Model following adaptive control for nodes in complex dynamical network via the state observer of links

Xiaoxiao Li[1], Yinhe Wang[1], and Shengping Li[2]

[1]Guangdong University of Technology
[2]Shantou University

September 26, 2022

Abstract

Geometrically, a complex dynamical network (CDN) can be regarded as the interconnected system composed of the node subsystem (NS) and the link subsystem (LS) coupled with each other. Guided by this idea, in order to achieve the goal of each node following asymptotically its own reference target in a CDN, this paper investigates the model following adaptive control (MFAC) problem of NS via the dynamics of links, which implies that the LS plays the important dynamic auxiliary role in the MFAC realization of nodes. Meanwhile, we focus on the condition that the links state information is unavailable, due to sensor practical application and measurement cost constraints. To obviate this restriction, we construct the asymptotical state observer for the LS. Next, to achieve the control goal of this paper, an appropriate Lyapunov candidate function is selected, by which the MFAC scheme for NS is synthesized based on the state observer of LS. Finally, the simulation example is performed to demonstrate the theoretical results in this paper.

Received: Added at production | Revised: Added at production | Accepted: Added at production

DOI: xxx/xxxx

RESEARCH ARTICLE

Model following adaptive control for nodes in complex dynamical network via the state observer of links

Xiaoxiao Li*[1] | Yinhe Wang[1] | Shengping Li[2]

[1]School of Automation, Guangdong University of Technology, Guangzhou, 510006, China
[2]MOE Key Laboratory of Intelligent Manufacturing, Shantou University, Shantou, 515063, China

Correspondence
*Xiaoxiao Li, School of Automation, Guangdong University of Technology, Guangzhou,510006, China. Email: xiaoxiaoli_gdut@sina.com

Present Address
School of Automation, Guangdong University of Technology, Guangzhou 510006, China

Summary

Geometrically, a complex dynamical network (CDN) can be regarded as the interconnected system composed of the node subsystem (NS) and the link subsystem (LS) coupled with each other. Guided by this idea, in order to achieve the goal of each node following asymptotically its own reference target in a CDN, this paper investigates the model following adaptive control (MFAC) problem of NS via the dynamics of links, which implies that the LS plays the important dynamic auxiliary role in the MFAC realization of nodes. Meanwhile, we focus on the condition that the links state information is unavailable, due to sensor practical application and measurement cost constraints. To obviate this restriction, we construct the asymptotical state observer for the LS. Next, to achieve the control goal of this paper, an appropriate Lyapunov candidate function is selected, by which the MFAC scheme for NS is synthesized based on the state observer of LS. Finally, the simulation example is performed to demonstrate the theoretical results in this paper.

KEYWORDS:
Complex dynamical network (CDN), model following adaptive control (MFAC), the state observer of links.

1 | INTRODUCTION

A complex dynamical network (CDN) can be viewed geometrically as a graph-theoretic model consisting of dynamically interacting nodes and links between nodes, which can be used to describe many practical scenes in the real-world, for example, social networks[1], biological neural networks[2], transportation networks[3], cellular and metabolic networks[4], etc. Therefore, in order to better understand the structure and function of real networks, considerable attention has been devoted to the study of CDN from various science and engineering fields including physics, mathematics, bioinformatics, management science, and so on[5,6].

From the perspective of large-scale systems, and exploit the relevant knowledge of graph theory, a CDN can be seen as an interconnected system consisting of node subsystem (NS) and link subsystem (LS) coupled with each other. Since NS and LS assist and influence each other, the complete dynamic characteristics (DCs) of the CDN should be reflected in two aspects, one is the NS, and the other is the LS. Most of the current studies on the DCs of the CND were mainly on the DCs of nodes, for instance, consensus[7], synchronization[8], tracking[9], etc. In fact, all the links are regarded wholly as the dynamic system (LS), and thus it also has some DCs, and some theoretical results have emerged to explain this in recent years, such as, structural balance[10,11], and so on.

[0]**Abbreviations:** ANA, anti-nuclear antibodies; APC, antigen-presenting cells; IRF, interferon regulatory factor

According to the above view for the existing literature, the synchronization and tracking control of CDN can be regarded as the collective dynamic behavior of nodes with the assistance of links. The synchronization of CDN means that all nodes tend to be in the same state, while tracking usually means that all nodes tracking the given single time signal, and many mature theoretical research results have been achieved in this field [12,13]. However, it is worth noting that under the background of some practical engineering applications, each node requires its own following target with the dynamic assistance of links, and the following target has its own internal dynamics. It can be seen that the synchronization and tracking control do not meet this requirement. For instance, the formation control of multiple non-holonomic wheeled mobile robots [14,15,16], in which each robot is considered as a controlled node, the reference model is the path it follows, and the information exchange between robots is considered as the links. The grid-connected generator [17], in the power grid, after the grid-connected generator sets, according to the requirements (reference target), each generator set (controlled node) needs to adjust and optimize its output speed and voltage in real time. The generator sets are connected by high-voltage transmission lines (links). Multiple robots present a certain physical dynamic posture according to their respective reference robots (reference target), and there exist the communication protocol (links) between robots [18], etc. From the viewpoint of control theory on the CDN, the above problems can be regarded as the model following adaptive control (MFAC) problem of nodes assisted by the links dynamics.

MFAC as one type important method in control theory, which can adaptively adjust the controller gain to ensure the stability of the system, and has the advantages of flexibility, adaptability and robustness. Therefore, it has attracted extensive attention on many scholars and has been applied in numerous fields [19,20,21,22,23]. Such as, Landau et al. [19] applied MFAC to optimal control theory to solve the difficulty of constructing system performance indicators. Zhang et al. [21] applied MFAC to the robot mechanism and designed a suitable control scheme, so that the robot mechanism could achieve the expected movement. Shyu et al. [22] for the single-phase shunt active power filter, propose the MFAC scheme to improve line power factor and to reduce line current harmonics. However, it is worth noting that the above-mentioned studies on MFAC are conducted on the single system and are limited by the model matching conditions. On the other hand, although literature [17] studied the MFAC problem of NS in CDN, it did not consider the important dynamic auxiliary role played by links, which should not be ignored. Because the dynamic change of links can promote the controlled nodes to emerge some DCs, for example, the synchronization and tracking [24,25]. Accordingly, inspired by the above discussions, this paper proposes the MFAC problem of NS in CDN via the links dynamics, which can be used to fill the insufficiencies above mentioned.

In addition, the issues should be considered that the state information of links in CDN is usually unavailable, due to the sensor limitations of technology and measurement cost in engineering applications. For example, the synapse strength between neurons in biological neural network [26,27]; the competition intensity changes between species in biological communities [28,29]; the information exchange between swarming or flocking robots [30,31] etc. As far as we know, in order to solve the problem that the state information of the links is unavailable in the CDN, two processing methods have emerged in the existing literature. The first method is to construct the dynamic auxiliary tracking targets of links firstly, then use the state information of auxiliary targets for links to design an appropriate control scheme for NS, then promote the nodes to emerge the required DCs through mutual coupling [25,32]. Another method is to design an asymptotical state observer for links [33], and use the state information of observer to achieve required control target. Guided by the above involved methods, when studying the MFAC of NS in CDN via the links dynamics, this paper adopts the second method to solve the problem that the state information of links is unavailable. That is, an asymptotical state observer of links is firstly designed, then use the state information of observer to assist nodes to achieve the MFAC goal.

Sum up the above discussions, this paper investigates the MFAC problem of NS in CDN based on the state observer of links. In this paper, the CDN is considered to be an interconnected system composed of NS and LS coupled with each other, where exploit the vector differential equation and Riccati matrix differential equation to establish the dynamics model of the NS and the LS, respectively. Based on the constructed mathematical model, in the case that the nodes state information is available but the links state information is unavailable, an asymptotical state observer of links is constructed firstly, then according to the Lyapunov stability theory, give some mathematical assumptions, and use the state information of observer to synthesize the MFAC scheme for NS, so as to accomplish the asymptotically following between the NS and its reference target. Compared with the most existing works on study for CDN, this paper has the following contributions. (1). This paper mainly studies the MFAC problem of nodes in CDN, and discusses the auxiliary role of links dynamics in the realization of this goal. (2). Compared with the existing literature on synchronization and tracking control for CDN, the main distinctive point of MFAC in this paper is that each node in CDN has its own reference following target with the assistance of links dynamics, and the reference target also has its own internal dynamics. (3). In this paper, an asymptotical state observer for links is designed to overcome the technical difficulty that its state information is usually unmeasurable precisely. (4). Compared with MFAC on single system, this paper does not require strict model matching conditions (e. g. Erzeberger conditions [19]), only simple mathematical assumptions need to be satisfied, due to the useful coupling information between NS and LS.

The structure of the rest of this paper is as follows. In Section 2, we propose the mathematical model for CDN, which is considered to be formed by the mutual coupling of the NS and the LS, and give relevant mathematical assumptions. In Section 3, construct the asymptotical state observer of LS. In Section 4, design the MFAC scheme for NS in CDN with the state information of links observer to achieve its MFAC target. The illustrate example is given in Section 5 to validate the correctness and effectiveness of the proposed control scheme in this paper. Give the conclusion in Section 6.

2 | MODEL DESCRIPTION OF CDN AND GIVE THE RELEVANT MATHEMATICAL ASSUMPTIONS

Consider the time-varying controlled CDN with N nodes, and the dynamics of each controlled isolated node is depicted by

$$\dot{z}_i = Az_i + f_i(z_i, t) + u_i \tag{1}$$

where $z_i = [z_{i1}, z_{i2}, \dots, z_{in}]^T \in R^n$ is the state variable of the ith node, $f_i(z_i, t) = [f_{i1}(z_i, t), f_{i2}(z_i, t), \dots, f_{in}(z_i, t)]^T \in R^n$ is the continuous vector function,$A \in R^{n\times n}$ is the constant matrix, and the control input of the ith node $u_i = [u_{i1}, u_{i2}, \dots, u_{in}]^T \in R^n$, $i = 1, 2, \dots, N$.

In consideration of the mutual coupling among nodes, the model of the ith controlled nodes can be depicted by

$$\dot{z}_i = Az_i + f_i(z_i, t) + \alpha \sum_{j=1}^{N} l_{ji}(t)H_j(z_j, t) + u_i \tag{2}$$

where $l_{ji}(t)$ denotes the time-varying link weight from the jth node pointing to the ith node, and the self-connection $l_{ii}(t)$ is allowed in this paper. $\alpha \in R$ represents the common coupling strength, the continuous inner coupling vector function $H_j(z_j, t) = [h_{j1}(z_j, t), h_{j2}(z_j, t), \dots, h_{jn}(z_j, t)]^T \in R^n$, $i, j = 1, 2, \dots, N$.

Assumption 1. Consider Equation (2), $H_j(z_j, t)$ is known and bounded, the constant matrix A is unknown. The continuous vector function $f_i(z_i, t)$ is unknown but bounded, that is to say, there exist N known nonnegative continuous functions $\delta_i(z_i, t)$ such that $\|f_i(z_i, t)\| \le \delta_i(z_i, t)$, where $\|\bullet\|$ represents the Euclidean norm of vector or matrix '$\bullet$', $i, j = 1, 2, \dots, N$.

In order to make the mathematical derivation concise, introduce the following vector and matrices symbols, $z = [{z_1}^T, {z_2}^T, \dots, {z_N}^T]^T \in R^{nN}$, $Z = [z_1, z_2, \dots, z_N] \in R^{n\times N}$, $F = F(z, t) = [f_1(z_1, t), f_2(z_2, t), \dots, f_N(z_N, t)] \in R^{n\times N}$, $H = H(z, t) = [H_1(z_1, t), H_2(z_2, t), \dots, H_N(z_N, t)] \in R^{n\times N}$, $L = L(t) = [l_{ji}(t)]_{N\times N} \in R^{N\times N}$, $U = [u_1, u_2, \dots, u_N] \in R^{n\times N}$. Therefore, according to the abovementioned symbols, Equation (2) can be rewritten as follows.

$$\dot{Z} = AZ + F(z, t) + \alpha HL + U \tag{3}$$

Remark 1. (a). Different from the processing method of NS in the Refs. [24,25,32], this paper introduces the matrix form Z in Equation (3) to describe the state of NS for MFAC research. This helps simplify mathematical analysis and synthesizing controller. (b). The constant matrix A in this research is unknown, which is different from the researches on synchronization and tracking control of CDN in the Refs. [34,35,25]. (c). If Assumption 1 holds, then the following inequality is true.

$$\|F(z, t)\| = \sqrt{\sum_{i=1}^{N} \|f_i(z_i, t)\|^2} \le \delta(z, t) \tag{4}$$

where $\delta(z, t) = \sqrt{\sum_{i=1}^{N} \delta_i(z_i, t)^2}$.

The following vector differential equation is given as the model following target for the ith node (2).

$$\dot{z}_{ri} = A_r z_{ri} + B_r u_{ri} \tag{5}$$

where $z_{ri} = [z_{ri1}, z_{ri2}, \dots, z_{rin}]^T \in R^n$ is the reference state of the ith node, the reference input of the ith node $u_{ri} = [u_{ri1}, u_{ri2}, \dots, u_{rim}]^T \in R^m$, $A_r \in R^{n\times n}$ and $B_r \in R^{n\times m}$ are the known constant matrices.

Likewise, for the convenience of mathematical derivation, we rewrite Equation (5) as the following matrix differential equation.

$$\dot{Z}_r = A_r Z_r + B_r U_r \tag{6}$$

where $Z_r = [z_{r1}, z_{r2}, \dots, z_{rN}] \in R^{n\times N}$, $U_r = [u_{r1}, u_{r2}, \dots, u_{rN}] \in R^{m\times N}$.

Assumption 2. Consider Equation (5), the known constant matrix $A_r$is the Hurwitz matrix.

Remark 2. (a). It is usually assumed that the system matrix of the reference model is the Hurwitz matrix, which is a basic assumption[19,22,23], when studying the MFAC of the controlled plant. Therefore, Assumption 2 in this paper is reasonable. (b). It is easy to see that if Assumption 2 holds, then we can get the following Lyapunov equation.

$$A_r^T T + T A_r = -S \tag{7}$$

where $S \in R^{n\times n}$ is a given symmetric positive definite matrix, and $T \in R^{n\times n}$ is the corresponding symmetric positive definite matrix solution to the above equation.

In this paper, we regard all links as a dynamic subsystem, which called LS. Therefore, we propose the following Riccati matrix differential equation to describe the dynamics of the LS.

$$\begin{cases} \dot{L} = PL + \Phi(z) \\ Y = \Upsilon L \end{cases} \tag{8}$$

where $P \in R^{N\times N}$ is a constant matrix, $Y \in R^{m\times N}$ is the output state matrix of LS, $\Upsilon \in R^{m\times N}$ is the output matrix, $\Phi(z) \in R^{N\times N}$ denotes the coupling term with states of the nodes.

Assumption 3. Consider the LS (8), the matrix pair (P, Υ) is completely observable.

Remark 3. (a). If Assumption 3 holds, then there exists a matrix $K \in R^{N\times m}$ such that matrix $P + K\Upsilon$ is a Hurwitz matrix. That is to say, for any given symmetric positive definite matrix $Q \in R^{N\times N}$, the following Lyapunov equation have corresponding symmetric positive definite matrix solution $W \in R^{N\times N}$.

$$(P + K\Upsilon)^T W + W(P + K\Upsilon) = -Q \tag{9}$$

(b). The matrices K and W in the above Lyapunov Equation (9) can be obtained by solving the following linear matrix inequality $P^T W + WP + R_1 + R_1{}^T < 0$, in which $K\Upsilon = W^{-1}R_1$. The specific method of solving the above matrices can refer to the following steps given by the toolbox Matlab: Define the variables W and $R_1 = W(K\Upsilon) \in R^{N\times N}$; Then describe the linear matrix inequality $P^T W + WP + R_1 + R_1{}^T < 0$, in which $W > 0$; End the description of the linear matrix inequality with the command getlmis and name it lmis; Call the solver feasp in the linear matrix inequality; converting the values of the determination variables into the form of matrix to obtain matrices W and R_1; then according to $R_1 = W(K\Upsilon)$ with W is the invertible matrix, we can obtain that $K\Upsilon = W^{-1}R_1$[25,36,37].

3 | DESIGN THE ASYMPTOTICAL STATE OBSERVER OF THE LS

In most practical engineering applications, due to the measurement cost and sensor technical limitations, the weight values of the links are usually can't be directly and accurately measured by suitable sensors, that is, the state information of the LS is unavailable. This increases the difficulty of the related control scheme design. To this end, it is necessary to design an asymptotical state observer for LS to make estimate values of its state information is available. We give the definition of asymptotical state observer as follows.

Definition 1[33]. Consider LS (8) and a given dynamic system $\dot{\hat{L}} = G(\hat{L}, Y, Z)$, in which if state $\hat{L}$ can be measured and $L - \hat{L} \xrightarrow{t\to+\infty} O_{N\times N}$, where $O_{N\times N}$ denotes the $N \times N$ dimension zero matrix, then the given dynamic system is called an asymptotical state observer for LS (8).

Assumption 4. The coupling term $\Phi(z)$ in LS (8) satisfies $\Phi(z) = W^{-1}\Upsilon^T\Gamma(z)$, in which, $\Gamma(z) \in R^{m\times N}$ and $\|\Gamma(z)\| \le \beta(z)$, $\beta(z)$ is the known function.

Remark 4. (a). Assumption 4 is the matching condition required to design an asymptotical state observer for LS (8), which corresponds to the matching conditions required when design the asymptotical state observer for nonlinear system in the literature [33,38,39]. (b). By Assumption 4, it can be clearly seen that the coupling term $\Phi(z)$ in LS (8) is related to the symmetric positive definite matrix W, the output matrix Υ and the state of nodes. In which, the matrix W is determined by the Lyapunov equation (9), the matrix Υ can be selected in the engineering application, and the state information of nodes is available. Hence, the form of the coupling term given in Assumption 4 is reasonable.

According to Definition 1 and combined with Assumption 4, propose the following asymptotical state observer for LS (8).

$$\begin{cases} \dot{\hat{L}} = P\hat{L} + \Psi(\hat{Y}, Y, z) - K(Y - \hat{Y}) \\ \hat{Y} = \Upsilon\hat{L} \end{cases} \tag{10}$$

where $\hat{L}$ represents the estimated value of the state L in LS (8), the robust term $\Psi(\hat{Y}, Y, z) = \begin{cases} \beta(z)W^{-1}\Upsilon^T\Omega, & \hat{Y} \neq Y \\ 0, & \hat{Y} = Y \end{cases}$, and $\Omega = \frac{Y-\hat{Y}}{\|Y-\hat{Y}\|}$.

The observer estimation error of the LS is defined as $E_L = L - \hat{L}$. According to Equations (8) and (10), we can obtain the following formula.

$$\begin{aligned} \dot{E}_L =& \dot{L} - \dot{\hat{L}} \\ =& PL + \Phi(z) - P\hat{L} - \Psi(\hat{Y}, Y, z) + K(Y - \hat{Y}) \\ =& PL + \Phi(z) - P\hat{L} - \Psi(\hat{Y}, Y, z) + K\Upsilon L - K\Upsilon\hat{L} \\ =& (P + K\Upsilon)(L - \hat{L}) + \Phi(z) - \Psi(\hat{Y}, Y, z) \\ =& (P + K\Upsilon)E_L + \Phi(z) - \Psi(\hat{Y}, Y, z) \end{aligned} \tag{11}$$

Lemma 1. If Assumptions 3 and 4 hold, then the dynamic system (10) is the asymptotical state observer of LS (8).

Proof. Refer to the results in Ref. [24], the $tr\{E_L{}^T W E_L\}$ is a positive definite function about the element E_L with the matrix W is the positive definite symmetric matrix. Choose the positive definite function $V_1 = tr\{E_L{}^T W E_L\}$. It is well known that $tr\{CD\} = tr\{DC\}$, $tr\{C + D\} = tr\{C\} + tr\{D\}$, $a = tr\{a\}$, $tr\{C^T\} = tr\{C\}$, $tr\{CD\} \leq \|C\| \cdot \|D\|$ hold for any matrices C and D with compatible dimensions, and a is any real number. Then its trajectory derivative with time along the error dynamic Equation (11) can be obtained as follows.

$$\begin{aligned} \dot{V}_1 =& tr\{\dot{E}_L^T W E_L + E_L{}^T W \dot{E}_L\} \\ =& tr\{[(P + K\Upsilon)E_L + \Phi(z) - \Psi(\hat{Y}, Y, z)]^T W E_L\} + tr\{E_L{}^T W[(P + K\Upsilon)E_L + \Phi(z) - \Psi(\hat{Y}, Y, z)]\} \\ =& tr\{E_L{}^T[(P + K\Upsilon)^T W + W(P + K\Upsilon)]E_L\} + 2tr\{E_L{}^T W\Phi(z)\} + 2tr\{-E_L{}^T W\Psi(\hat{Y}, Y, z)\} \\ =& tr\{-E_L{}^T Q E_L\} + 2tr\{E_L{}^T W W^{-1}\Upsilon^T\Gamma(z)\} + 2tr\{-E_L{}^T W\beta(z)W^{-1}\Upsilon^T \frac{Y - \hat{Y}}{\left\|Y - \hat{Y}\right\|}\} \\ \leq& -tr\{E_L{}^T Q E_L\} + 2\left\|\Upsilon E_L\right\| \cdot \|\Gamma(z)\| - 2\beta(z) \cdot \left\|\Upsilon E_L\right\| \\ =& -tr\{E_L{}^T Q E_L\} + 2\left\|\Upsilon E_L\right\| \cdot [\|\Gamma(z)\| - \beta(z)] \\ \leq& -tr\{E_L{}^T Q E_L\} \\ \leq& 0 \end{aligned} \tag{12}$$

Further, according to the inequality (12), which show that $\dot{V}_1$ is the negative definite function about E_L, therefore, the error dynamic system (11) is asymptotically stable in the Lyapunov sense, that is to say, $\lim_{t\to+\infty} E_L \to O_{N\times N}$. This completes the proof of Lemma 1.

4 | DESIGN THE MFAC SCHEME FOR NS BASED ON THE STATE OBSERVER OF LINKS

The model following adaptive error of NS is defined as $e_i = z_i - z_{ri}$ and in matrix form as $E = Z - Z_r$, where $E = [e_1, e_2, \dots, e_N] \in R^{n\times N}$, $i = 1, 2, \dots, N$. By using these definitions, the error dynamic equation of NS can be obtained as follows.

$$\begin{aligned} \dot{E} =& \dot{Z} - \dot{Z}_r \\ =& AZ + F(z, t) + \alpha HL + U - A_r Z_r - B_r U_r \\ =& A_r E + \alpha H E_L + (A - A_r)Z + F(z, t) + \alpha H\hat{L} + U - B_r U_r \end{aligned} \tag{13}$$

The control objective. Consider the controlled CDN with (2) and (8), the following model of the ith node is proposed as (5). Suppose that the state z_i of NS is available and the state L of LS is unavailable. By employing the asymptotical state observer (10) of LS (8), design the adaptive model following controller for the NS (2) such that the NS can achieve MFAC target, that is

$\lim_{t\to+\infty} e_i = O_{nN\times 1}$, which is equivalent to $\lim_{t\to+\infty} E = O_{n\times N}$. Furthermore, the other involved variables are ensured to be bounded. $O_{nN\times 1}$ and $O_{n\times N}$ denote $nN \times 1$ dimension zero vector and $n \times N$ dimension zero matrix, respectively.

Proposing the following matrix signal function $sign(TE) = \begin{cases} \frac{TE}{\|E^T T\|}, & E \neq O_{n\times N} \\ O_{n\times N}, & E = O_{n\times N} \end{cases}$, where the positive definite matrix T is determined by Equation (7). It is easily known that $tr\{E^T T sign(TE)\} = \|E^T T\|$.

By Assumption 1, the matrix A in the controlled NS is unknown, therefore, we denote $G_p{}^* = A - A_r$, the control gain matrix G_p represents an estimate of $G_p{}^*$, the controller gain estimation error matrix $\hat{G}_p = G_p - G_p{}^*$. Then in order to achieve the above proposed control objective, we synthesize the following MFAC scheme for NS.

$$U = -G_p Z + B_r U_r - \alpha H \hat{L} - \delta(z,t) sign(TE) \tag{14}$$

with the following adaptive law.

$$\dot{G}_p = \Delta_p T E Z^T \tag{15}$$

where $\Delta_p \in R^{n\times n}$ is the adjustable positive definite symmetric matrix.

Substituting the MFAC scheme (14) into the error dynamic equation (13) of NS, we can obtain the following equation.

$$\begin{aligned} \dot{E} =& A_r E + \alpha H E_L + (G_p{}^* - G_p) Z + F(z,t) - \delta(z,t) sign(TE) \\ =& A_r E + \alpha H E_L - \hat{G}_p Z + F(z,t) - \delta(z,t) sign(TE) \end{aligned} \tag{16}$$

Remark 5. Equation (14) gives the clearly structure of the MFAC scheme for NS and is divided into four parts. The first part $-G_p Z$ is the state feedback of the nodes, in which G_p is the estimated value matrix of the matrix $A - A_r$, and can be adjusted adaptively through the adaptive law (15). The second part $-\alpha H \hat{L}$ is the state feedback based on the asymptotical state observer for LS, and $\hat{L}$ is determined by Equation (10). The third part $B_r U_r$ is related to the reference model of NS and all information is known. The fourth part $-\delta(z,t) sign(TE)$ is the robust term, which is used to overcome the nonlinear bounded uncertainty term in the dynamics of NS.

Theorem 1. Consider the controlled CDN with Equation (2) and (8), the reference model of the ith node is given by Equation (5). If Assumptions 1-4 and the inequality $\|H\| < \min\left\{\frac{\varepsilon\lambda_{\min}(S)}{\alpha\|T\|}, \frac{\lambda_{\min}(Q)}{\alpha\varepsilon\|T\|}\right\}$ are satisfied, in which $\varepsilon > 0$ is an adjustable parameter, $\lambda_{\min}(S)$ and $\lambda_{\min}(Q)$ represent the minimum eigenvalues of matrices S and Q, respectively. Propose the asymptotical state observer (10) for LS (8), then by employing the designed MFAC scheme (14) and (15), the NS in CDN can achieve the MFAC target.

Proof. Consider the positive definite function $V = V(t) = tr\{E^T T E\} + tr\{\hat{G}_p^T \Delta_p{}^{-1} \hat{G}_p\} + V_1$, where $T \in R^{n\times n}$ is determined by Equation (7). If Assumptions 1-4 hold, by employing the asymptotical state observer (10) of LS (8), and the MFAC scheme (14) and (15) for NS. Then the orbit derivative of $V = V(t)$ along the error dynamic systems (11) and (16) can be obtained by the following equation.

$$\begin{aligned} \dot{V} =& tr\{\dot{E}^T T E\} + tr\{E^T T \dot{E}\} + tr\{\dot{\hat{G}}_p{}^T \Delta_p{}^{-1} \hat{G}_p\} + tr\{\hat{G}_p^T \Delta_p{}^{-1} \dot{\hat{G}}_p\} + \dot{V}_1 \\ =& tr\{[A_r E + \alpha H E_L - \hat{G}_p Z + F(z,t) - \delta(z,t) sign(TE)]^T T E\} \\ &+ tr\{E^T T [A_r E + \alpha H E_L - \hat{G}_p Z + F(z,t) - \delta(z,t) sign(TE)]\} \\ &+ tr\{\dot{\hat{G}}_p{}^T \Delta_p{}^{-1} \hat{G}_p\} + tr\{\hat{G}_p^T \Delta_p{}^{-1} \dot{\hat{G}}_p\} + \dot{V}_1 \\ =& tr\{E^T (A_r{}^T T + T A_r) E\} + tr\{\alpha E_L{}^T H^T T E + \alpha E^T T H E_L\} + tr\{-Z^T \hat{G}_p^T T E - E^T T \hat{G}_p Z\} \\ &+ 2tr\{E^T T [F(z,t) - \delta(z,t) sign(TE)]\} + 2tr\{\dot{\hat{G}}_p{}^T \Delta_p{}^{-1} \hat{G}_p\} + \dot{V}_1 \\ =& - tr\{E^T S E\} - tr\{E_L{}^T Q E_L\} + 2tr\{\alpha E^T T H E_L\} \\ &+ 2tr\{-Z E^T T \hat{G}_p\} + 2tr\{\dot{\hat{G}}_p{}^T \Delta_p{}^{-1} \hat{G}_p\} + 2tr\{E^T T [F(z,t) - \delta(z,t) sign(TE)]\} \\ \leq& - tr\{E^T S E\} - tr\{E_L{}^T Q E_L\} + 2tr\{\alpha E^T T H E_L\} \\ &+ 2tr\{(\dot{\hat{G}}_p{}^T \Delta_p{}^{-1} - Z E^T T) \hat{G}_p\} + 2\left\|E^T T\right\| \cdot [\|F(z,t)\| - \delta(z,t)] \\ \leq& - tr\{E^T S E\} - tr\{E_L{}^T Q E_L\} + 2tr\{\alpha E^T T H E_L\} \\ \leq& - \lambda_{\min}(S)\|E\|^2 - \lambda_{\min}(Q)\left\|E_L\right\|^2 + 2\alpha \|T\| \cdot \|H\| \cdot \|E\| \cdot \left\|E_L\right\| \end{aligned}$$

$$\leq -\lambda_{\min}(S)\|E\|^2 - \lambda_{\min}(Q)\|E_L\|^2 + 2\alpha\|T\|\cdot\|H\|\cdot[\frac{1}{2\varepsilon}\|E\|^2 + \frac{\varepsilon}{2}\|E_L\|^2]$$
$$= -\|E\|^2[\lambda_{\min}(S) - \frac{\alpha}{\varepsilon}\|T\|\cdot\|H\|] - \|E_L\|^2[\lambda_{\min}(Q) - \alpha\varepsilon\|T\|\cdot\|H\|] \tag{17}$$

Denote $\beta_1 = \lambda_{\min}(S) - \frac{\alpha}{\varepsilon}\|T\|\cdot\|H\|$, $\beta_2 = \lambda_{\min}(Q) - \alpha\varepsilon\|T\|\cdot\|H\|$, then we can obtain that the following formula.

$$\dot{V} = -\beta_1\|E\|^2 - \beta_2\|E_L\|^2 \tag{18}$$

If inequality $\|H\| < \min\left\{\frac{\varepsilon\lambda_{\min}(S)}{\alpha\|T\|}, \frac{\lambda_{\min}(Q)}{\alpha\varepsilon\|T\|}\right\}$ is satisfied, then $\beta_1 > 0$ and $\beta_2 > 0$. Therefore, we can obtain that $\dot{V} \leq 0$ through Equation (18), which means that $\dot{V}$ is the semi-negative definite function about elements E, E_L, $\hat{G}_p$. Thus, we can know that the model following adaptive error E of the NS , the state observer estimation error E_L of the LS and the controller gain estimation error matrix $\hat{G}_p$ are bounded. Furthermore, according to the above obtained results with Equations (11) and (16), it can be clearly seen that $\dot{E}_L$ and $\dot{E}$ are also bounded. Therefore, by the Barbalat Lemma[40], we can obtain that $\lim_{t\to+\infty} E_L = O_{N\times N}$ and $\lim_{t\to+\infty} E = O_{n\times N}$, where the latter is equivalent to $\lim_{t\to+\infty} e_i = O_{nN\times 1}$. Therefore, the NS in CDN can achieve the MFAC target which is shown in Theorem 1.

Remark 6. In order to apply Theorem 1 to achieve the MFAC of NS in CDN, we give the following steps.

Step (i). Give the controlled CDN composed with (2) and (8), the reference model of the ith node is given by Equation (5). Then rewrite Equations (2) and (5) as Equations (3) and (6), respectively.

Step (ii). Determine the Hurwitz constant matrix A_r, constant matrix B_r, the reference input matrix U_r, the inner coupling matrix function $H(z,t)$, the common coupling strength α, the known nonnegative continuous function $\delta(z,t)$ which satisfied the inequality (4), the output matrix Υ, and the adjustable positive definite symmetric matrix Δ_p.

Step (iii). Obtain the matrices T, K, and W from Lyapunov Equations (7) and (9), respectively. Then substituting the above parameters into the proposed asymptotical state observer (10) for LS (8), and construct the model following adaptive controller of NS shown as Equations (14) and (15).

Step (iv). By adjusting the parameter ε, make the inequality $\|H\| < \min\left\{\frac{\varepsilon\lambda_{\min}(S)}{\alpha\|T\|}, \frac{\lambda_{\min}(Q)}{\alpha\varepsilon\|T\|}\right\}$ is satisfied. Then the MFAC of NS can be implemented, that is to say, the state z_i of NS can asymptotically following the state z_{ri} of the reference model. At the same time, the other involved parameters can also be guaranteed to be bounded.

5 | THE NUMERICAL SIMULATION

A simulation example with practical engineering application background is given to verify the theoretical results. Consider the CDN with N ($N = 20$) nodes, in which, the dynamics of each isolated node is depicted by the 2-DOF (Degree of Freedom) ($n = 2$) helicopter[41], and the communication between helicopters is seen as links. Consequently, propose the following equation as the dynamic model of the 2-DOF helicopter is shown below, $i = 1, 2, \ldots, N$.

$$J_p\ddot{\theta}_i + D_p\dot{\theta}_i + K_{spi}\theta_i = K_{ppi}V_{pi} + K_{pyi}V_{yi} \tag{19}$$

$$J_u\ddot{\psi}_i + D_y\dot{\psi}_i = K_{ypi}V_{pi} + K_{yyi}V_{yi} \tag{20}$$

where θ_i and ψ_i denote the pitch angle and yaw angle of the ith 2-DOF helicopter, respectively. V_{pi} and V_{yi} denote the control input voltages to the DC-motors that control the pitch and yaw propellers of the ith 2-DOF helicopter, respectively. The definitions of the remaining parameters are given in Table 1.

In this simulation, we mainly discuss the speed following of pitch and yaw for the N 2-DOF helicopters. Therefore, let us consider the speed state vector $z_i = [\dot{\theta}_i, \dot{\psi}_i]^T$ and the control input $v_i = [V_{pi}, V_{yi}]^T$, then Equations (19) and (20) can be combined and expressed as follows.

$$\dot{z}_i = G^{-1}Dz_i + G^{-1}g_i(z_i,t) + G^{-1}K_iv_i \tag{21}$$

where $g_i(z_i,t) = [-K_{spi}\theta_i, 0]^T$, the 2-order matrices $G = \begin{bmatrix} J_p & 0 \\ 0 & J_u \end{bmatrix}$, $D = \begin{bmatrix} -D_p & 0 \\ 0 & -D_y \end{bmatrix}$ and $K_i = \begin{bmatrix} K_{ppi} & K_{pyi} \\ K_{ypi} & K_{yyi} \end{bmatrix}$.

In this paper, we consider the given communication protocol $\alpha\sum_{j=1}^{N} l_{ji}(t)H_j(z_j,t)$, where $l_{ji}(t)$ denote the communication strength from the jth 2-DOF helicopter to the ith 2-DOF helicopter, and the dynamics of which is given by Equation (8).

Therefore, Equation (21) can be rewritten as.

$$\dot{z}_i = G^{-1}Dz_i + G^{-1}g_i(z_i,t) + \alpha \sum_{j=1}^{N} l_{ji}(t)H_j(z_j,t) + G^{-1}K_iv_i \tag{22}$$

By using these transformations $A = G^{-1}D$, $f_i(z_i,t) = G^{-1}g_i(z_i,t)$ and $u_i = G^{-1}K_iv_i$, Equation (22) can be expressed in the form of Equation (2). In this simulation, according to the literature [41], the definition and selection of the above involved parameters shown in Table 1.

Symbol	Parameter	Value	Unit
J_p	Moment of Inertia about the pitch axis	0.0215	$kg.m^2$
J_u	Moment of Inertia about the yaw axis	0.0215	$kg.m^2$
K_{spi}	Stiffness about the pitch axis	$0.0374 + 0.0001sin(it)$	$N.m/rad$
D_p	Pitch viscous friction constant	0.0071	$N.m.s/rad$
D_y	Yaw viscous friction constant	0.0220	$N.m.s/rad$
K_{ppi}	Thrust-torque gain acting on pitch axis from pitch propeller	$0.0011 + 0.0001sin(it)$	$N.m/V$
K_{yyi}	Thrust-torque gain acting on yaw axis from yaw propeller	$0.0022 + 0.0001sin(it)$	$N.m/V$
K_{pyi}	Thrust-torque gain acting on pitch axis from yaw propeller	$0.0021 + 0.0001sin(it)$	$N.m/V$
K_{ypi}	Thrust-torque gain acting on yaw axis from pitch propeller	$-0.0027 + 0.0001sin(it)$	$N.m/V$

TABLE 1 The definition and selection of the ith 2-DOF helicopter parameters.

The speed reference model of pitch and yaw for the ith 2-DOF helicopter is given by Equation (6). Based on the above description, in this paper, the simulation is completed with Matlab Toolbox. The reference input u_{ri}, the matrices B_r, A_r in Equation (6), the matrices P, Υ in Equation (8), and the common coupling strength α, the continuous inner coupling vector function $H_j(z_j,t)$ in Equation (2) are given by the following rules, respectively.

(i). Let $u_{ri} = rand(1)\left[\sin(u_{ri1}\pi t), \sin(u_{ri2}\pi t), \sin(u_{ri3}\pi t)\right]^T$, in which u_{ri1}, u_{ri2} and u_{ri3} are randomly generated in the interval $[0,3]$, and $B_r = \ell rand(n,m)$ $(m=3)$, where ℓ is an adjustable parameter.

(ii). Give a diagonal matrix $\Xi = diag(\sigma_1, \sigma_2, \ldots, \sigma_n)$, where $\sigma_1, \sigma_2, \ldots, \sigma_n$ are negative real numbers randomly selected in the range $[-3,-1]$. Then, randomly generate an n-order invertible matrix by using the command $\mathrm{M} = \varsigma randn(n,n)$ with ς is an adjustable parameter. Therefore, the Hurwitz matrix $A_r = \mathrm{M}^{-1}\Xi\mathrm{M}$.

(iii). Use the command 'randn' to generate matrices P and Υ randomly, and each element in which is required to be any real number in the range $[-1,2]$.

(iv). α is randomly selected within $[0,1]$, and $H_j(z_j,t) = [5\cos(z_{j1}z_{j2}), 5\cos(z_{j1}z_{j2})]^T$, $j = 1,2,\ldots,N$.

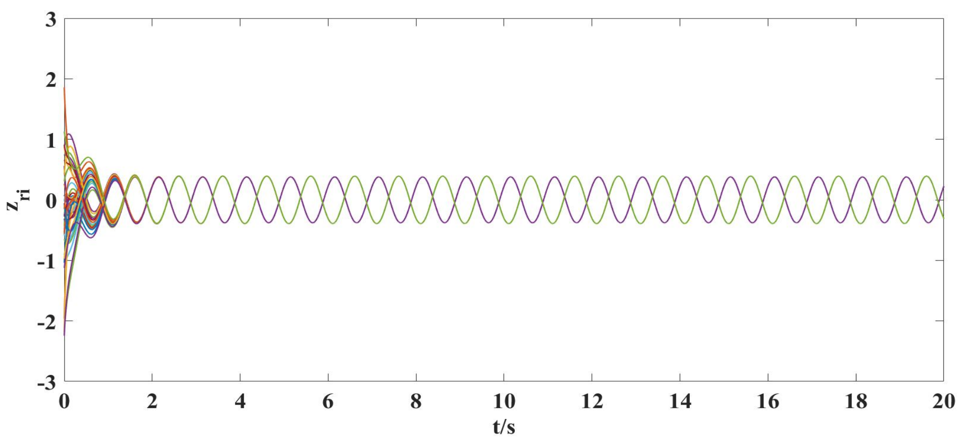

FIGURE 1 The speed following target curves z_{ri} of 2-DOF helicopters.

Apart from this, the matrices T and W are obtained by solving the Lyapunov Equations (7) and (9), respectively, in which, the matrix $S = 100 * eye(n,n)$. The initial states of the nodes and the links are chosen as $z_i(0) = randn(n,1)$, the state matrices

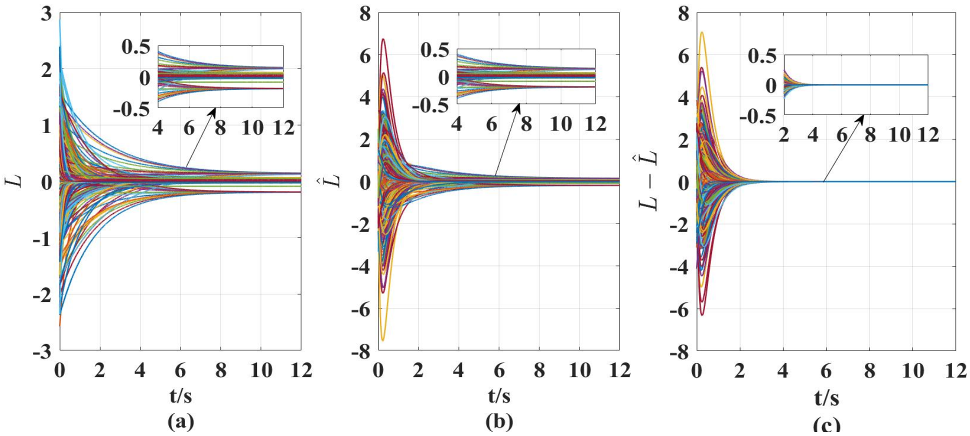

FIGURE 2 (a). The communication strength curves L between 2-DOF helicopters. (b). The observer curves $\hat{L}$ of communication strength between 2-DOF helicopters. (c). The communication strength observer error curves $L-\hat{L}$ between 2-DOF helicopters.

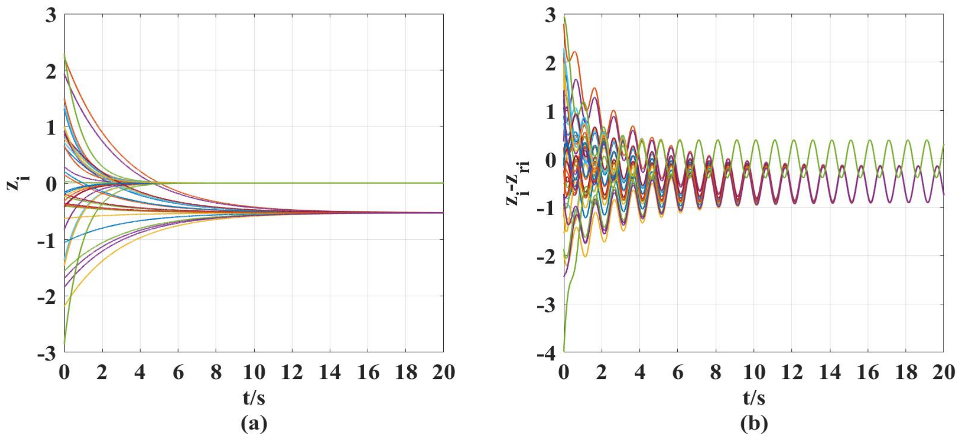

FIGURE 3 (a). The speed curves z_i of 2-DOF helicopters without controller. (b). The speed following error curves $z_i - z_{ri}$ of 2-DOF helicopters without controller.

$Z(0) = [z_1(0), z_2(0), \ldots, z_N(0)] \in R^{n\times N}$, and $L(0) = randn(N, N)$, $i = 1, 2, \ldots, N$. According to the above parameters selection, using the asymptotical state observer (10) for LS (8) designed in this paper and the control scheme (14) and (15) for NS (3), we can obtain the following simulation results shown in Figs.1-5.

By analyzing the simulation results in Figs 1-5, we can make the following observations.

(i). In Fig.2, it can be seen that the state error between the communication strength (8) between 2-DOF helicopters and the observer (10) designed for it can quickly approach 0 with time. This fully demonstrates the effectiveness of the designed observer in this paper.

(ii). Fig.3 shows that the speeds of 2-DOF helicopters cannot asymptotically follow the given speed following targets without controller. Conversely, Fig.4 shows that the speed following errors of 2-DOF helicopters can gradually converge to 0 with controller. Therefore, the effectiveness of the controller designed in this paper is illustrated in conjunction with Figs. 3 and 4.

(iii). From Fig. 2 (a) and Fig. 4, it is observed that when the speeds of 2-DOF helicopters achieve the MFAC target, that is, when the speeds of 2-DOF helicopters asymptotically following the given reference speeds, there also exists information exchange between the 2-DOF helicopters, and its communication strength change curves are reflected by (a) in Fig. 2.

(iv). From Fig. 5, we can obtain that the gain estimation matrix in controller is bounded, which is required in the control goal of this paper.

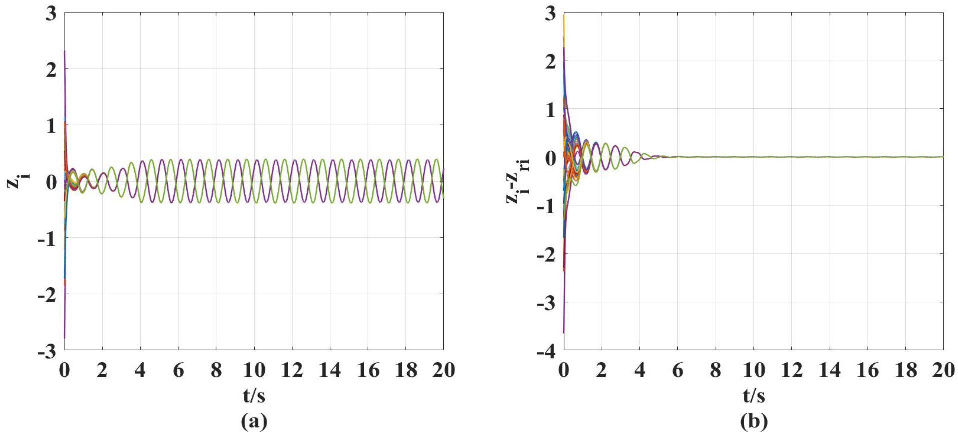

FIGURE 4 (a). The speed curves z_i of 2-DOF helicopters with controller. (b). The speed following error curves $z_i - z_{ri}$ of 2-DOF helicopters with controller.

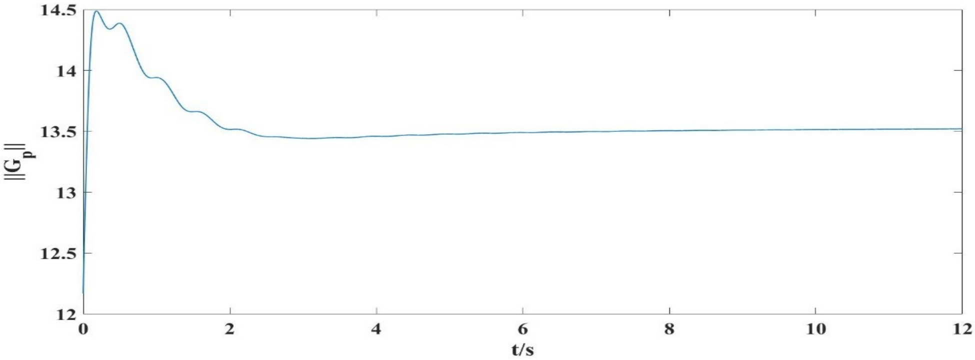

FIGURE 5 The norm curve of controller gain estimation matrix G_p.

6 | CONCLUSION

This paper mainly focuses on the design of the state observer for LS and the synthesis of the controller for NS based on the links state estimation value to realize the MFAC of NS in CDN. Firstly, the dynamics models of NS and LS are established, which are described by the vector differential equation and the Riccati matrix differential equation, respectively. Then, under some mathematical assumptions, the asymptotical state observer of LS is designed so that its state estimation information is available. Furthermore, based on the state estimation information of LS, combined with the Lyapunov stability theory to synthesize the adaptive controller of NS, so that the NS can asymptotically follow the given reference target, that is, the NS realizes MFAC. Finally, according to the obtained simulation results, the effectiveness of the state observer for LS and the controller for NS designed in this paper is illustrated. By using the observer state information of the LS, the LS can be directly controlled, therefore, a related problem is how to design the control scheme for LS to realize the MFAC of LS, which needs to be further discussed in the future work.

ACKNOWLEDGMENTS

This work was supported by the Key Laboratory of Intelligent Manufacturing Technology (Shantou University), Ministry of Education of China under Grant (202109242), the National Natural Science Foundation of China under Grant (61673120).

DECLARATION OF COMPETING INTEREST

Conflict of interest. The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

1. Cui Q, Li L, Lu J, Alofi A. Finite-time synchronization of complex dynamical networks under delayed impulsive effects. *Applied Mathematics and Computation.* 2022;430:127290. doi: https://doi.org/10.1016/j.amc.2022.127290

2. Bonabi SY, Asgharian H, Safari S, Ahmadabadi MN. FPGA implementation of a biological neural network based on the Hodgkin-Huxley neuron model. *Frontiers in Neuroscience.* 2014;8(379):1-12. doi: https://doi.org/10.3389/fnins.2014.00379

3. Chen GR. Pinning control and controllability of complex dynamical networks. *International Journal of Automation and Computing.* 2017;14(1):1-9. doi: https://doi.org/10.1007/s11633-016-1052-9

4. Li Q, Shen B, Wang ZD, Huang TW, Luo J. Synchronization control for a class of discrete time-delay complex dynamical networks: a dynamic event-triggered approach. *IEEE Transactions on Cybernetics.* 2019;49(5):1979-1986. doi: 10.1109/TCYB.2018.2818941

5. Wang YF, Wu XQ, Feng H, Lu JA, Lu JH. Topology inference of uncertain complex dynamical networks and its applications in hidden nodes detection. *Science China Technological Sciences.* 2016;59(8):1232-1243. doi: 10.1007/s11431-016-6050-1

6. Tang Y, Qian F, Gao HJ, Kurths J. Synchronization in complex networks and its application-A survey of recent advances and challenges. *Annual Reviews in Control.* 2014;38(2):184-198. doi: 10.1016/j.arcontrol.2014.09.003

7. Liu ZW, Wen GH, Yu XH, Guan ZH, Huang TW. Delayed impulsive control for consensus of multiagent systems with switching communication graphs. *IEEE Transactions on Cybernetics.* 2020;50(7):3045-3055. doi: 10.1109/TCYB.2019.2926115

8. Yang HL, Wang X, Zhong SM, Shu L. Synchronization of nonlinear complex dynamical systems via delayed impulsive distributed control. *Applied Mathematics and Computation.* 2018;320:75-85. doi: 10.1016/j.amc.2017.09.019

9. Vega CJ, Suarez OJ, Sanchez EN, Chen GR, Elvira-Ceja S, Rodriguez-Castellanos D. Trajectory tracking on complex networks via inverse optimal pinning control. *IEEE Transactions on Automatic Control.* 2019;64(2):767-774. doi: 10.1109/TAC.2018.2835412

10. Heider F. Attitudes and cognitive organization. *Journal of Psychology.* 1946;21(1):107-112. doi: 10.1080/00223980.1946.9917275

11. Gao ZL, Wang YH, Peng Y, Liu LZ, Chen HG. Adaptive control of the structural balance for a class of complex dynamical networks. *Journal of Systems Science and Complexity.* 2020;33:725-742. doi: 10.1007/s11424-020-8093-4

12. Xi Q, Liu XZ, Li XD. Finite-time synchronization of complex dynamical networks via a novel hybrid controller. *IEEE Transactions on Neural Networks and Learning Systems.* 2022;1-10. doi: 10.1109/TNNLS.2022.3185490

13. Liu LZ, Wang YH, Gao ZL. Tracking control for the dynamic links of discrete-time complex dynamical network via state observer. *Applied Mathematics and Computation.* 2020;369:124857. doi: 10.1016/j.amc.2019.124857

14. Peng ZX, Yang SC, Wen GG, Rahmani A, Yu YG. Adaptive distributed formation control for multiple nonholonomic wheeled mobile robots. *Neurocomputing.* 2016;173(3):1485-1494. doi: 10.1016/j.neucom.2015.09.022

15. Qin DD, Liu AD, Zhang D, Ni HJ. Formation control of mobile robot systems incorporating primal-dual neural network and distributed predictive approach. *Journal of the Franklin Institute.* 2020;357(17):12454-12472. doi: 10.1016/j.jfranklin.2020.09.025

16. Hou R, Cui LZ, Bu XH, Yang JQ. Distributed formation control for multiple non-holonomic wheeled mobile robots with velocity constraint by using improved data-driven iterative learning. *Applied Mathematics and Computation.* 2021;395:125829. doi: 10.1016/j.amc.2020.125829

17. Yu WS. Design of a power system stabilizer using decentralized adaptive model following tracking control approach. *International Journal of Numerical Modeling Electronic Networks, Devices and Fields.* 2010;23(2):63-87. doi: 10.1002/jnm.722

18. Lu Q, Zhang D, Ye WJ, Fan JY, Liu S, Su CY. Targeting posture control with dynamic obstacle avoidance of constrained uncertain wheeled mobile robots including unknown skidding and slipping. *IEEE Transactions on Systems, Man and Cybernetics: Systems.* 2021;51(11):6650-6658. doi: 10.1109/TSMC.2019.2962732

19. Landau ID, Courtiol B. Design of multivariable adaptive model following control systems. *IFAC Proceedings.* 1973;6(2):315-322. doi: 10.1016/S1474-6670(17)68205-3

20. Ahmadian N, Khosravi A, Sarhadi P. Integrated model reference adaptive control to coordinate active front steering and direct yaw moment control. *ISA Transactions.* 2020;106:85-96. doi: 10.1016/j.isatra.2020.06.020

21. Zhang D, Wei B. A review on model reference adaptive control of robotic manipulators. *Annual Reviews in Control.* 2017;43:188-198. doi: 10.1016/j.arcontrol.2017.02.002

22. Shyu KK, Yang MJ, Chen YM, Lin YF. Model reference adaptive control design for a shunt active-power-filter system. *IEEE Transactions on Industrial Electronics.* 2008;55(1):97-104. doi: 10.1109/TIE.2007.906131

23. Yucelen T, Haddad WM. Low-frequency learning and fast adaptation in model reference adaptive control. *IEEE Transactions on Automatic Control.* 2013;58(4):1080-1085. doi: 10.1109/TAC.2012.2218667

24. Wang YH, Wang WL, Zhang LL. State synchronization of controlled nodes via the dynamics of links for complex dynamical networks. *Neurocomputing.* 2020;384:225-230. doi: 10.1016/j.neucom.2019.12.055

25. Gao PT, Wang YH, Peng Y, Zhang LL, Huang YY. Tracking control for the nonlinear complex dynamical network assisted with outgoing links dynamics. *International Journal of Robust and Nonlinear Control.* 2022;1-13. doi: 10.1002/rnc.6282

26. Aljadeff J, Gillett M, Obilinovic UP, Brunel N. From synapse to network: models of information storage and retrieval in neural circuits. *Current Opinion in Nuerobiology.* 2021;70:24-33. doi: 10.1016/j.conb.2021.05.005

27. Dong T, Gong XM, Huang TW. Zero-Hopf Bifurcation of a memristive synaptic Hopfield neural network with time delay. *Neural Networks.* 2022;149:146-156. doi: 10.1016/j.neunet.2022.02.009

28. Maciel GA, Garcia RM. Enhanced species coexistence in Lotka-Volterra competition models due to nonlocal interactions. *Journal of Theoretical Biology.* 2021;530:110872. doi: 10.1016/j.jtbi.2021.110872

29. Aliyu MB, Mohd MH. The interplay between mutualism, competition and dispersal promotes species coexistence in a multiple interactions type system. *Ecological Modelling.* 2021;452:109595. doi: 10.1016/j.ecolmodel.2021.109595

30. Martin S. Multi-agent flocking under topological interactions. *System and Control Letters.* 2014;69:53-61. doi: 10.1016/j.sysconle.2014.04.004

31. Fan MC, Zhang HT, Wang MM. Bipartite flocking for multi-agent systems. *Communications in Nonlinear Science Numerical Simulation.* 2014;19(9):3313-3322. doi: 10.1016/j.cnsns.2013.10.009

32. Gao PT, Wang YH, Liu LZ, Zhang LL, Tang X. Asymptotical state synchronization for the controlled directed complex dynamic network via links dynamics. *Neurocomputing.* 2021;448(11):60-66. doi: 10.1016/j.neucom.2021.03.095

33. Gao ZL, Wang YH, Xiong J, Zhang LL, Wang WL. Robust state observer design for dynamic connection relationships in complex dynamical networks. *International Journal of Control, Automation and Systems.* 2019;17:336-344. doi: 10.1007/s12555-018-0315-3

34. Kazemy A, Shojaei K. Synchronization of complex dynamical networks with dynamical behavior links. *Asian Journal of Control.* 2020;22(1):474-485. doi: 10.1002/asjc.1910

35. Meng L, Bao H. Synchronization of delayed complex dynamical networks with actuator failure by event-triggered pinning control. *Physica A*. 2022; doi: 10.1016/j.physa.2022.128138

36. Homaeinezhad MR, Yaqubi S. Two-sided linear matrix inequality solution of affine input matrix for feasible discrete finite-time sliding mode control of uncertain nonlinear mechanical machines. *Journal of Vibration and Control.* 2020;26(23-24):2243-2260. doi: 10.1177/1077546320916637

37. Arceo JC, Sanchez M, Estrada-Manzo V, Bernal M. Convex stability analysis of nonlinear singular systems via linear matrix inequalities. *IEEE Transactions on Automatic Control.* 2019;64(4):1740-1745. doi: 10.1109/TAC.2018.2854651

38. Hmidi R, Brahim AB, Hmida FB, Sellami A. Robust fault tolerant control design for nonlinear systems not satisfying matching and minimum phase conditions. *International Journal of Control, Automation and Systems.* 2020;18(X):1-14. doi: 10.1007/s12555-019-0516-4

39. Yi BW, Ortega R, Zhang WD. On state observers for nonlinear systems: A new design and a unifying framework. *IEEE Transactions on Automatic Control.* 2019;64(3):1193-1200. doi: 10.1109/TAC.2018.2839526

40. Luo L, Wang YH, Fan YQ, Zhang Y. Novel adaptive control design for nonlinear system with extended partition of unity method. *Asian Journal of Control.* 2013;15(3):911-918. doi: 10.1002/asjc.576

41. Lopes AND, Arcese L, Guelton K, Cherifi A. Sampled-data controller design with application to the quanser AERO 2-DOF helicopter. *2020 IEEE International Conference on Automation, Quality and Testing, Robotics (AQTR).* 2020;1-6. doi: 10.1109/AQTR49680.2020.9129983

How to cite this article: Li X, Wang Y, Li S Model following adaptive control for nodes in complex dynamical network via the state observer of links, *Math Meth Appl Sci*.