Stochastic Diffusion of Electrons interacting with Whistler-mode Waves in the Solar Wind

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Abstract

Whistler-mode waves have often been proposed as a plausible mechanism for pitch angle scattering and energization of electron populations in the solar wind. Recent studies reported observations of obliquely propagating and narrowband waves consistent with the whistler mode at 1 AU. Close to 0.3 AU, similar waves have also been observed in PSP data, where evidence of strong scattering of strahl electrons indicates that these waves regulate the electron heat flux. At both radial distances, the wave amplitude can be as high as 10% of the ambient magnetic field. The oblique propagation angle enables resonant interactions without requiring that the electrons counter-stream with the waves. Self-consistent PIC simulations by Roberg-Clark et al (2019) and Micera et al (2020) studied the strahl scattering and subsequent halo formation due to anomalous resonant interactions enabled by oblique whistlers generated from the heat flux fan instability. Observational studies of whistlers near the Sun have also concluded that they are connected to this instability. Cattell and Vo (2021) also demonstrated the same features of the scattering from a particle tracing simulation, one advantage of which is the ability to calculate kinetic quantities such as the diffusion coefficients. Also, the tracing code includes variational calculations to ensure energy conservation in the presence of highly chaotic dynamics. In this study, we investigate in more detail the resonant interactions of electrons with these high amplitude and oblique whistlers. We will show that these waves at 0.3 AU may exceed the stochasticity condition where resonance overlap occurs. Furthermore, the stochastic width around the primary islands might be large enough that diffusion is enabled even before they overlap. In simulations with 1 AU parameters, the particle motion is strongly stochastic where all harmonics significantly overlap, leading to an isotropic pitch angle diffusion which forms the halo population. Our calculations also indicate the presence of higher-order effects, allowing for sub- and super-harmonic resonant interactions.

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Abstract

Effects of increasing whistler amplitude and propagation angle on electron diffusion are studied through a variational test particle simulation. We compare the simulation results with theoretical calculations of the resonance width. While high amplitude and oblique whistlers in typical 1 AU solar wind parameters are capable of forming an isotropic population without any additional processes, anomalous interactions with quasi-parallel whistlers can scatter strahl electrons to lower velocities (less than the wave phase velocity) to form a halo population, as long as their amplitude is sufficiently high. We also present a careful treatment of the sensitivity to initial conditions based on calculations of the phase space volume, which is necessary for numerical calculations when the motion is highly stochastic due to resonant interactions with large amplitude waves. These calculations have a wide application in both PIC and test particle simulations.

I. Introduction

Whistler-mode waves are often proposed as a plausible mechanism for the pitch angle scattering of solar wind electrons [1-4]. Recently, self-consistent PIC simulations [5,6] have been used to study the strahl scattering and halo formation in 0.3 AU solar wind conditions (see Fig. 1). In the early stages (a-d), large amplitude ($\delta B/B_0 \sim 0.1$) oblique whistlers are generated and scatter highly field-aligned electrons. However, the oblique whistler heat flux instability later saturates. Instead, large amplitude quasi-parallel whistlers are generated (e-f) and continue to isotropize the distribution.

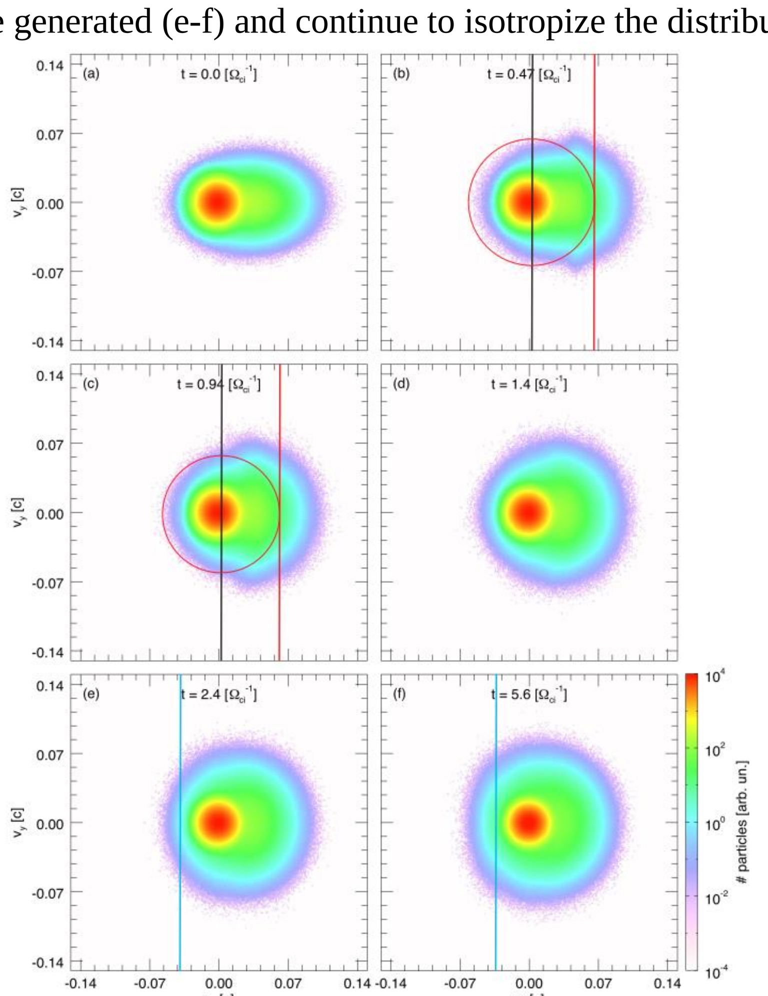

Figure 1. Evolution of the velocity distribution function (VDF) in Micera et al (2020).

Resonant interactions between whistlers and electrons occur at the resonant velocities defined by the resonant condition $\omega - k_\parallel v_\parallel - n\Omega/\gamma = 0$ (vertical lines in Fig. 1). Resonant electrons oscillate along constant energy surfaces (circular lines with centers at $v_z = \omega/k_\parallel$) with amplitude $\Delta v_\parallel$ (see definition in paper). For each harmonic n, $\Delta v_\parallel$ is called the resonance width, which indicates where the resonant interaction with each harmonic n ceases to be effective.

In this study, we perform test particle simulations with wave profiles and particle behaviors consistent with the evolution in Micera's simulations. We focus on describing the diffusion of electrons along the energy surfaces and parameterizing this with varying wave amplitude and propagation angle. This would reveal the role of large amplitude, quasi-parallel/oblique whistlers in the processes of strahl scattering and halo formation.

II. Measuring chaos

PIC simulations usually do not need to consider chaotic motion because it is self-consistently averaged in the results. However, ***when the interest is the kinetic information of the particle, it is necessary to ensure that the conservation laws are properly simulated when the motion is highly stochastic, which occurs when the wave is high amplitude***. In our simulations, we use the *variational* calculations of the Lyapunov characteristic exponents (LCEs) to estimate the expansion of the phase space volume around the particle trajectory in time

$$\frac{\Delta V}{V_0} = \exp(ht) - 1$$

where $h = \sum_i h_i$ is the total Lyapunov exponent and h_i is the growth rate in the i$^{\text{th}}$ dimension (see Fig. 2a). ***The condition for Liouville theorem (and subsequently, zeroth order energy conservation) applies is that ΔV is small (or ideally, zero).***

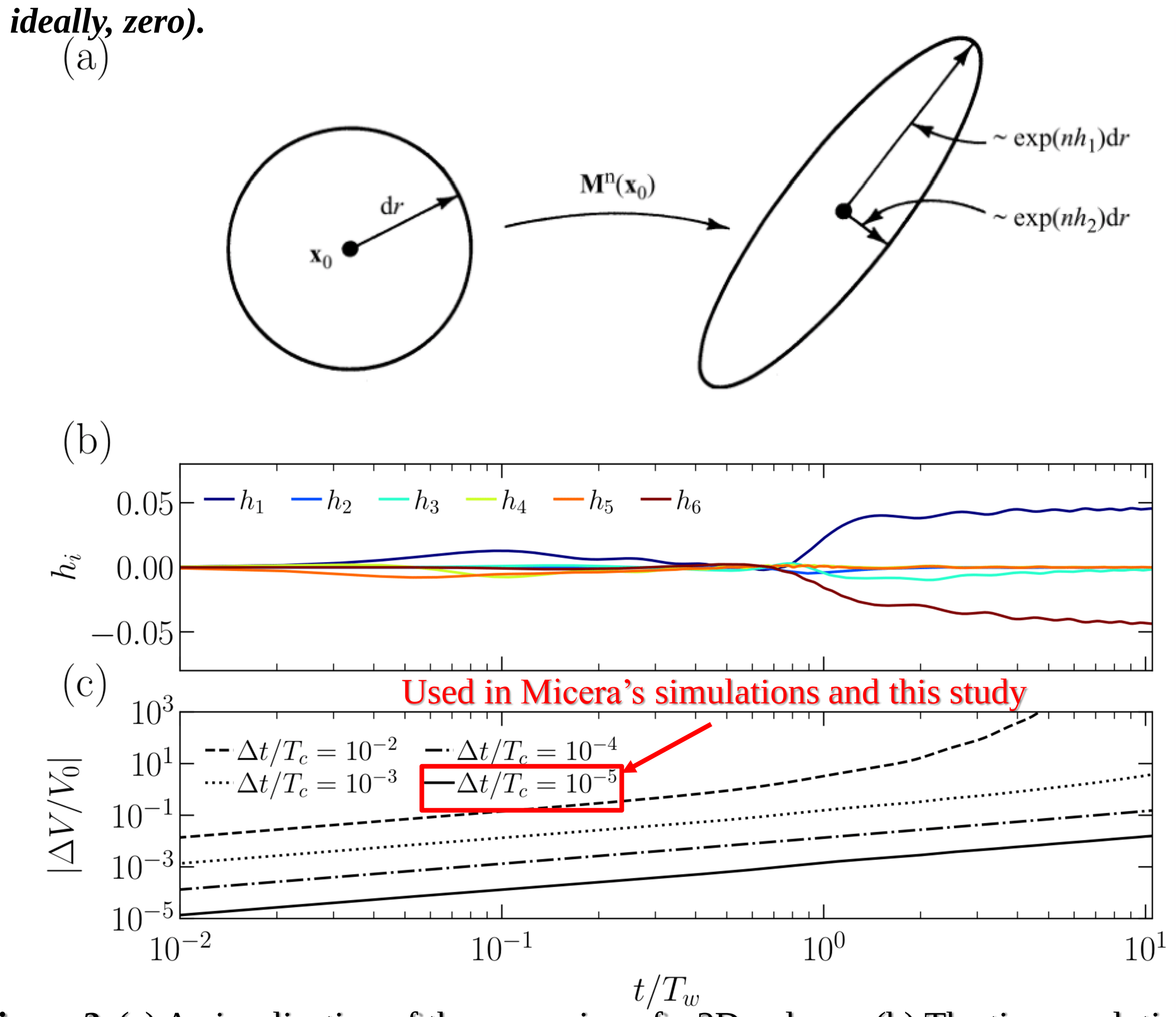

Figure 2. (a) A visualization of the expansion of a 2D volume. (b) The time evolution of a 6D LCE spectrum of a typical electron interacting with a whistler wave. (c) A comparison in the volume expansion among simulations with different step sizes.

From Fig. 2c, ***only simulations with $\Delta t < 10^{-4}$ have well-behaved volume expansion*** around the numerical solutions. For consistency, it is then necessary to pick a step size such that all particles in a range of initial conditions have similar and sufficiently small ΔV. In Fig. 3, we see the effects of increasing amplitude. ΔV becomes larger for particles near the resonances. However, they all have the same order of magnitude because the step size is sufficiently small. Otherwise, ΔV will be drastically different among different initial conditions.

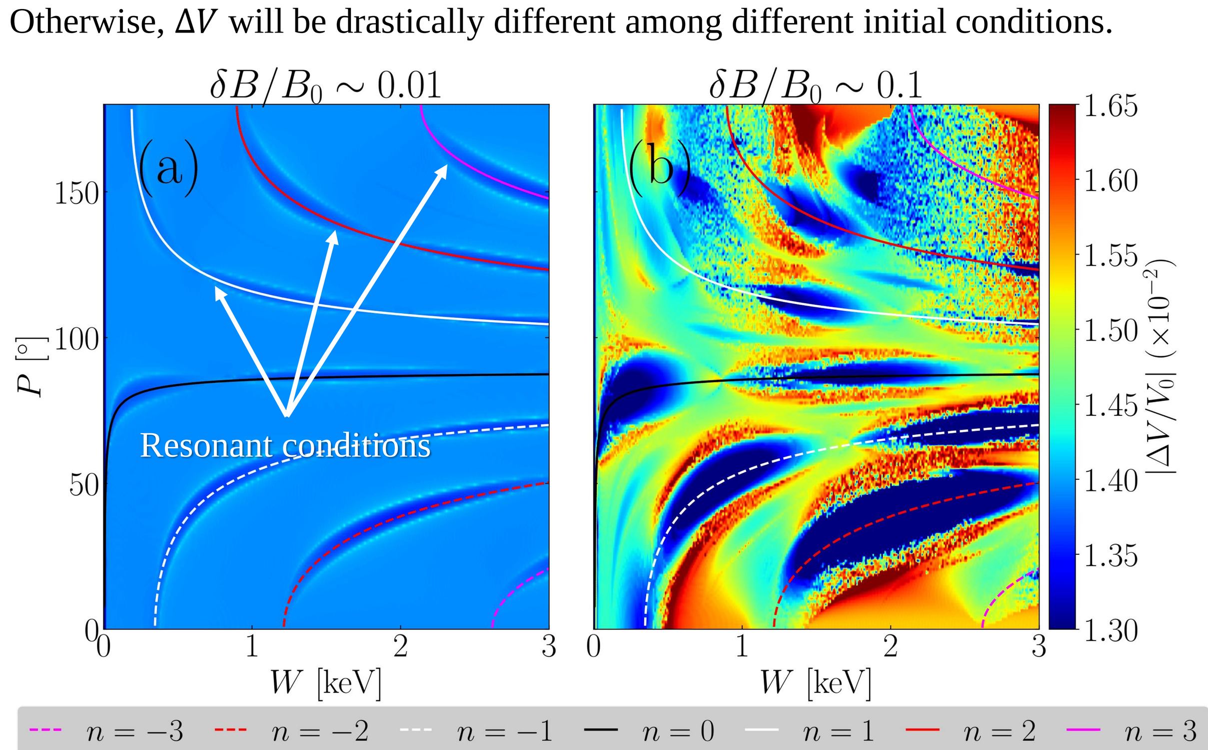

Figure 3. ΔV around electrons with initial kinetic energy W and pitch angle P.

III. Comparison between interactions with oblique and quasi-parallel whistlers

- Resonant electrons oscillate with regular motion inside each resonant "island" bounded by $v_{\parallel,res} \pm \Delta v_\parallel$.
- The islands are destroyed as they gradually overlap when the amplitude increases (see a1-d1), enabling electrons to diffuse stochastically across different harmonics.
- In 1 AU parameters (e), the islands overlap so much that the electrons are scattered everywhere in phase space.
- The islands of quasi-parallel whistlers reside in the middle of those of oblique whistlers. Thus, they can scatter electrons from one island of oblique whistlers to another.
- The pitch angle scattering of quasi-parallel whistlers can be very large because the fundamental island (solid red line) is wide.

1. Oblique whistlers (increasing amplitude)

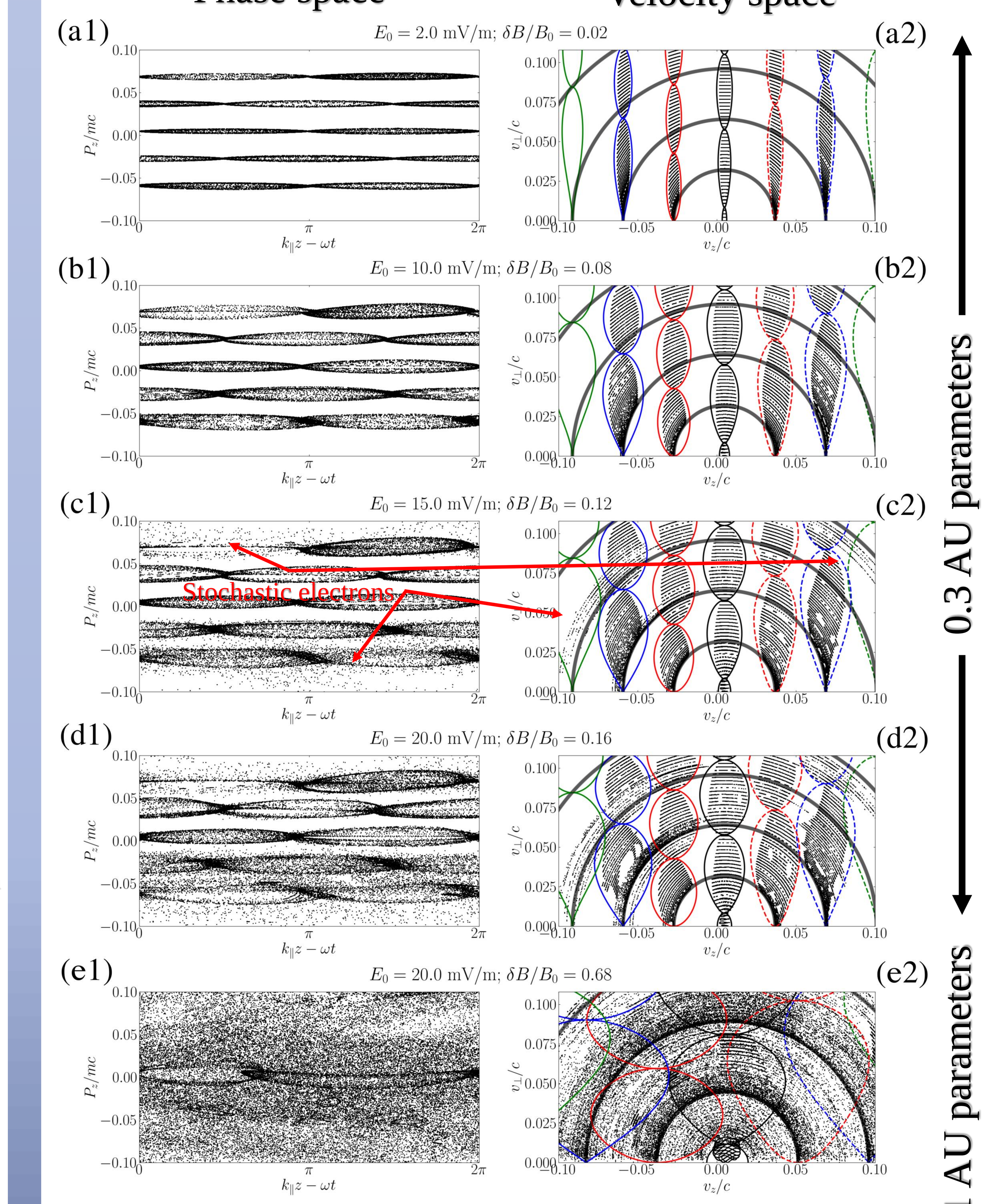

Figure 4. Poincare sections from the trajectories of electrons interacting with an oblique ($\alpha = 65°$) whistler in phase space (left column) and velocity space (right column). The amplitude is increased from (a)-(e).

2. Quasi-parallel whistlers (increasing propagation angle)

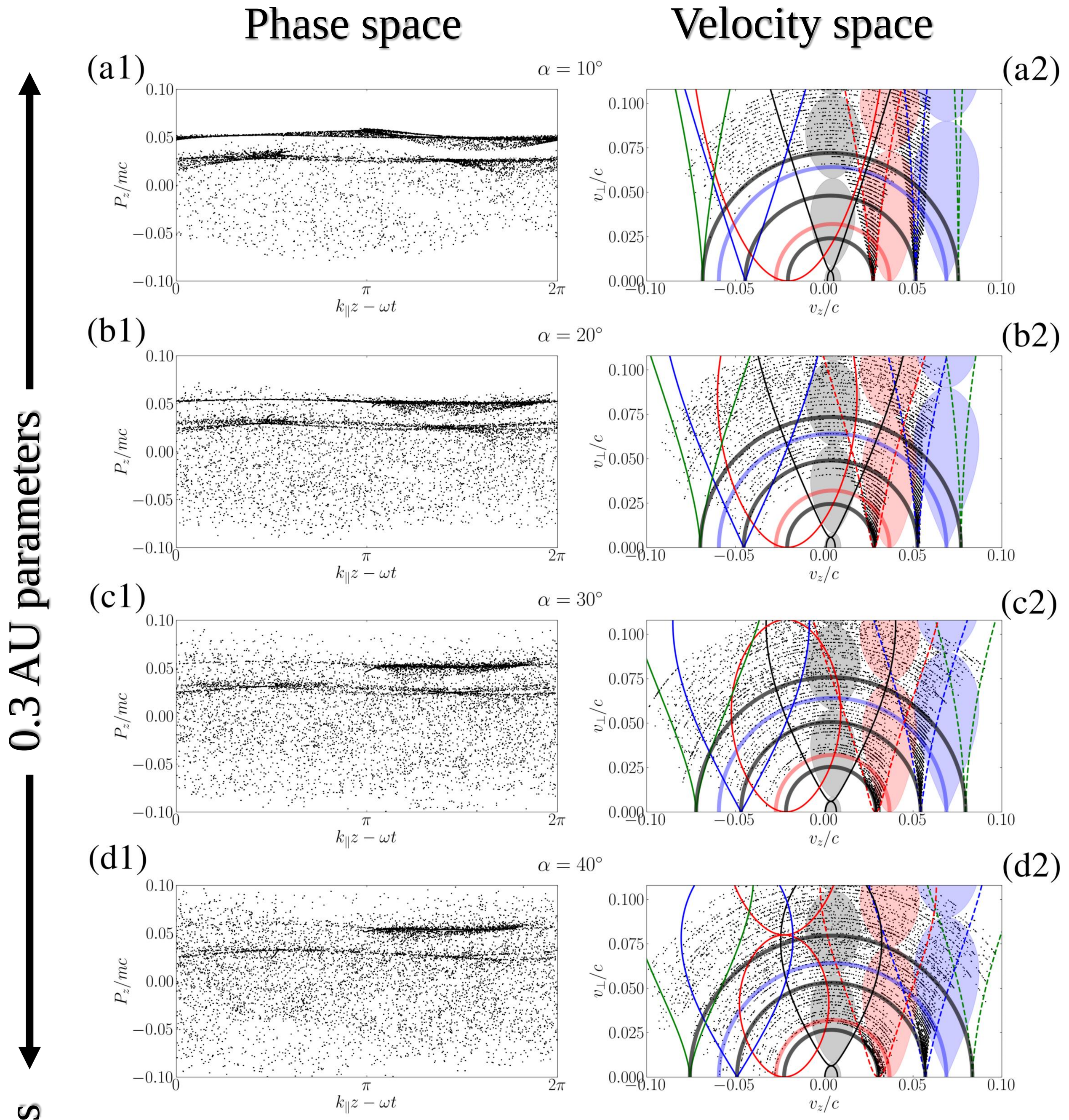

Figure 5. Poincare sections from the trajectories of electrons interacting with high amplitude ($\delta B/B_0 \sim 0.1$) whistler in phase space (left column) and velocity space (right column). The propagation angle α is increased from the quasi-parallel to slightly-oblique range. ***The shaded regions are islands from the large amplitude, oblique whistler in Fig. 4d.***

- 0.3 AU parameters: $n_e = 350cm^{-3}, B_0 = 60\,nT, \omega_{pe}/\omega_{ce} = 100$
- 1 AU parameters: $n_e = 5cm^{-3}, B_0 = 10\,nT, \omega_{pe}/\omega_{ce} = 71$

VI. Conclusions

- The limitation in strahl scattering for whistlers in 0.3 AU solar wind can be explained by the island separations (see Fig. 4a2-d2).
- ***Whistlers in 1 AU solar wind can isotropize the VDF on their own*** (see Fig. 4e2).
- ***Large amplitude, quasi-parallel whistlers can isotropize scattered strahl electrons***, as long as they have ***sufficiently high perpendicular velocities*** after interacting with oblique whistlers (see Fig. 5a-d).
- Considering that the only waves present in Micera's simulation are oblique (early stages) and quasi-parallel (later stages) whistlers, we might conclude that ***large amplitude, quasi-parallel whistlers are essential to the isotropization of the VDF near the Sun***, because they help strahl electrons cross different resonant islands of oblique whistlers.

References & Link to Paper

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