Measuring Wavelength with a Ruler

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Abstract

In this experiment we will find that the wavelength of light from a laser can be measured fairly well with the clever use of a ruler. This is a fine way to introduce more advanced forms of error propagation, as well as some introductory statistical analysis tools, to students in an upper division physics lab who are looking to understand the appropriate ways to interpret and report data. The evaluation methods used come directly from John R. Taylor's "**An Introduction to Error Analysis**"

Introduction

This report discusses how to determine the wavelength of light from a laser using two rulers, one that is highly reflective, and another just a simple meter stick. The first will be used by setting up the laser in such a way that the laser will incident the ruler at a glancing angle, and within the area marked for metric measurements, so that we get an interference pattern by reflection from the strips between the measurement marks on the ruler. The equation

$$\lambda = \frac{d}{n}(\cos\alpha - \cos\beta_n) \tag{1}$$

was then used to determine the wavelength of the light. This equation will be explained in more detail in the procedure section. In this equation d will be the distance between the marks on your ruler, namely one millimeter, and n will be a specified bright spot in the interference pattern. We will see later that the brightest spot will be specified as $(n = 0)$. Then α will be the original incidence angle with the reflective ruler which is also β_0, β_n in general will be the angle from the ruler to every bright spot on the wall.

Procedure

First we will need to set up the reflective ruler in such a way that it is as close to perpendicular with the wall as can be managed. The laser will then be incident with the ruler at a small enough angle that the interference pattern will be present on the wall. It will be necessary to allow a small portion of your laser to pass the end of the ruler without reflecting so that we have a reference point to generate the lengths necessary to produce the angles α and β_n. Remember that the laser will also need to illuminate the ruler within the metric marks so that we have multiple places the laser will reflect from or we will not get a clean interference pattern, as well the original angle of incidence will need to be something on the order of 3 degrees or less or a pattern will not be created. Best results will occur if all equipment is maintained in the same plane relative to the floor. Your set up should look something like this.

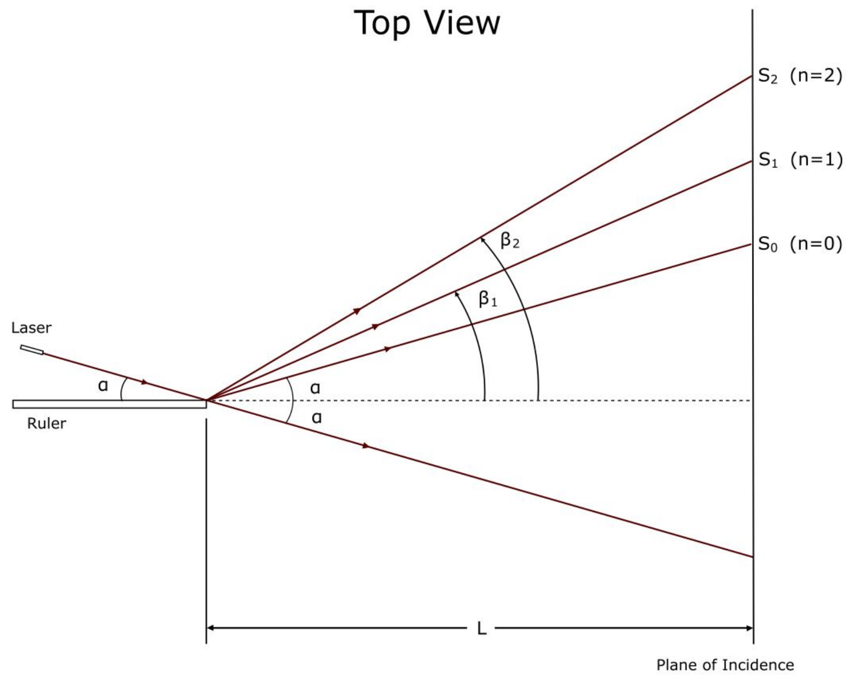

Figure 1: **Set up for orientation of laser**

Most of the light will refract at the same angle α that it was incident on the ruler so we will be able to establish our S_0 bright spot with this knowledge. Angles α and β_n will need to be determined using the length L to the wall and another length l_n, that will be the length from the central line to the each bright spot S_n . The central point can be generated with some careful consideration of the reference beam and the location of S_0 . It may be best to measure each point S_n from the reference beam and then subtract the extra distance later.

The equation that we use to generate the wavelength can be made with some consideration of how the laser illuminates the ruler.

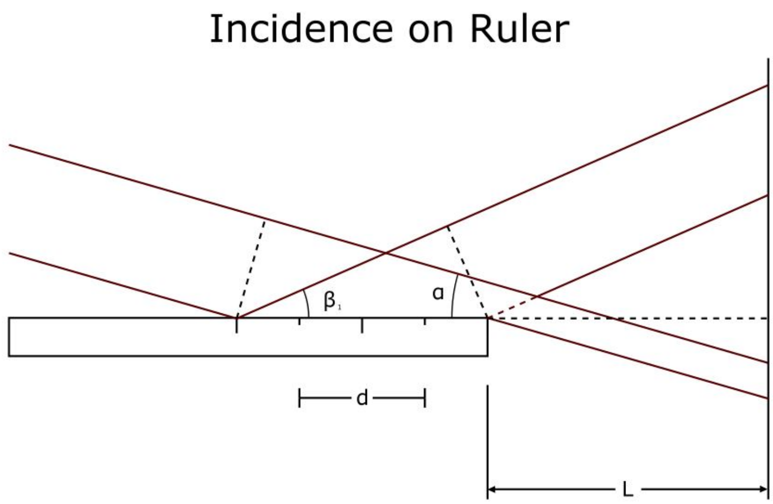

Figure 2: **Path length from the angles of reflection**

If we take **d** to be the central point of every millimeter marking then we can see that the path length difference between two beams incident at nearby millimeter marks and then continuing to the wall will be as shown in equation 1. Some hesitation may come from the knowledge that the end of the ruler is not necessarily cut so that there is only 1 millimeter left after the final measurement mark but in general the laser should incident with enough millimeter markings, given the small angle, that this final discrepancy does not affect our ability to measure an appropriate bright spot.

Data

Data was evaluated by two methods. First by rearranging equation 1 into its linear form and then by treating $\cos(\beta_n)$ as our y variable, n as our x variable, and λ as our slope and parameter to be fit too. A best fit was then made using the scipy.optimize package of python and the standard deviation of our best estimate was taken by the square root of the covariance also generated by the scipy.optimize package. Error in our measurements of $\cos(\beta_n)$ were included so that a visual estimate of the accuracy of the best fit could be made. These errors were created via equation 3.

$$\cos\beta_n = (\frac{-n}{d})\lambda + \cos\alpha \tag{2}$$

$$\sigma_{\cos\beta} = \sqrt{(\frac{\partial\cos\beta}{\partial L}(\delta L))^2 + (\frac{\partial\cos\beta}{\partial l_n}(\delta l_n))^2} \tag{3}$$

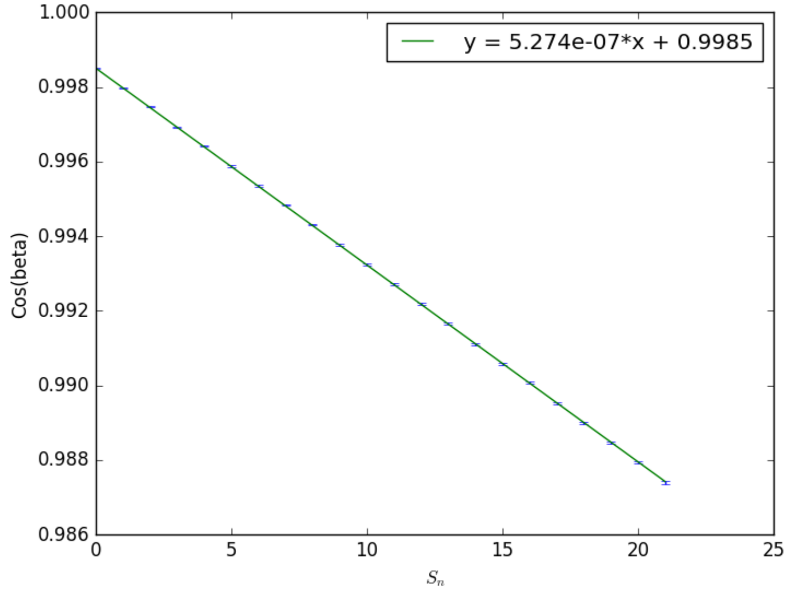

Figure 3: **Equation of best fit for linear equation**

Using this method a best estimate for the actual value of the wavelength of the laser was made with error as 527.4 $\pm$ 0.3 nm. In the second method it was necessary to find the final error in each estimate of the wavelength of the laser then use some statistical methods to evaluate a best guess for the final wavelength of the laser as shown in equations 4 - 7 .

$$\bar{\lambda} = \frac{1}{N}\sum \lambda_n \tag{4}$$

$$\sigma_{\lambda_n} = \sqrt{(\frac{\partial \lambda}{\partial l_0}\delta l_0)^2 + (\frac{\partial \lambda}{\partial l_n}\delta l_n)^2 + (\frac{\partial \lambda}{\partial L}\delta L)^2} \tag{5}$$

$$\sigma_{\lambda} = \frac{1}{N}\sum \sigma_{\lambda_n} \tag{6}$$

$$\sigma_{\bar{\lambda}} = \frac{1}{\sqrt{N}}\sigma_{\lambda} \tag{7}$$

Points	l_n (cm)	L (cm)	Wavelength (nm)
S_0	37.6 (.05)	343 (0.5)	
S_1	40.65 (.05)	343 (0.5)	524 (12)
S_2	43.25 (.05)	343 (0.5)	516 (7)
S_3	45.75 (.05)	343 (0.5)	523 (5)
S_4	47.9 (.05)	343 (0.5)	520 (4)
S_5	50.05 (.05)	343 (0.5)	525 (3)
S_6	52.0 (.05)	343 (0.5)	525 (3)
S_7	53.8 (.05)	343 (0.5)	523 (3)
S_8	55.55 (.05)	343 (0.5)	524 (3)
S_9	57.25 (.05)	343 (0.5)	525 (2)
S_{10}	58.9 (.05)	343 (0.5)	526 (2)
S_{11}	60.45 (.05)	343 (0.5)	527 (2)
S_{12}	61.95 (.05)	343 (0.5)	527 (2)
S_{13}	63.4 (.05)	343 (0.5)	527 (2)
S_{14}	64.85 (.05)	343 (0.5)	528 (2)
S_{15}	66.2 (.05)	343 (0.5)	528 (2)
S_{16}	67.5 (.05)	343 (0.5)	527 (2)
S_{17}	68.58 (.05)	343 (0.5)	528 (2)
S_{18}	70.1 (.05)	343 (0.5)	528 (2)
S_{19}	71.35 (.05)	343 (0.5)	528 (2)
S_{20}	72.55 (.05)	343 (0.5)	528 (2)
S_{21}	75.75 (.05)	343 (0.5)	528 (2)

Table 1: **Wavelength Measurement with Error Propagation for S_n**

In our evaluation through statistical means we find that our best guest for the actual wavelength of the laser with error is 525.7 ± 0.7 nm. In this method it was necessary to omit the S_0 wavelength as a divide by zero error occurs.

Conclusion

Though our data and uncertainty did not enclose the actual wavelength of the laser which was 532 nm our best estimate for the laser was only 1% from the actual value of the laser which leads us to believe that there was some systematic error that caused our evaluation of the actual wavelength of the laser to shift down slightly. This is likely do to the difficulty in keeping the ruler exactly perpendicular to the wall. A better guess could be made by maintaining the laser perpendicular to the wall and offsetting the ruler at some necessary angle. In this way a square could be used to make sure the unmodified light coming from the laser maintains normal incidence with the wall.