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A New Method for Linear Antenna Array Design

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May 12, 2022

Abstract

The inherent relation between the linear antenna array and the finite impulse response filter of digital signal processing is revealed and the least square linear-phase finite impulse response filter design method is introduced for the linear antenna array to obtain a similar main lobe width of uniform linear antenna array but much lower sidelobe level.

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Introduction: The linear antenna array (LAA) is a widely used for airborne target detection. The main efforts are focusing on reducing its sidelobe level while trying to keep the main lobe being narrow. In addition, the phase control concept was introduced to allow the LAA scanning, and the uniform linear antenna array (ULAA) is always the research platform [1]. From the LAA phase point of view, the ULAA is just the finite impulse response (FIR) filter of digital signal processing (DSP) [2] with the rectangle window function (RWF). Beside RWF, there are many high-performance ones available for LAA design. However, the availability of low sidelobe level is at the price of the wider main lobe for the window functions based LLA. A more effective FIR filter design method called least-square linear-phase design can be employed for the LLA design to balance the tradeoff between the low sidelobe level and narrow main lobe.

The relation between LLA and FIR Filer: The LAA can be shown as Fig.1. The distance d between the adjacent elements is $\lambda/2$, where λ is the wavelength; and the incident angle isθ. So, the phase difference between the adjacent elements is kdcosθ, where k is $2\pi/\lambda$.

Ποστεδ ον Αυτηορεα 12 Μαψ 2022. Τηε ςοπψριγητ ηολδερ ις τηε αυτηορ/φυνδερ. Αλλ ριγητς ρεσερεδ. Νο ρευσε ωιτηουτ περμισσιον. ηττπς://δοι.οργ/10.22541/αυ.165237498.82069415/1. Τηις α πρεπριντ ανδ ηας νοτ βεεν πεερ ρειεωεδ. Δατα μαψ βε πρελιμιναρψ.

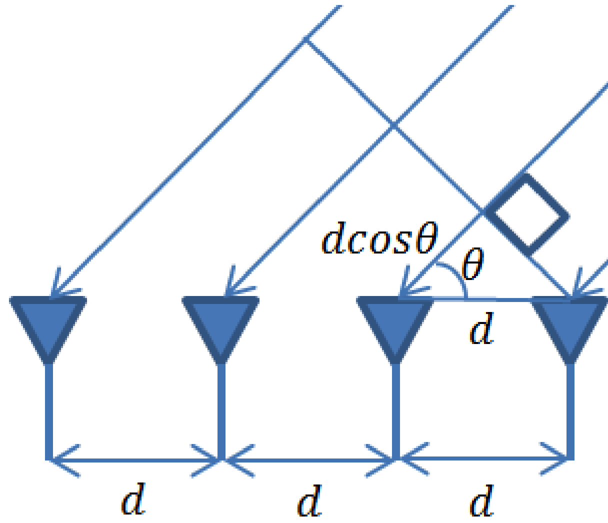

Fig. 1. Uniform Linear Antenna Array diagram

Then, the LLA has the antenna factor (AF) :

$AF(\theta) = \sum_{n=0}^{N-1} I_n e^{-jnkdcos\theta}$ (1)

where I_n is the exciting current of the nth element, N is the total number of the elements of the LLA.

Let $\varphi = kdcos\theta = \pi cos\theta$, then formular (1) becomes:

$AF(\theta) = \sum_{n=0}^{N-1} I_n e^{-jn\varphi}$(2)

On the other hand, the frequency response definition of the FIR filter is:

$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$ (3)

When z is $e^{\vartheta\omega}$, φορμυλαρ (2) μεετς τηις δεφινιτιον. Σο, τηε ΛΛΑ ςαν βε ρεγαρδεδ ας τηε ΦΙΡ φιλτερ οφ ΔΣΠ, ανδ τηε ΥΛΑΑ ις ϑυστ τηε ΦΙΡ φιλτερ ωιτη ΡΩΦ.

ΛΛΑ δεσιγν ωιτη λεαστ-σχυαρε λινεαρ-πηασε μετηοδ: Λετ τηε ΦΙΡ φιλτερ λενγτη βε Λ+1 σαμπλες, ωιτη Λ εεν. Ινιτιαλλψ ιτ ωιλλ βε ςεντερεδ αβουτ τηε τιμε οριγιν, ι.ε., ζερο πηασε. Τηεν τηε φρεχυενςψ ρεσπονσε ςαν βε ςαλςυλατεδ ατ τηε σπεςιφιεδ φρεχυενςψ ω_k βψ

$H(\omega_k) = \sum_{n=-L/2}^{L/2} h(n) e^{-j\omega_k n}, \quad k = 0$ το $N-1$.ωηερε N>>Λ (4)

Ρεγαρδινγ εεν σψμμετρψ ιν τηε ιμπυλσε ρεσπονσε, ι.ε., η(ν)=η(-ν), γιες α ζερο-πηασε ΦΙΡ φιλτερ τηατ ςαν βε λατερ ριγητ-σηιφτεδ Λ/2 σαμπλες το μακε α ςαυσαλ, λινεαρ πηασε φιλτερ. τηεν, τηε φρεχυενςψ ρεσπονσε ρεδυςες το α συμ οφ ςοσινες:

$H(\omega_k) = h(0) + 2\sum_{n=1}^{L/2} h(n)\cos(\omega_k n)$,ωηερε

ϰ=0,1, 2, . . . , N-1, N>>Λ. (5)

Λετ τηε δεσιρεδ ΦΙΡ φιλτερ φρεχυενςψ ρεσπονσε βε $D(\omega_k)$, τηεν τηε δεσιγν ταργετ ις το μινιμιζε τηε φολλοωινγ σχυαρε ερρορ:

$\min ||H(\omega_k) - D(\omega_k)||^2$ (6)

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Ωηεν τηε λεαστ σχυαρε ερρορ ις ρεαςηεδ, τηε ρελατεδ η(ν) ςαν βε οβταινεδ, ανδ η(ν) ις ρεαλ νυμβερ σινςε τηε ΦΙΡ φιλτερ ις ενφορςεδ το βε εεν σψμμετρψ. Ιν ουρ ςασε, τηις η(ν) ςαν βε ρεγαρδεδ ας τηε εξςιτινγ ςυρρεντ οφ εαςη ελεμεντ οφ τηε ΛΑΑ.

Σο, τηε ΛΑΑ δεσιγν προςεδυρε οφ υσινγ μετηοδ ςαν βε:

1. Σπεςιφψ τηε τοταλ ελεμεντ νυμβερ οφ ΛΛΑ.
2. Σπεςιφψ τηε νορμαλιζεδ ςυτ-οφφ πηασε οφ τηε μαιν λοβε.
3. Σπεςιφψ τηε νορμαλιζεδ σταρτ πηασε οφ τηε σιδελοβε.
4. Υσε τηε φιρλς φυνςτιον ιν σιγναλ προςεσσινγ τοολβοξ οφ ΜΑΤΛΑΒ [3],

Ηερε ις α δεσιγν εξαμπλε ωιτη τηε ΜΑΤΛΑΒ σουρςε ςοδες φορ αν 8-ελεμεντ ΛΑΑ. Φορ τηε νορμαλιζεδ πηασε, ι.ε., φ is between 0 and 1, set the cut-off phase of the main lobe to 0.1, the start phase of the sidelobe from 0.2 to 1. By the exciting current of each element for this LAA is as the follows:

$$I_0 = I_7 = 0.0804,$$

$$I_1 = I_6 = 0.1147,$$

$$I_2 = I_5 = 0.1422$$

$$I_3 = I_4 = 0.1574$$

The pattern diagram of this LAA vs that of ULAA is as the follows:

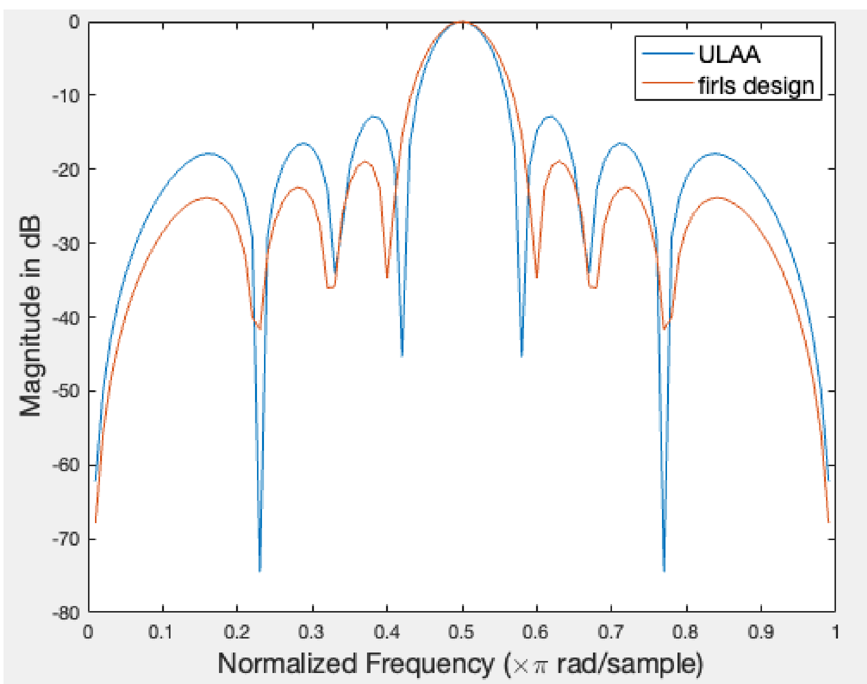

Fig. 2. Pattern diagram, ULAA vs Least square Linear Phase LAA

Obviously, the 3dB main lobe of this LAA is very close to that of ULAA, while the 1st side lobe level of this LAA is about 7dB lower than that of ULAA.

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The MATLAB source codes is listed as the follows:

```
N=8;
% 8-element LAA, the distance between two adjacture elements is 1/2 wavelen
A=zeros(1,8); %Initial the exciting current of the ULAA
B=zeros(1,8); % Initial the exciting current of the LAA
rectangleoutput=zeros(1,99); % Initial the frequency response of ULAA
leastsquareoutput=zeros(1,99); % Initial the frequency response of LAA
theta=zeros(1,99); % initial the incident angles
sum1=0;% DC component of the ULAA
sum2=0;% DC component of the LAA

f = [0 0.1 0.2 1];         % desired and normalized Frequency band edges
a = [1 1 0 0];           % desired and normalized Amplitudes
B = firls(7,f,a); % Least Square Linear phase function in the toolbox

for j=1:N
    A(j)=1; % assign 1 for each element of ULAA
    sum1=sum1+A(j); % calculate DC component of the ULAA
    sum2=sum2+B(j); % calculate DC component of the LAA

end

theta_step=pi/100; % The step for the incident angle

for K=1:99
    theta(K)=theta_step*K; % obtain all of the incident angles
end

for K=1:99

    temp_variable1=0;
    temp_variable2=0;
    for j=1:N
       temp_variable1=temp_variable1+A(j)*exp(-1i*pi*cos(theta(K))*(j-1));
       temp_variable2=temp_variable2+B(j)*exp(-1i*pi*cos(theta(K))*(j-1));
   end
       rectangleoutput(K)=20*log10(abs(temp_variable1)/sum1);
       leastsquareoutput(K)=20*log10(abs(temp_variable2)/sum2);
end
plot(theta/pi,rectangleoutput,theta/pi,leastsquareoutput);
legend('ULAA','firls design','FontSize',14);
xlabel('Normalized Incident Angle (\times\pi rad/sample)','FontSize',16);
ylabel('Magnitude in dB','FontSize',16);
```

Conclusion: Generally, the LAA can be regarded as the FIR filter of DSP with respect of its phase. And ULAA is just the FIR filter with RWF. To have the better performance, square linear-phase design method for FIR filter can be directly employed for the LAA design.

References

1. COLLIN, R. E.:' Antennas and Radiowave Propagation'. (McGraw-Hill, 1985)

2. OPPENHEIM, A. v., and SCHAFER. R.W.: 'Digital signal processing' (Prentice-Hall, New Jersey, 1975)

3. MATLAB R2018 User Manual

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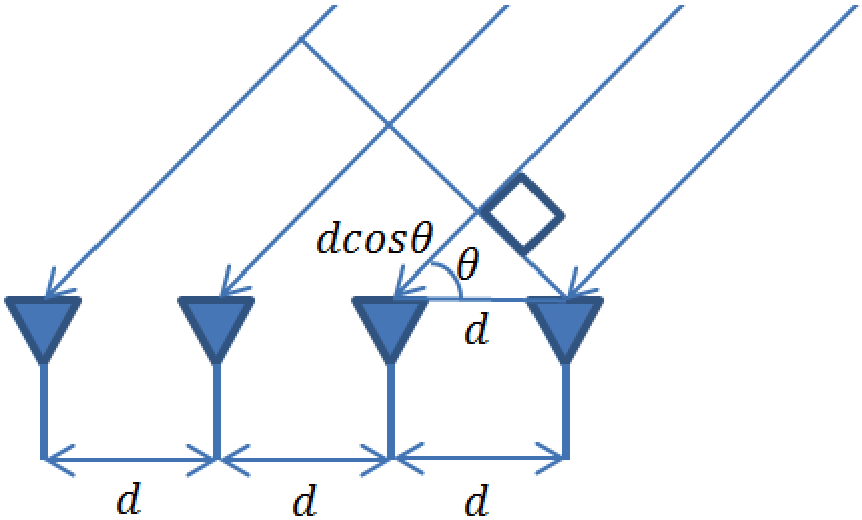

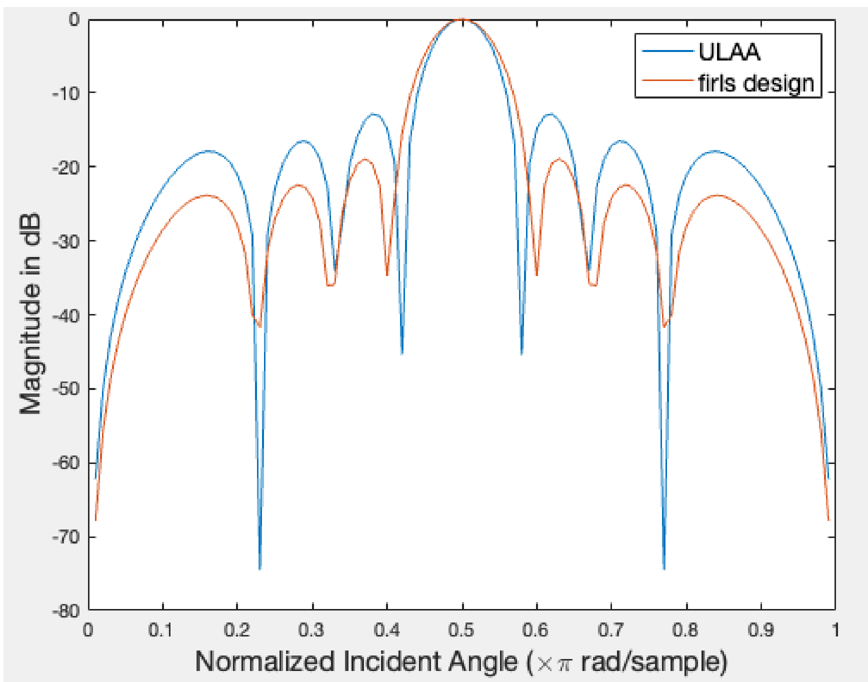

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    theta(K)=theta_step*K; % obtain all of the incident angles
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end
plot(theta/pi,rectangleoutput,theta/pi,leastsquareoutput);
legend('ULAA','firls design','FontSize',14);
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ylabel('Magnitude in dB','FontSize',16);
```