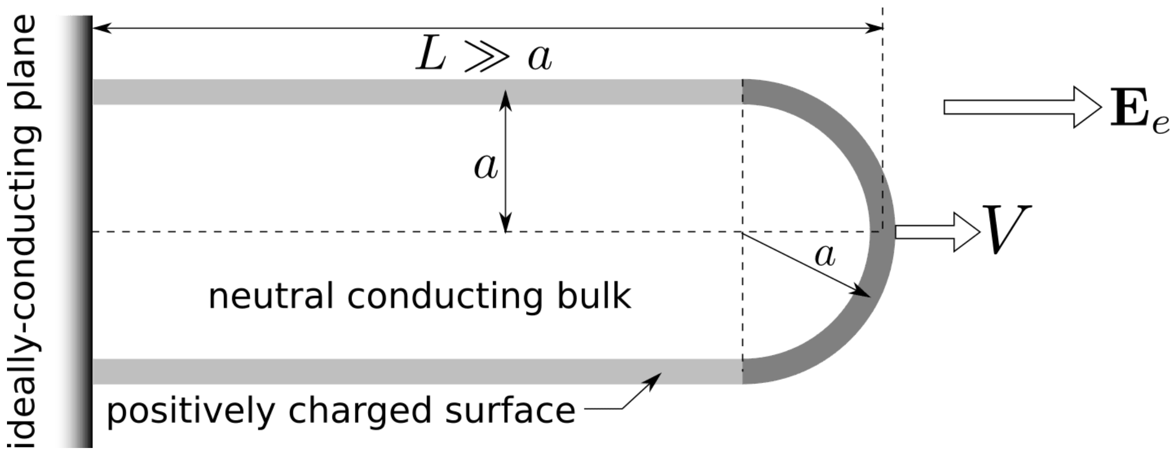
Nikolai Lehtinen

University of Bergen, Birkeland Centre for Space Science

PRESENTED AT:

STREAMER PARAMETER MODEL (SPM)

The streamer is modelled as an ionized column, i.e. a cylinder with a hemispherical cap of the same radius [Lehtinen, 2021] (https://dx.doi.org/10.1007/s11141-021-10108-5).



External parameters:

- Electric field Ee
- Streamer length L

Streamer parameters:

- Radius *a*
- Velocity *V*
- Electron number density in the channel *n*s (assumed constant - we neglect attachment in the channel)
- Tip electric field E_m
- Channel field E_s (assumed constant, dependent on J_s=*const*)

Relations between parameters:

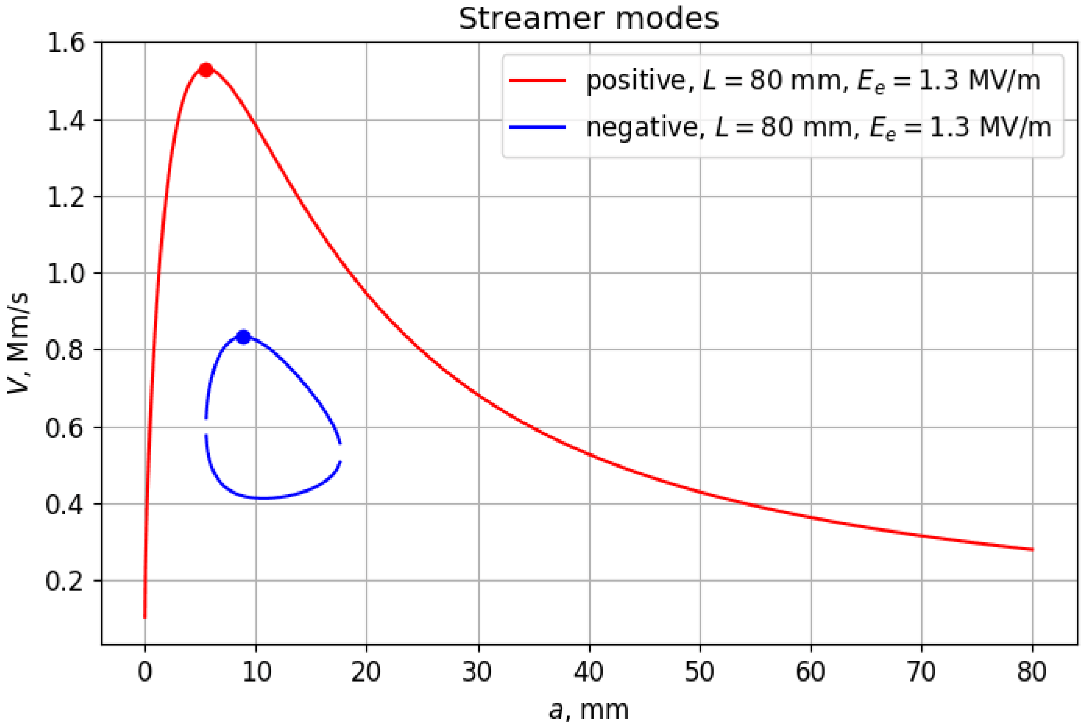
SPM1 Electrostatic, connecting Ee, Es, Em

SPM2 Current continuity at the tip: conductivity current becomes displacement current

SPM3 Ionization time is approximately equal to Maxwellian relaxation time

SPM4 Balance between the number of seeds (created by photoionization or background ionization) and the electron number density in the channel

From these relations, we get the *dispersion function V(a)*. The **maximum** *V* corresponds to the *preferred mode*, or the physical streamer.

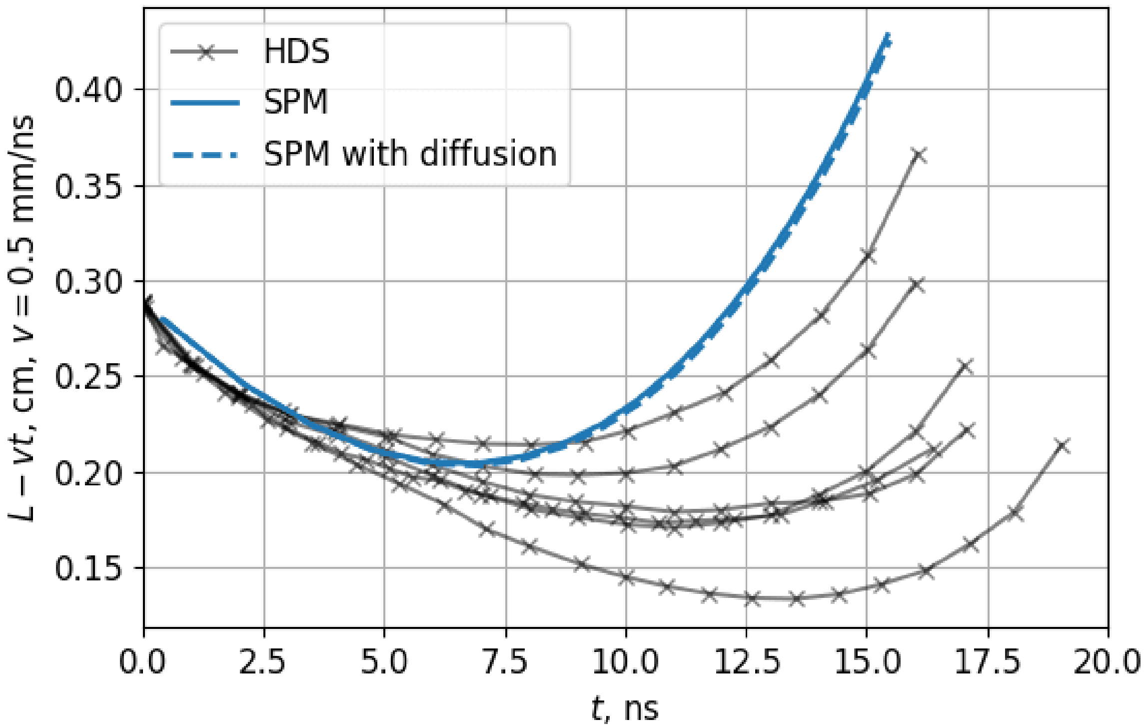


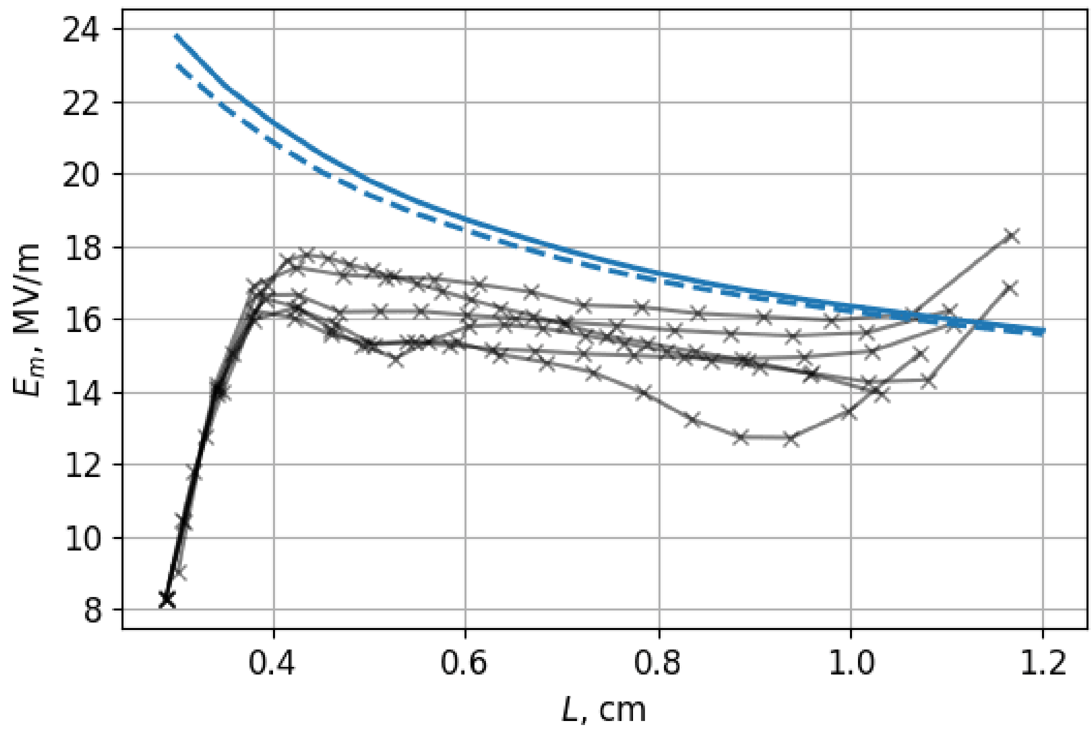
Streamer modes
positive, $L = 80$ mm, $E_e = 1.3$ MV/m
negative, $L = 80$ mm, $E_e = 1.3$ MV/m
V, Mm/s
a, mm
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
10
20
30
40
50
60
70
80

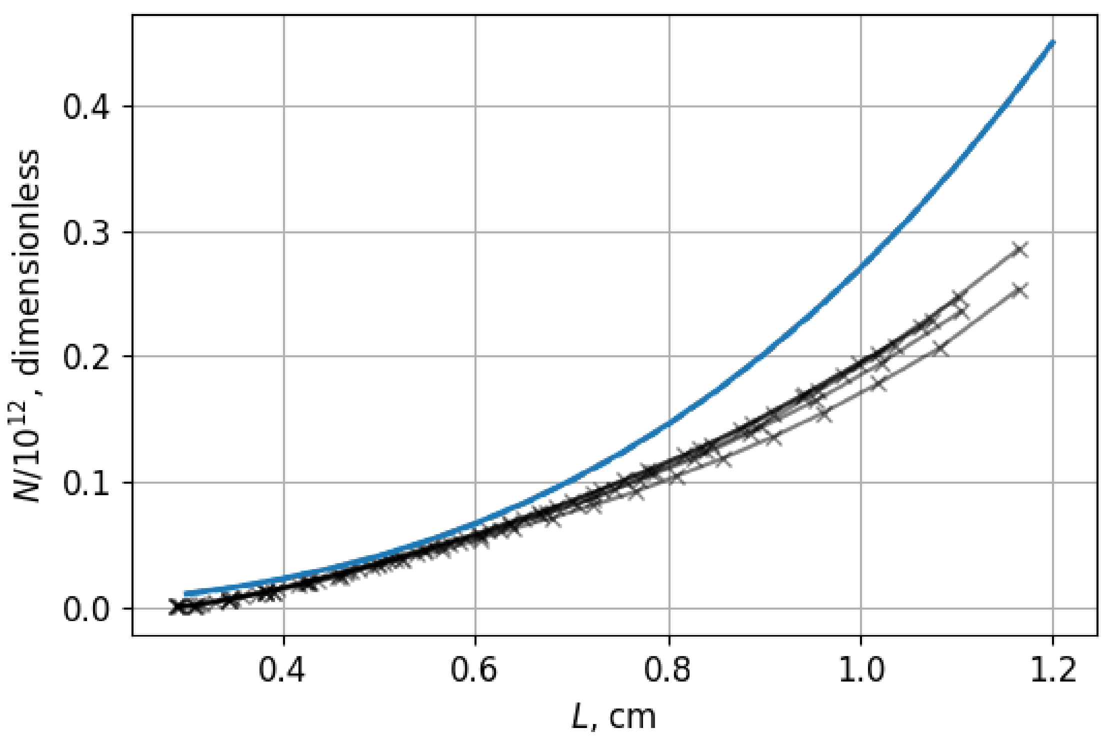
DYNAMIC STREAMERS: COMPARISON OF SPM TO HYDRODYNAMIC SIMULATIONS (HDS)

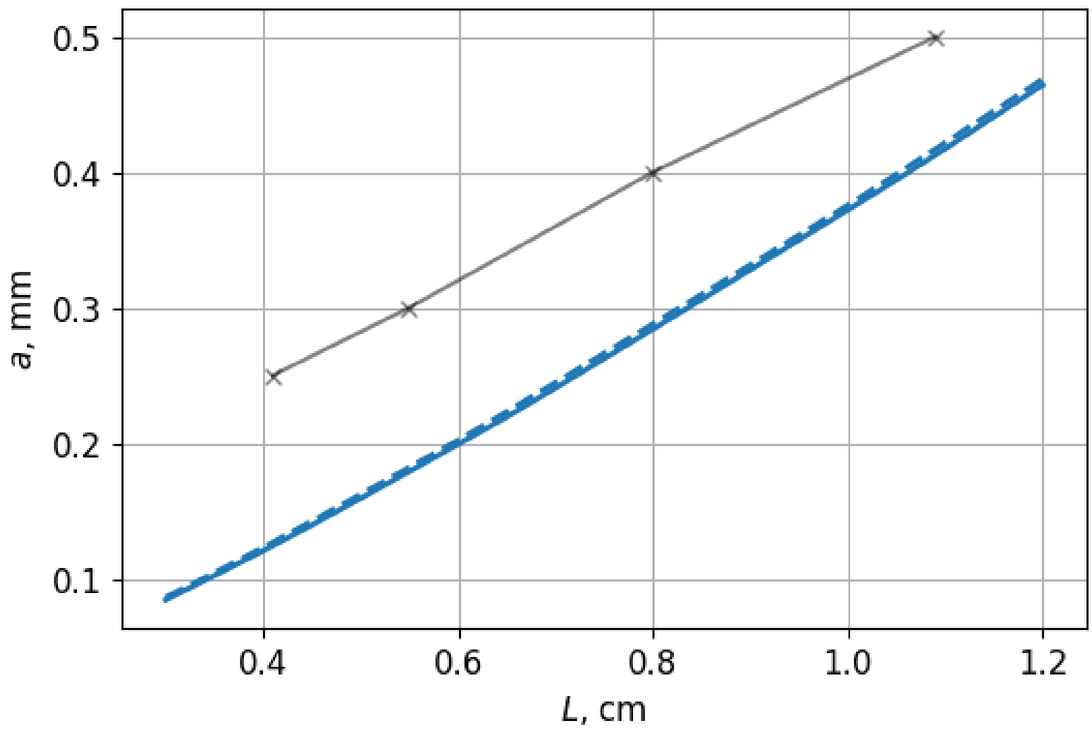
Detailed comparisons for several cases describing various propagation conditions were performed in Lehtinen and Marskar [2021] (https://www.mdpi.com/2073-4433/12/12/1664).

Here, we present results for the case without photoionization but with background electron density $n_e=10^{13}$ m^{-3}, which provides the avalanche seeds for streamer propagation. The relation **SPM4** was modified accordingly. Electron diffusion was also included (dashed lines), but its effect is small.




24
22
20
18
16
14
12
10
8
E_m, MV/m
0.4
0.6
0.8
1.0
1.2
L, cm


0.4
0.3
0.2
0.1
0.0
$N/10^{12}$, dimensionless
0.4
0.6
0.8
1.0
1.2
L, cm


0.5
0.4
0.3
0.2
0.1
a, mm
0.4
0.6
0.8
1.0
1.2
L, cm

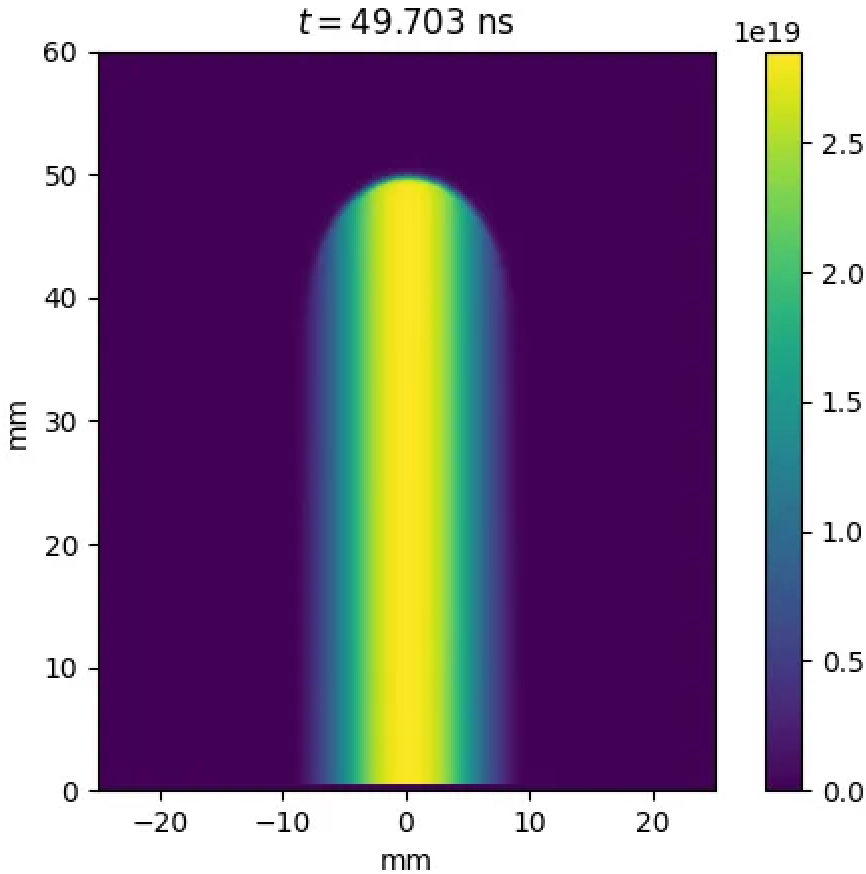
STATIC STREAMERS: COMPARISON OF SPM TO HDS

Two possible ways to create a steady-state streamer:

- Positive streamer propagating at the threshold field. This requires electron attachment process in the channel
- Without electron attachement, we can get a steady-state streamer if we limit its growth in some other way: e.g., let the electrode move together with the streamer (only theoretical study is possible because streamers are very fast, $V \sim 10^6$-10^7 m/s)

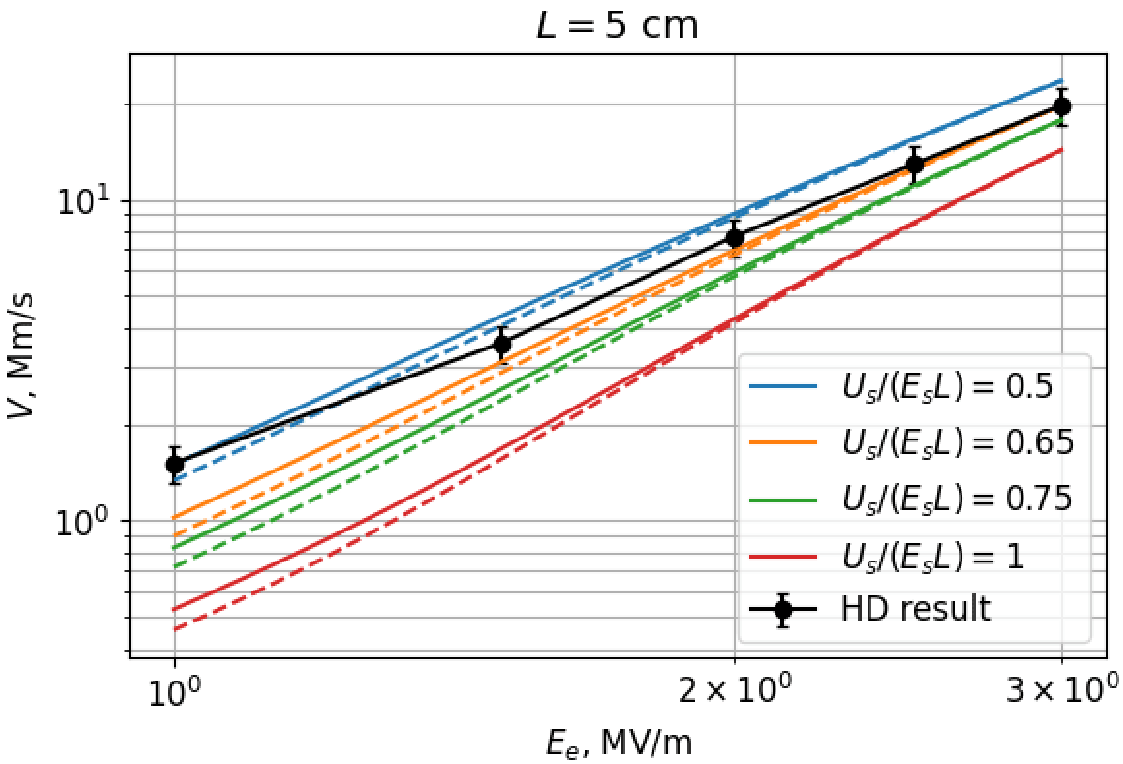
In both cases, the assumption J_s=*const* becomes invalid because the charge distribution changes in the reference frame associated with air. The current is higher at the tip than at the tail. Without attachment, we still have n_s=*const* so it means that E_s is not constant. In SPM, E_s enters **SPM2** as its value at the tip, and **SPM1** as its integral over the channel length $U_s = E_s L$, which we must now change to $U_s = \alpha E_s L$, where $\alpha < 1$.

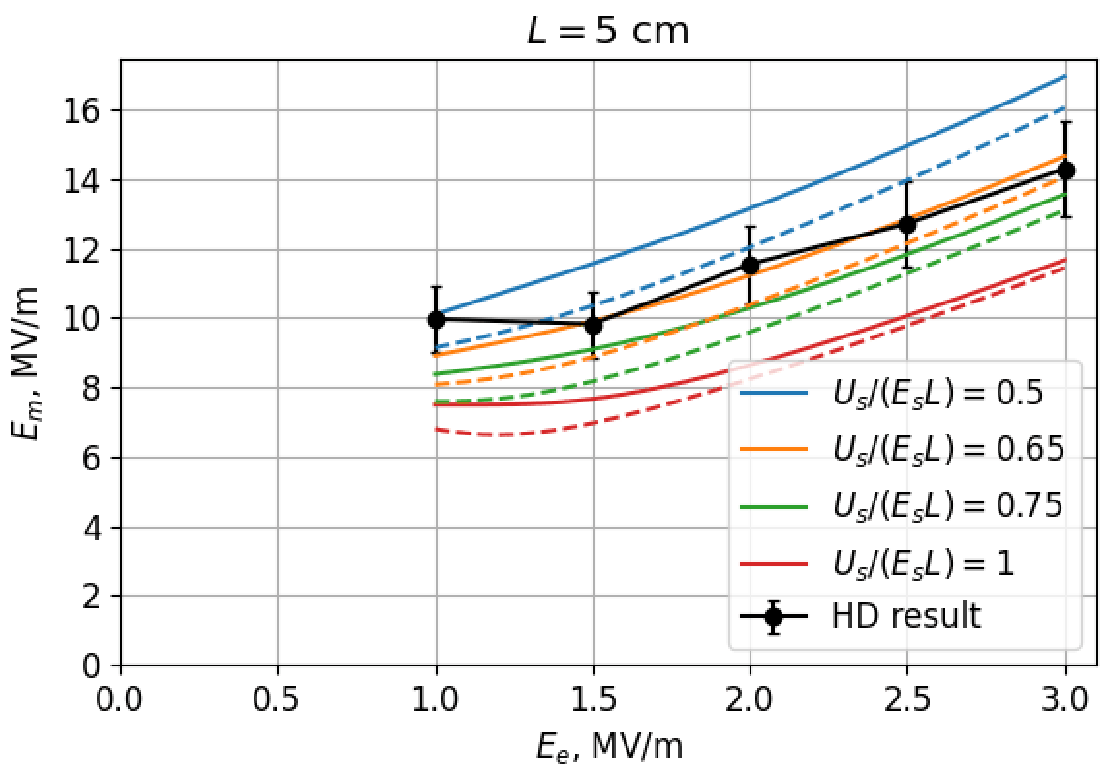
HD simulations were performed using a variable-step grid. Here is a typical steady steate (L=5 cm, Ee=2 MV/m):

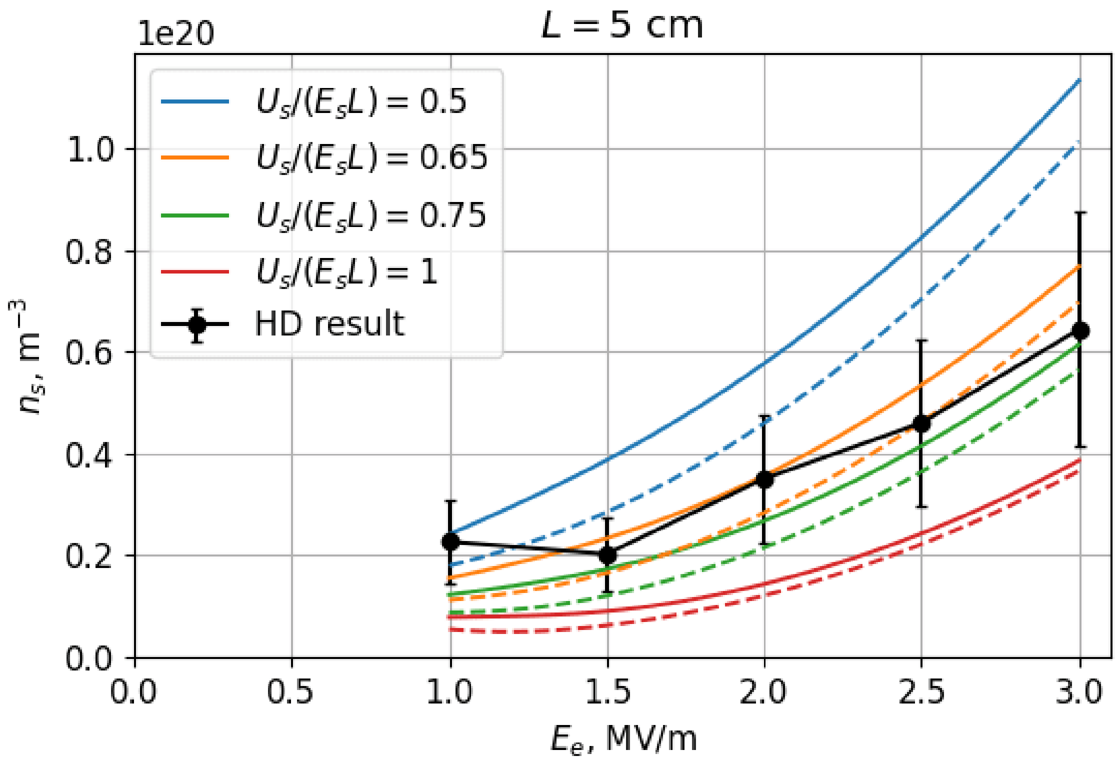


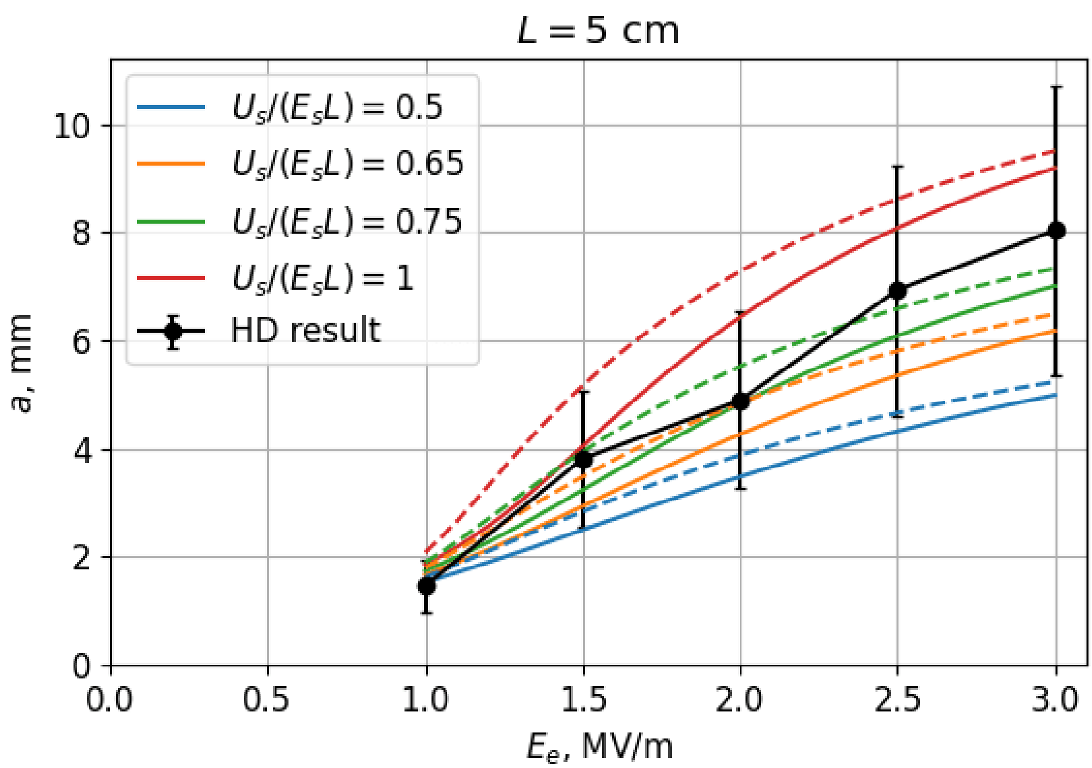
Errors are estimated by comparing to results with grid with a twice larger step Δx. The most accurate results are for $d/\Delta x$ = 7-12, where d is the width of the ionization front.

Results for L = 5 cm:

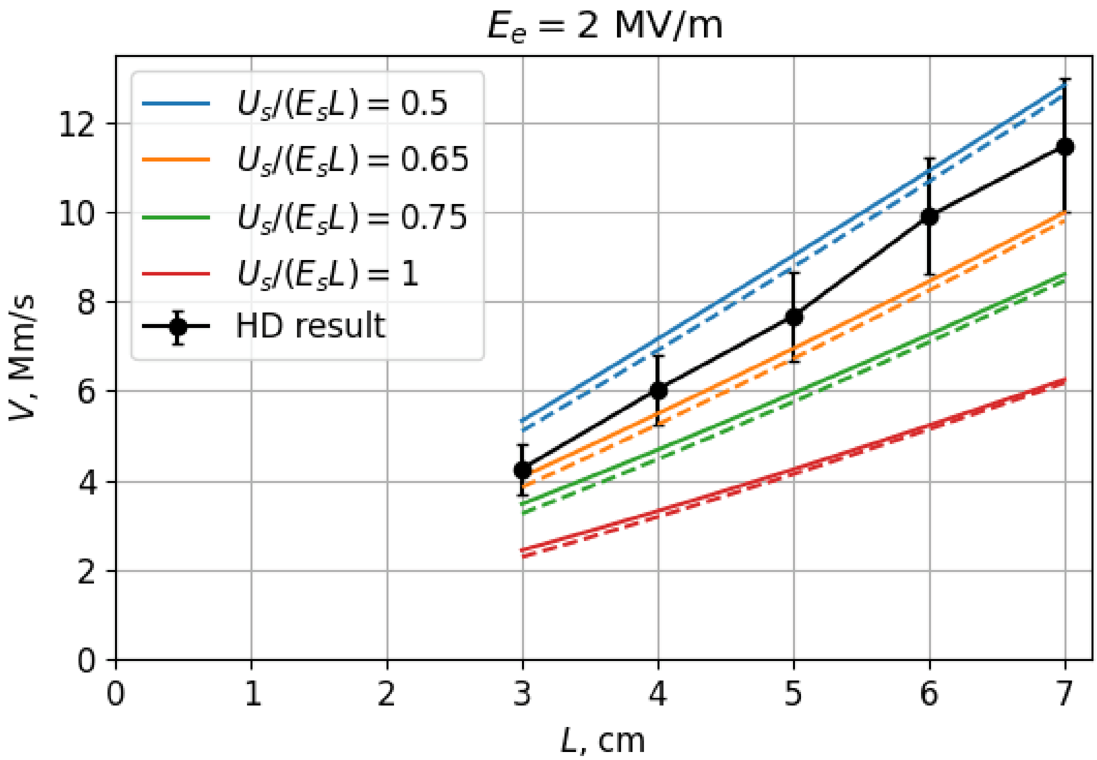

$L = 5$ cm
V, Mm/s
E_e, MV/m
10^1
10^0
2×10^0
3×10^0
$U_s/(E_s L) = 0.5$
$U_s/(E_s L) = 0.65$
$U_s/(E_s L) = 0.75$
$U_s/(E_s L) = 1$
HD result


$L = 5$ cm
E_m, MV/m
E_e, MV/m
0
2
4
6
8
10
12
14
16
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$U_s/(E_s L) = 0.5$
$U_s/(E_s L) = 0.65$
$U_s/(E_s L) = 0.75$
$U_s/(E_s L) = 1$
HD result

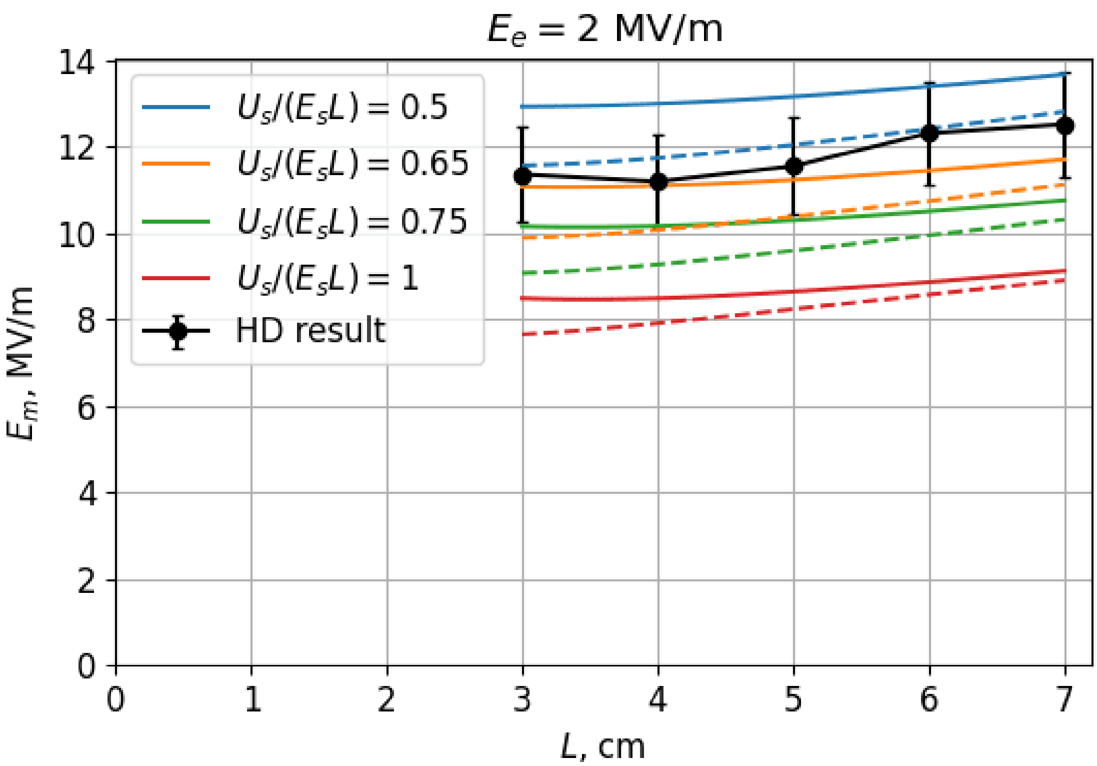




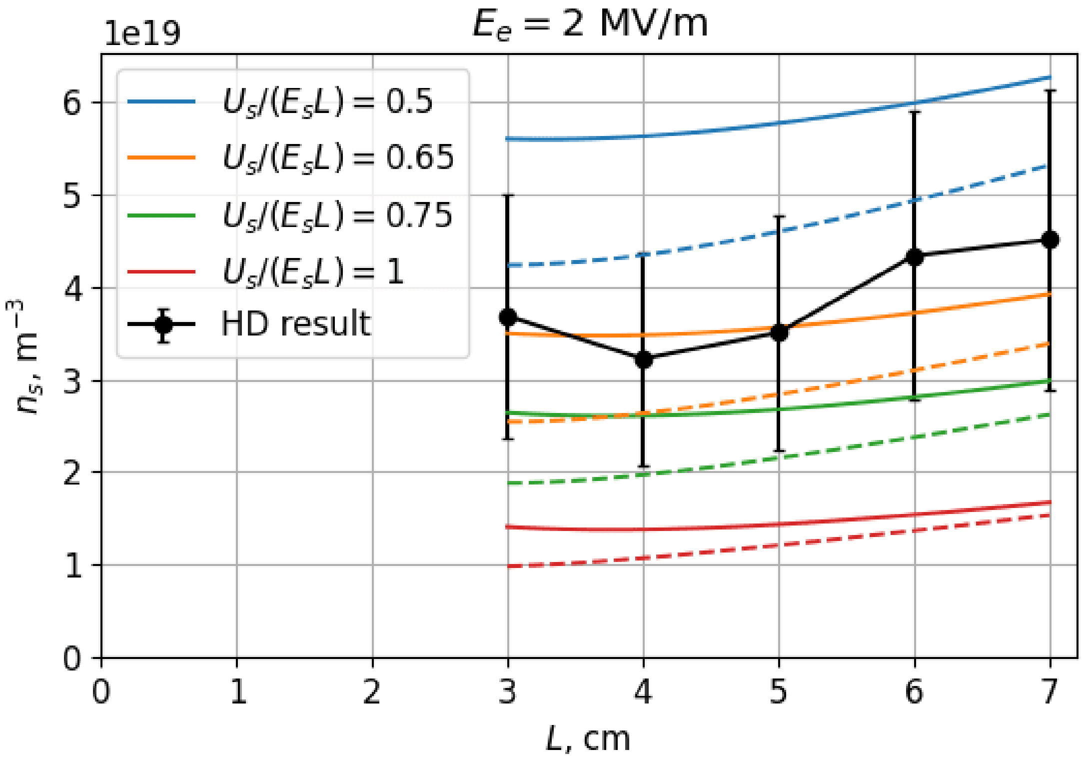
Results for $E_e = 2$ MV/m:

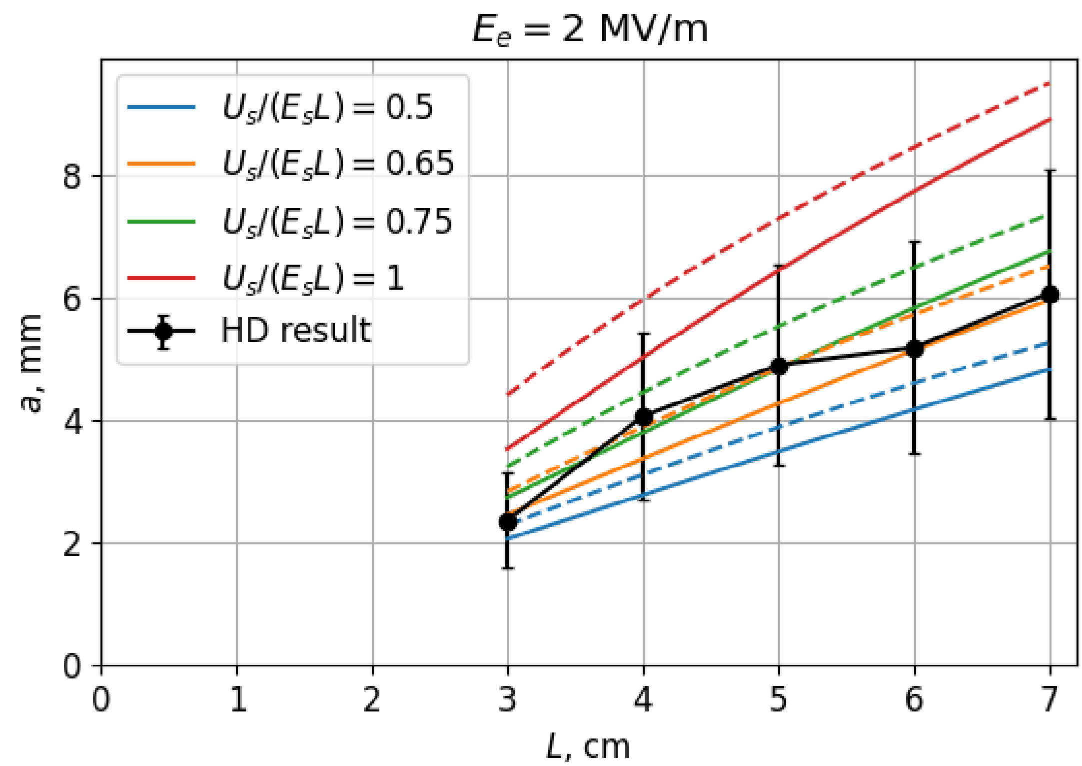


$E_e = 2$ MV/m
$U_s/(E_sL) = 0.5$
$U_s/(E_sL) = 0.65$
$U_s/(E_sL) = 0.75$
$U_s/(E_sL) = 1$
HD result
V, Mm/s
L, cm



$E_e = 2$ MV/m
$U_s/(E_sL) = 0.5$
$U_s/(E_sL) = 0.65$
$U_s/(E_sL) = 0.75$
$U_s/(E_sL) = 1$
HD result
E_m, MV/m
L, cm


$E_e = 2$ MV/m
1e19
$U_s/(E_sL) = 0.5$
$U_s/(E_sL) = 0.65$
$U_s/(E_sL) = 0.75$
$U_s/(E_sL) = 1$
HD result
n_s, m^{-3}
L, cm
0
1
2
3
4
5
6
7


$E_e = 2$ MV/m
$U_s/(E_sL) = 0.5$
$U_s/(E_sL) = 0.65$
$U_s/(E_sL) = 0.75$
$U_s/(E_sL) = 1$
HD result
a, mm
L, cm
0
1
2
3
4
5
6
7
8

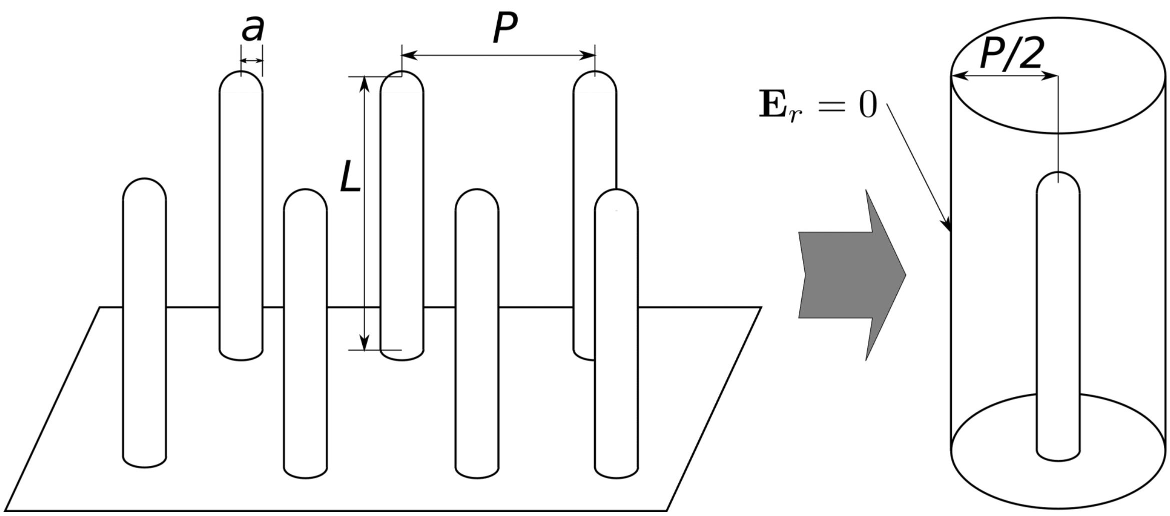
EFFECTS OF THE BACKGROUND CONDITIONS

We have considered two cases:

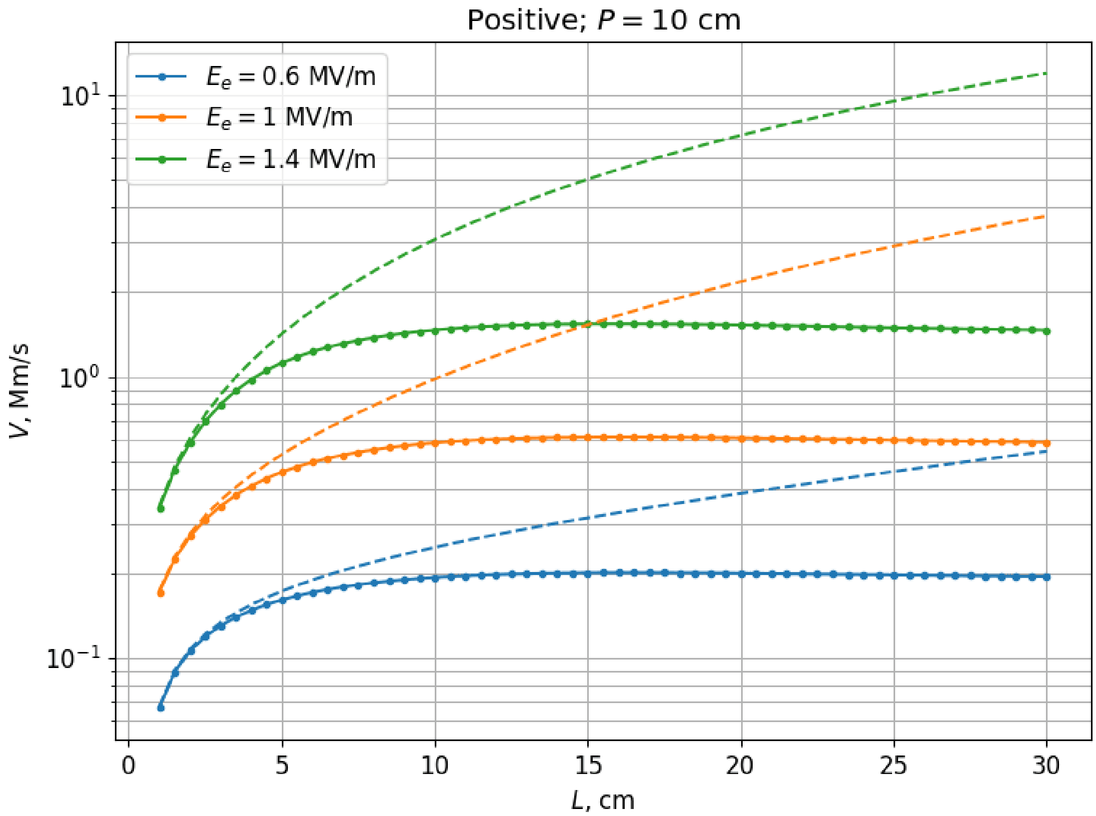
1. Effect of neighboring streamers, such as streamers propagating parallel to each other in a complex branched structure.
2. Effect of diverging field, such as, e.g., from a spherical electrode.

Effects of branching on streamer speed

We model a set of parallel streamers, with distance P between them, approximately by a single streamer propagating in a cylindrical region of radius P/2 with Neumann boundary conditions:



Because the effect of the cylinder walls is to limit the field at the tip E_m, the resulting effect on streamer parameters is stabilization of velocity and radius as L grows above P:



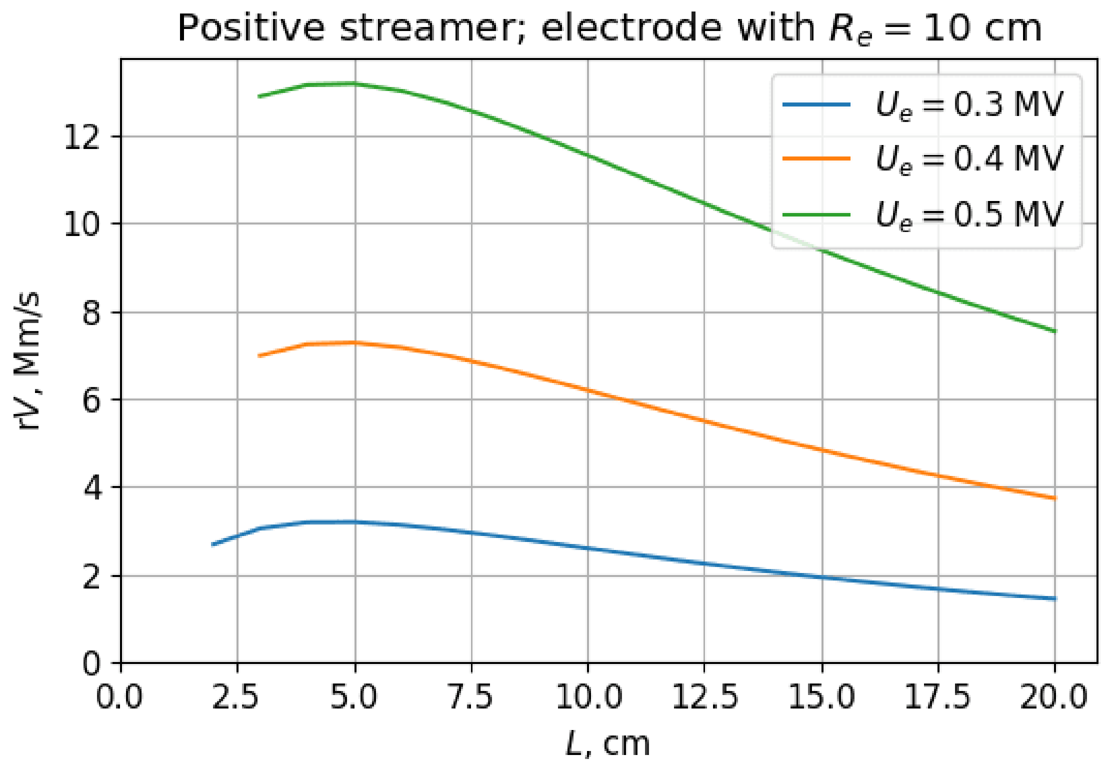
Here, the dashed lines are for isolated streamers. The reason for the observed stabilization is that the external field E_e does not penetrate beyond distance ~P behind the streamer tips, so the effective length of the streamer remains to be of the order of P (or $P/2$).

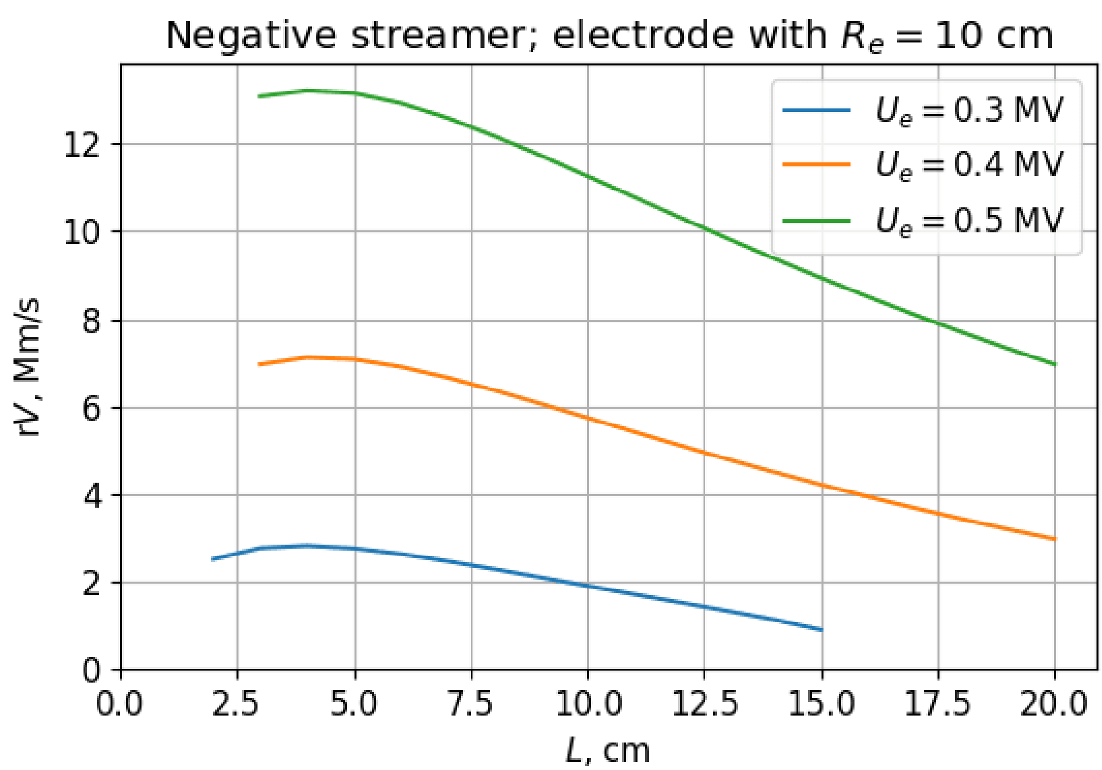
Streamers in spherical electrode field

We **do not** take into account:

- Attachment inside the channel
- The effect of neighboring streamers (see above)

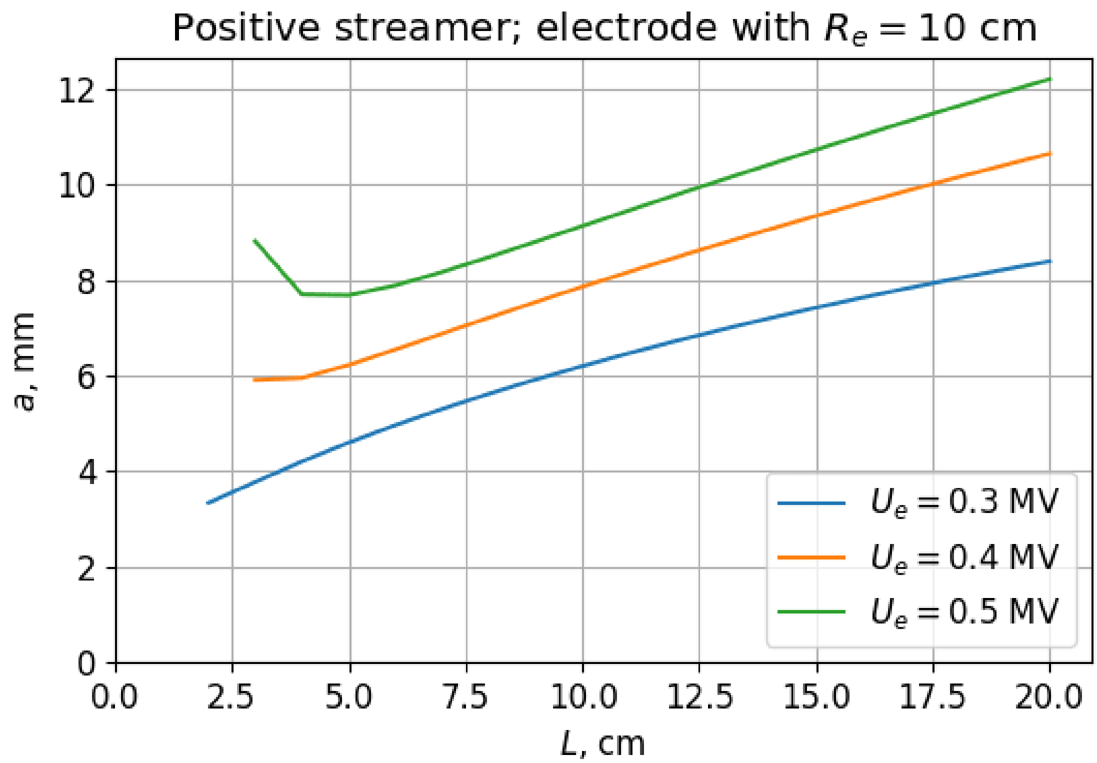
The velocity slows down despite the fact that the streamer is getting longer, for both positive and negative streamers:

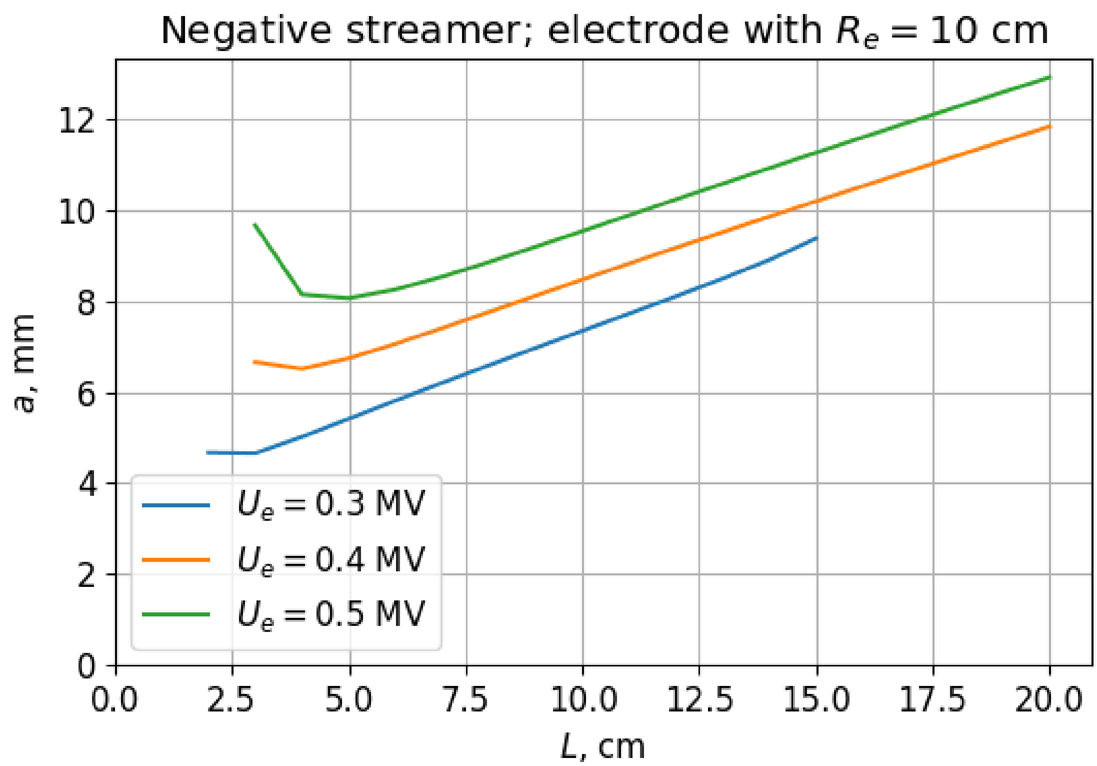




In the negative streamer case, the streamer stops because the field falls below the propagation threshold.

The streamer radius is getting wider, but slower than in the uniform field case:


Positive streamer; electrode with $R_e = 10$ cm
a, mm
L, cm
$U_e = 0.3$ MV
$U_e = 0.4$ MV
$U_e = 0.5$ MV


Negative streamer; electrode with $R_e = 10$ cm
a, mm
L, cm
$U_e = 0.3$ MV
$U_e = 0.4$ MV
$U_e = 0.5$ MV

ABSTRACT

Positive and negative electric streamers are column-shaped discharges which are an important stage in lightning development. The mechanism, by which the streamer acquires a certain velocity and radius of its head, had been a long-standing puzzle. In [1], we proposed a streamer parameter model (SPM), which may explain this mechanism, as well as the mechanism of the streamer threshold electric field. The SPM results for a positive isolated streamer in constant uniform external electric field are verified by comparing with hydrodynamic calculations of a `steady-state' streamer, namely a streamer whose length is kept constant by synchronizing the position of the electrode to which it is attached with the moving streamer head. For this particular streamer configuration, we found that both velocity and radius of a streamer increase with its length, a results which was also observed in previously performed hydrodynamic calculations and experiments.

Beside the velocity and radius of the streamer, SPM allows to quickly estimate all other streamer parameters, such as the maximum field at the streamer tip and the field inside the streamer channel. The relatively low, compared to, e.g., hydrodynamic (HD) or particle-in-cell (PIC) models, computational costs of SPM suggest that generalizion of PSM principles to more complicated streamer structures may be very valuable.

In the present work, we generalize SPM to evaluate how different background conditions affect streamer propagation. In particular, we find that, unlike the uniform-field case described above, (1) for the non-uniform electric field such as that of a spherical electrode, the streamer velocity decreases with length; (2) for a streamer propagating parallel to other similar streamers (as in a streamer bunch"), both velocity and radius stabilize to an approximately constant value.

[1] N. G. Lehtinen, Physics and mathematics of electric streamers," Radiophysics and Quantum Electronics, 2021, doi: 10.1007/s11141-021-10108-5.

REFERENCES

[Lehtinen, 2020] (https://arxiv.org/abs/2003.09450) Lehtinen, N.G. Electric streamers as a nonlinear instability: The model details. arXiv 2020, arXiv:2003.09450.

[Lehtinen, 2021] (https://dx.doi.org/10.1007/s11141-021-10108-5) Lehtinen, N.G. Physics and Mathematics of Electric Streamers. Radiophys Quantum El 64, 11–25 (2021). doi:10.1007/s11141-021-10108-5

[Lehtinen and Marskar, 2021] (https://www.mdpi.com/2073-4433/12/12/1664) Lehtinen, N.G.; Marskar, R. What Determines the Parameters of a Propagating Streamer: A Comparison of Outputs of the Streamer Parameter Model and of Hydrodynamic Simulations. Atmosphere 2021, 12, 1664. https://doi.org/10.3390/atmos12121664