Should input rainfall dictate model timestep? - A 'commensurability' perspective

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November 26, 2022

Abstract

Application of fixed timestep numerical schemes in engineering has long been criticized for their inaccuracy, inefficiency, and inconsistency across time-scales. Yet, to date, most hydrological models fix their timestep to the input rainfall resolution, instead of using adaptive schemes. Aside from their known maladies, we argue that fixed timestep schemes also suffer from 'commensurability' errors: errors that emerge when comparing quantities that are not precisely at the same spatial/temporal scales. At least at <= hourly resolutions, the observed discharge is a set of discrete measurements of an otherwise time-continuous (TC) quantity, but the modelled discharge is time-averaged (TA) across the fixed timestep. Hence the commensurability error when compared against one another during calibration. (In)significance of such errors simultaneously depends on the nonlinearity of the discharge within that timestep, and the timestep size. Consequently, these errors are the largest where they are potentially least acceptable to ignore, i.e., around peaks. Also, they tend to grow with timestep size (data resolution), unless timestep is detached from data resolution using adaptive schemes, which produce a TC solution. Importantly, since modern calibration procedures revolve around 'fitting' to observed discharge, such errors are likely undetectable in model's curve-fitting performance, and instead are to be found in calibrated parameter-sets. Here, in a novel approach within the Generalize Likelihood Uncertainty Estimation (GLUE) framework with Limits of Acceptability (LOA) defined a-priori, and for a micro-catchment case study, we calibrate a TA and a TC version of Dynamic-TOPMODEL to datasets at different resolutions. Through experimentation with the calibrated parameter-sets, we estimate the relative (to TC version) magnitude of the time-commensurability errors resulting from fixing the timestep to input rainfall. Our findings confirm the overall insufficient accuracy, inefficiency of timestepping, and inconsistency across resolution when fixing the timestep. We find that for calibration data resolution >10min, time-commensurability errors become very significant.

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1. Commensurability error

(i)At <hourly resolutions, observed discharge is a set of discrete measurments of a time-continuous quantity, i.e., river level measurements converted to discharge through a rating curve.

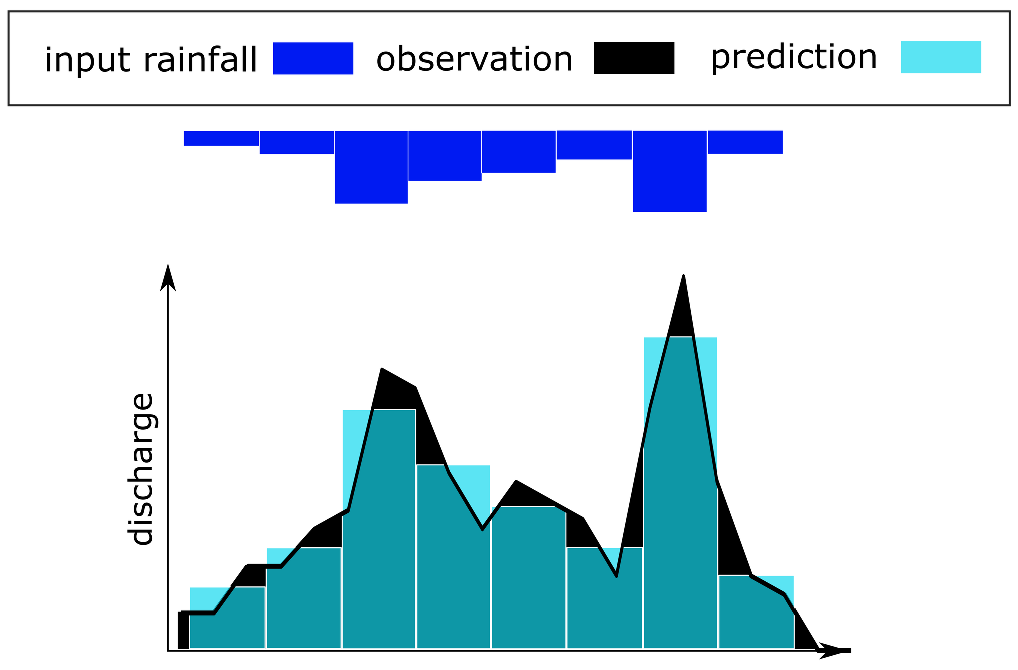
(ii)But, model predictions are time-averaged across its timestep.

(iii)So, comparing the two is associated with some error since they are not at the same timescale.

-This is the 'time-commensurability' error.

2. How can it be quantified?

-Most hydrological models use **fixed timesteps** equal to the input rainfall resolution:



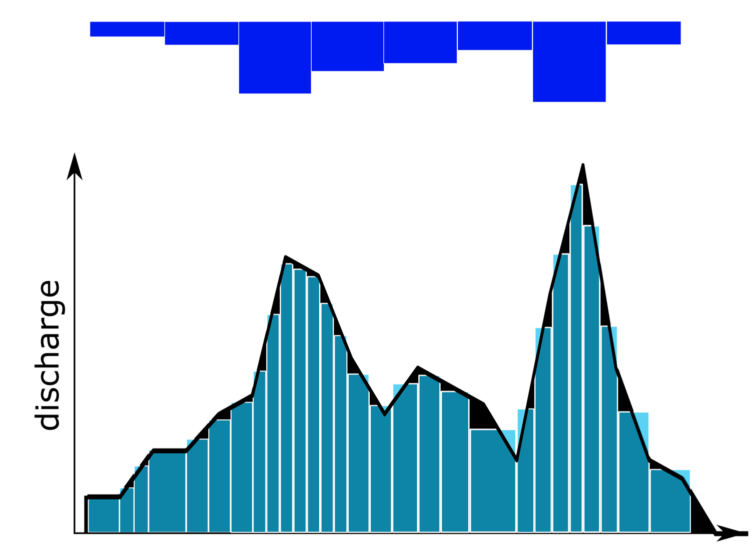
fixed timesteps ~ time-averaged (TA)

-Within each timestep, if the observed discharge is truely constant, then there is no commensurability error, because the interval-averaged and the point-measured values will be the same.

-But generally that is not the case. So when fixing the timestep, more commensurability error results as discharge deviates more from being 'constant' in that timestep.

Problem: predicted discharge ends up with different levels of error in different timesteps. This error propagates in time and infects the solution in future timesteps in an unpredictable way.

Solution: alternative models use **adaptive timesteps** to make sure the time-averaging always occurs on a small enough time-scale that is safe to assume discharge was constant. In other words they produce a time-continuous solution 'commensurate' with the observed discharge:



adaptive timesteps ~ time-continuous (TC)

-By comparing the TA and TC version of the same model we can estimate how much time-commensurability error results when fixing the timestep.

3. Methodology

-We compare TA and TC version of Dynamic-TOPMODEL, at 10min, 20min, 30min, and 60min resolutions.

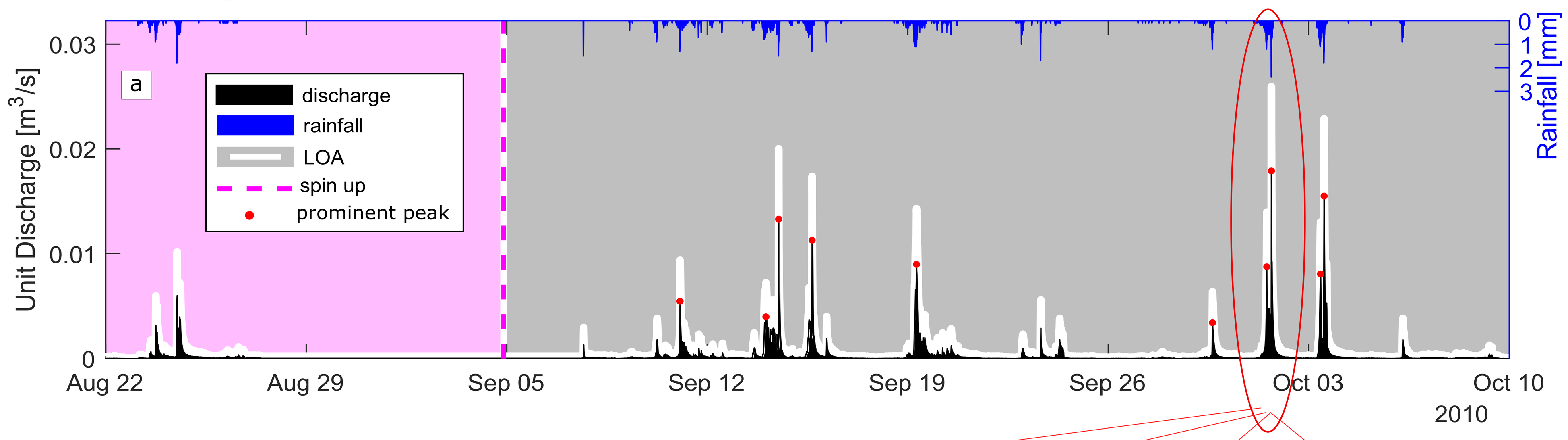
-TA uses fixed timesteps equal to rainfall resolution.

-TC uses MATLAB's adaptive ode solver independent of rainfall resolution.

-We calibrate both models to the same catchment/data using Generlaised Likelihood Uncertainty Estimation (GLUE) methodology, with Limits of Acceptability (LOA).

-LOA take into account observed data uncertainty. Model predictions outside LOA are rejected.

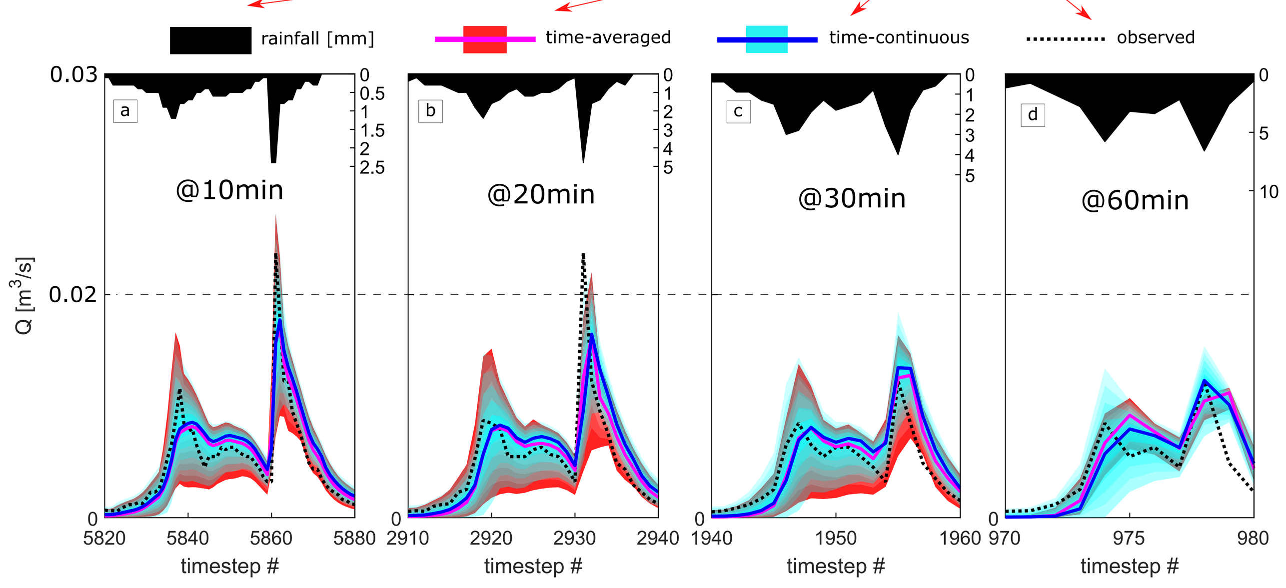
4. Discharge predictions



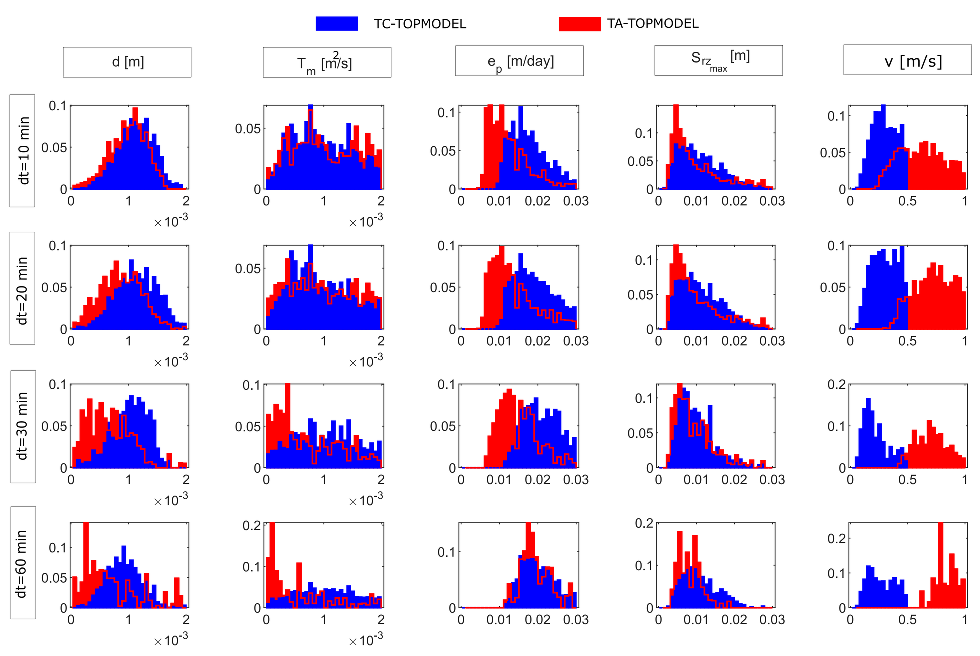
-Solid lines show the mean of all model predictions within LOA.

-Both TA and TC versions produce good fits to the observed discharge.

-Suggesting, either: commensurability errors are small, or, they are captured in model parameters.



5. Parameter Distributions



d [m]: exponential decay of hydraulic conductivity
Tm [m/s]: max hydraulic conductivity
ep[m/s]: potential evapotranspiration rate
Srzmax [m]: max rootzone storage
v [m/s]: overland flow velocity

-When calibrated, the TA and TC versions of the same model produce different parmater distributions.

-This suggests that any commensurability errors of TA model are to be found in its parameters. But, even TC parameters change with input rainfall resolution.

Question: if both TA and TC parameters change, how can we isolate the commensurability errors of TA from general model sensitivity to data resolution?

Answer: through a parameter-switch experiment

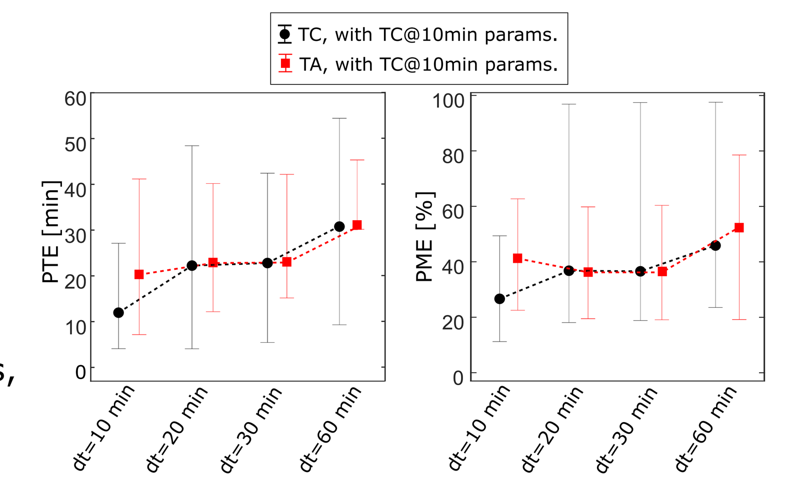
6. Parameter switch experiment

Assumption: parameters obtained by TC model @10min resolution input rainfall are, by definition, the most accurate, therefore:

(i) by running the TC model at different resolutions using TC@10min parameters, we control for TC's sensitivity to data resolution.

(ii) by running TA at different resolutions using TC@10min parmeters, we control for both TA's sensitivity to input rainfall resolution and its commensurability errors (relative to TC).

Conclusion: commensurability errors of fixed timestepping (i.e., TA) are significant when calibration data (rain&discharge) <20min resolution is used.

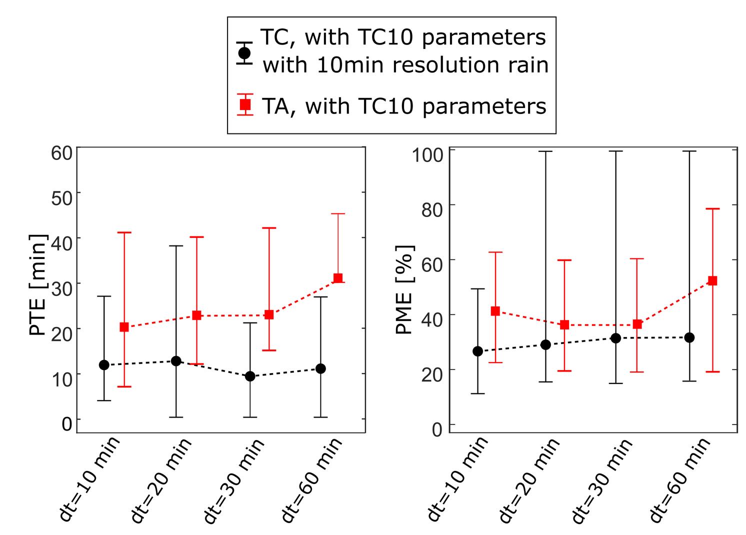


* PTE is the peak timing error for the 10 most prominent peaks in the record
** PME is peak magnitude error for the 10 most prominent peaks
*** Error bas denote min-mean-max of all model predictions

7. Realising the full potential of the TC model

Here we feed the TC model our finest input rainfall (10min) and use it to generate discharge at coarser resolutions. Note the TA does not allow this.

Conclusion: in situations where coarse calibration discharge, but fine rainfall data are available, use of TA model results in very significant commensurability errors even at coarser discharge resolutions, relative to a TC implementation.



8. So, should input rainfall dictate model timestep?

-**Generally, NO** - we did not talk about this here, but TC models are generally much faster to run than TA models, because they use very efficient adaptive timestepping schemes, available in many instances as publically available packaged sofware.

More specifically, NO, if:

(i) both rainfall and calibration discharge data are available at the same resolutions, then a TA model is arguebly justified for resolutions coarser than 20min. For resolutions finer than 20min, commensurability errors of TA amount to ~10 min peak timing error, and ~17% peak magnitude error.

(ii) finer rainfall data is available, commensurability errors of a TA implementation become significant even at coarser calibration discharge than 20min. In this case commensurability errors of a TA implementation amount to 10min-20min peak timing error, and 5%-25% peak magnitude errors.