New upper bounds of cumulative coherence for ℓ_{1-2}-minimization in compressed sensing

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Abstract

This paper focuses on the compressed sensing ℓ_{1-2}-minimization problem and develops new bounds on cumulative coherence $\mu_1(s)$. We point out that if cumulative coherence $\mu_1(s-1)$ and $\mu_1(2s-1)$ satisfy $(**??**)$, or cumulative coherence $\mu_1(2s-1)$ satisfies $(**??**)$ then the sparse signal can via ℓ_{1-2}-minimization problem stably recover in noise model and exact recovery in free noise model.

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Abstract

This paper focuses on the compressed sensing ℓ_{1-2}-minimization problem and develops new bounds on cumulative coherence $\mu_1(s)$. We point out that if cumulative coherence $\mu_1(s-1)$ and $\mu_1(2s-1)$ satisfy (2), or cumulative coherence $\mu_1(2s-1)$ satisfies (7) then the sparse signal can via ℓ_{1-2}-minimization problem stably recover in noise model and exact recovery in free noise model.

Keywords: ℓ_{1-2}-Minimization, Cumulative coherence, Sparse signal, Stably recove

1 Introduction

In recent years, compressed sensing (CS) has attracted considerable attention. It primarily reconstructs an unknown high-dimensional s-sparse signal $x \in R^n$ from lower-dimensional $y = Ax$ measurements, where $A \in R^{n\times m}, m << n$. For the reconstruction of x, the most intuitive approach is to find the sparsest signal in the feasible set of possible solutions, which leads to an ℓ_0-minimization problem as follows:

$$\min_{x\in R^n} \|x\|_0 \qquad subject \quad to \qquad y - Ax \in B,$$

where $B = \{0\}$ indicates a noiseless case, and $B = \{\epsilon\}$ indicates a noise case.

The ℓ_0-minimization problem is NP-hard, and thus computationally not feasible in high-dimensional sets [3]. To solve this problem, various methods have been proposed such as ℓ_1-minimization problem [1–3, 6], ℓ_p-minimization problem [5], ℓ_{1-2}-minimization problem [4,7], weighted ℓ_1-minimization problem.

There are numerous results on the $\ell_1$minimization problem in the literature. These results are mainly based on the null space property, coherence [4], cumulative coherence [6], restricted orthogonality constants [1], and restricted isometry properties [2,3].

Although the ℓ_1-minimization problem yields considerable results, it is not exactly equivalent to the ℓ_0-minimization problem. Hence, the ℓ_{1-2}-minimization problem [4,7] and ℓ_p-minimization problem [5] have been proposed to replace the ℓ_1-minimization problem in the case where the ℓ_1-minimization problem does not execute well.

In this paper, we mainly study the ℓ_{1-2}-minimization problem, and obtain sufficient conditions for stable recovery of any k sparse signals by using the cumulative coherence condition. The ℓ_{1-2}-minimization problem is the following model:

$$\min_{x\in R^n} \|x\|_1 - \|x\|_2 \qquad subject\ to \qquad \|y - Ax\|_2 \le \epsilon \tag{1}$$

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where $A \in R^{m\times n}(m \ll n)$ is the measurement matrix, $y \in R^m$ is the measurement vector, and $x \in R^n$ is the unknown vector to be recovered.

The structure of this paper is as follows: We introduce related concepts in Section II, and present our main results in Section III and conclude the paper in Section IV.

Notations: For $x \in R^n$, $\|x\|_0$ indicates the number of non-zero elements in x. $\|x\|_1 = \sum_{i=1}^n |x_i|$, $\|x\|_2 = \sqrt{\sum_{i=1}^n x_i^2}$, and $\|x\|_\infty = \max_{i\in[n]} |x_i|$, where $[n] = \{1, 2, 3, \cdots, n\}$. $s \in N^+$ and $x_{max(s)}$ is defined as the vector x with all but the largest s entries in absolute value set to zero, and $x_{-max(s)} = x - x_{max(s)}$. For $y \in R^n$, $\langle x, y\rangle = \sum_{i=1}^n x_i y_i$. $T \subset [n]$, x_T is defined as the vector $(x_T)_i = x_i$, if $i \in T$ and $(x_T)_i = 0$ otherwise.

2 Preliminary

Definition 1 ([3]) *Let $A \in R^{m\times n}$ be a matrix with ℓ_2-normalized columns $A_1, \ldots, A_n$, that is, $\|A_i\|_2 = 1$ for all $i = 1, ..., n$. The cumulative coherence function $\mu_1(s) = \mu_1(A, s)$ of matrix A is defined for $s \in [n-1]$ by*

$$\mu_1(s) = \max_{i\in[n]} max\{\sum_{j\in S} |\langle A_i, A_j\rangle|, S \subset [n], card(S) = s, i \notin S\}$$

When the cumulative coherence of a matrix grows slowly, we can informally say that the dictionary is quasi-incoherent.

The following lemmas are needed in the proof of our main results and we list them below.

Lemma 1 ([3]) *Let $A \in R^{m\times n}$ be a matrix with ℓ_2-normalized columns and $s \in [n]$. For all s-sparse vectors $x \in R^n$,*

$$(1 - \mu_1(s-1))\|x\|_2^2 \le \|Ax\|_2^2 \le (1 + \mu_1(s-1))\|x\|_2^2.$$

Lemma 2 ([6]) *Let $s_1, s_2 \le n$ and $\lambda \ge 0$. Suppose that $x, y \in R^n$ satisfies $supp(x) \cap supp(y) = \varnothing$, and x is s_1 sparse. If $\|y\|_1 \le \lambda s_2$ and $\|y\|_\infty \le \lambda$, then*

$$|\langle Ax, Ay\rangle| \le \lambda\sqrt{s_2}\mu_1(s_1 + s_2 - 1)\|x\|_2.$$

Lemma 3 ([6]) *Suppose that x is s-sparse and y is t-sparse; then,*

$$|\langle Ax, Ay\rangle - \langle x, y\rangle| \le \mu_1(s + t - 1)\|x\|_2\|y\|_2.$$

Moreover, if $supp(x) \cap supp(y) = \emptyset$, then

$$|\langle Ax, Ay\rangle| \le \mu_1(s + t - 1)\|x\|_2\|y\|_2.$$

3 Main result

In this section, we present the main results. Theorem 1 shows that when $\mu_1(s-1)$ and $\mu_1(2s-1)$ satisfy (2), the ℓ_{1-2}-minimization problem can stably recover an unknown signal.

Theorem 1 *Let $A \in R^n$ be a measurement matrix, $y = Ax$ be the measurement vector, and s be a given positive integer with $2 \le s < m$. If measurement matrix A satisfies*

$$\gamma(s) := (\sqrt{s}-1)\mu_1(s-1)+(\sqrt{2}+\sqrt{s}-1)\mu_1(2s-1) \le \sqrt{s}-1, \tag{2}$$

then, the solution $\overline{x}$ of (1) and the original signal x will have

$$\|\overline{x} - x\|_2 \le C_1\sigma_s(x)_1 + C_2\epsilon,$$

where $C_1 = \frac{2(1-\mu_1(s-1)+(\sqrt{2}-1)\mu_1(2s-1))}{\sqrt{s}-1-\gamma(s)}$, $C_2 = \frac{2\sqrt{2s(1+\mu_1(s-1))}}{\sqrt{s}-1-\gamma(s)}$.

Proof: Set $h = \overline{x} - x$ and from [7], we know

$$\|h_{-max(s)}\|_1 \leq \|h_{max(s)}\|_1 + 2\|x_{-max(s)}\|_1 + \|h\|_2. \tag{3}$$

Hence,

$$\|h_{-max(s)}\|_1 \leq s(\frac{\|h_{max(s)}\|_2}{\sqrt{s}} + \frac{2\|x_{-max(s)}\|_1 + \|h\|_2}{s}),$$

$$\|h_{-max(s)}\|_\infty \leq \frac{\|h_{max(s)}\|_1}{s} \leq \frac{\|h_{max(s)}\|_2}{\sqrt{s}}.$$

Based on Lemma 2, the following inequality holds

$$\langle Ah_{max(s)}, Ah_{-max(s)}\rangle \leq (\frac{\|h_{max(s)}\|_2}{\sqrt{s}} + \frac{2\|x_{-max(s)}\|_1 + \|h\|_2}{s})\sqrt{s}\mu_1(s+s-1)\|h_{max(s)}\|_2.$$

From Lemma 1 and the above inequality, we have

$$\begin{aligned}
&|\langle Ah, Ah_{max(s)}\rangle| \\
&= |\langle Ah_{max(s)}, Ah_{max(s)}\rangle + \langle Ah_{-max(s)}, Ah_{max(s)}\rangle| \\
&\geq (1-\mu_1(s-1))\|h_{max(s)}\|_2^2 - |\langle Ah_{-max(s)}, Ah_{max(s)}\rangle| \\
&\geq (1-\mu_1(s-1))\|h_{max(s)}\|_2^2 - (\frac{\|h_{max(s)}\|_2}{\sqrt{s}} + \\
&\frac{2\|x_{-max(s)}\|_1 + \|h\|_2}{s})\sqrt{s}\mu_1(2s-1)\|h_{max(s)}\|_2 \\
&= (1-\mu_1(s-1)-\mu_1(2s-1))\|h_{max(s)}\|_2^2 - \\
&(\frac{2\|x_{-max(s)}\|_1 + \|h\|_2}{s})\sqrt{s}\mu_1(2s-1)\|h_{max(s)}\|_2.
\end{aligned} \tag{4}$$

On the other hand,

$$\|Ah\|_2 \leq \|A\overline{x} - y\|_2 + \|Ax - y\|_2 \leq 2\epsilon.$$

This inequality combining Cauchy–Buniakowsky–Schwarz inequality and Lemma 1 yields

$$|\langle Ah, Ah_{max(s)}\rangle| \leq 2\epsilon\sqrt{(1+\mu_1(s-1))}\|h_{max(s)}\|_2. \tag{5}$$

It follows from (4) and (5) that

$$\begin{aligned}
&(1-\mu_1(s-1)-\mu_1(2s-1))\|h_{max(s)}\|_2 \\
&\leq \frac{2\|x_{-max(s)}\|_1 + \|h\|_2}{\sqrt{s}}\mu_1(2s-1) + 2\epsilon\sqrt{(1+\mu_1(s-1))}.
\end{aligned}$$

From condition (2), the above inequality can be simplified as

$$\begin{aligned}
&\|h_{max(s)}\|_2 \leq \frac{1}{1-\mu_1(s-1)-\mu_1(2s-1)} \\
&(\frac{2\|x_{-max(s)}\|_1 + \|h\|_2}{\sqrt{s}}\mu_1(2s-1) + 2\epsilon\sqrt{(1+\mu_1(s-1))}).
\end{aligned}$$

Applying [2, Lemma 5.5] on (4), we get

$$\|h_{-max(s)}\|_2 \leq \|h_{max(s)}\|_2 + \frac{2\|x_{-max(s)}\|_1 + \|h\|_2}{\sqrt{s}}.$$

It follows from the above two inequalities that

$$\begin{aligned}
&\|h\|_2 = \sqrt{\|h_{max(s)}\|_2^2 + \|h_{-max(s)}\|_2^2} \\
&\leq \sqrt{\|h_{max(s)}\|_2^2 + (\|h_{max(s)}\|_2 + \frac{2\|x_{-max(s)}\|_1 + \|h\|_2}{\sqrt{s}})^2} \\
&\leq \sqrt{2}\|h_{max(s)}\|_2 + \frac{2\|x_{-max(s)}\|_1 + \|h\|_2}{\sqrt{s}} \\
&\leq (\frac{\sqrt{2}}{1-\mu_1(s-1)-\mu_1(2s-1)})(\frac{2\|x_{-max(s)}\|_1 + \|h\|_2}{\sqrt{s}}\mu_1(2s-1) \\
&+2\epsilon\sqrt{1+\mu_1(s-1)}) + \frac{2\|x_{-max(s)}\|_1 + \|h\|_2}{\sqrt{s}}.
\end{aligned} \tag{6}$$

Applying condition (2) to inequality (6),

$$\begin{aligned}
&\|h\|_2 \leq \frac{\sqrt{s}(1-\mu_1(s-1)-\mu_1(2s-1))}{\sqrt{s}-1+(1-\sqrt{s})\mu_1(s-1)+(1-\sqrt{s}-\sqrt{2})\mu_1(2s-1)} \\
&(\frac{2-2\mu_1(s-1)+(2\sqrt{2}-2)\mu_1(2s-1)}{\sqrt{s}(1-\mu_1(s-1)-\mu_1(2s-1))}\|x_{-max(s)}\|_1 \\
&+\frac{2\sqrt{2}\epsilon\sqrt{1+\mu_1(s-1)}}{1-\mu_1(s-1)-\mu_1(2s-1)}).
\end{aligned}$$

Hence, we have

$$\begin{aligned}
&\|h\|_2 \leq \frac{2(1-\mu_1(s-1)+(\sqrt{2}-1)\mu_1(2s-1))}{\sqrt{s}-1-\gamma(s)}\|x_{-max(s)}\|_1 \\
&+\frac{2\sqrt{2s(1+\mu_1(s-1))}}{\sqrt{s}-1-\gamma(s)}\epsilon.
\end{aligned}$$

□

From the conclusion of Theorem 1, we can easily get the following result.

Theorem 2 *Assume $\epsilon = 0$ in model (1), if the cumulative coherence of the measurement matrix A satisfies (2), then ℓ_{1-2}-minimization problem can accurately recover any $s-$sparse vector.*

Theorem 1 requires two cumulative coherence parameters to ensure that model (1) can stably recover sparse vectors. Whether it is possible to include only one cumulative coherence parameter to ensure that the sparse vector can be recovered stably via model (1). The following theorem gives a positive answer. Before giving the relevant theorem, we need to give a lemma.

Lemma 4 *For $s \geq 2$, if*

$$\mu_1(2s-1) \leq \frac{(\sqrt{2}-1)\sqrt{s}+1-\sqrt{2}}{\sqrt{s}+3-2\sqrt{2}},$$

then

$$\sqrt{s}-1+(1-\sqrt{2})\mu_1(2s-1)-(1+\sqrt{2})\sqrt{s}\mu_1(2s-1) > 0,$$

$$\mu_1(2s-1) \leq \sqrt{2}-1.$$

Proof: Setting $f(s) = \frac{(\sqrt{2}-1)\sqrt{s}+1-\sqrt{2}}{\sqrt{s}+3-2\sqrt{2}}$, we can easily assume that $f(s)$ monotonically increases. Hence, we have

$$\mu_1(2s-1) \leq \lim_{s\to+\infty}\frac{(\sqrt{2}-1)\sqrt{s}+1-\sqrt{2}}{\sqrt{s}+3-2\sqrt{2}} = \sqrt{2}-1.$$

In the first part of the result, it is sufficient to prove that

$$(1-\sqrt{2}-(1+\sqrt{2})\sqrt{s})\mu_1(2s-1) \geq 1-\sqrt{s}.$$

Hence, it is sufficient to prove

$$\mu_1(2s-1) \leq \frac{\sqrt{s}-1}{(1+\sqrt{2})\sqrt{s}-(1-\sqrt{2})} = \frac{(\sqrt{2}-1)\sqrt{s}+1-\sqrt{2}}{\sqrt{s}+3-2\sqrt{2}}.$$

□

Theorem 3 *For $s \geq 2$, assume that*

$$\mu_1(2s-1) \leq \frac{(\sqrt{2}-1)\sqrt{s}+1-\sqrt{2}}{\sqrt{s}+3-2\sqrt{2}}, \tag{7}$$

then the solution $\overline{x}$ of (1) and the original signal x obeys

$$\begin{aligned}
&\|\overline{x}-x\|_2 \leq \frac{4\sqrt{s(1+\mu_1(2s-1))}\epsilon}{\sqrt{s}-1+(1-\sqrt{2})\mu_1(2s-1)-(1+\sqrt{2})\sqrt{s}\mu_1(2s-1)} \\
&+\frac{2+(2\sqrt{2}-2)\mu_1(2s-1)}{\sqrt{s}-1+(1-\sqrt{2})\mu_1(2s-1)-(1+\sqrt{2})\sqrt{s}\mu_1(2s-1)}\|x_{T_0^C}\|_1.
\end{aligned}$$

Proof: Set $h = \overline{x} - x$ and decompose h into the sum of vectors $h_{T_0}, h_{T_1}, h_{T_2}, ...$, with each sparsity of these vectors at s, and the sparsity of the last vector being less than s. Here, T_0 corresponds to the locations of the s largest coefficients of x, and T_1 to the locations of the s largest coefficients of $h_{T_0^C}$, and T_2 to the locations of the next s largest coefficients of $h_{T_0^C}$. Now, note that for each $j \geq 2$,

$$\|h_{T_j}\|_2 \leq \sqrt{s}\|h_{T_j}\|_\infty \leq s^{-\frac{1}{2}}\|h_{T_{j-1}}\|_1,$$

and thus

$$\sum_{j\geq 2}\|T_{T_j}\|_2 \leq s^{-\frac{1}{2}}(\|h_{T_1}\|_1 + \|h_{T_2}\|_1 + ...) \leq s^{-\frac{1}{2}}\|h_{T_0^C}\|_1. \quad (8)$$

This gives the useful estimation

$$\|h_{(T_0\cup T_1)^C}\|_2 = \|\sum_{j\geq 2} h_{T_j}\|_2 \leq \sum_{j\geq 2}\|h_{T_j}\|_2 \leq s^{-\frac{1}{2}}\|h_{T_0^C}\|_1. \quad (9)$$

From the definition of $\overline{x}$ and h, we have

$$\|x\|_1 - \|x\|_2 \geq \|x+h\|_1 - \|x+h\|_2.$$

Thus,

$$\|h\|_2 + \|x\|_1 \geq \|x+h\|_2 - \|x\|_2 + \|x\|_1 \geq \|x+h\|_1.$$

Additionally,

$$\begin{aligned}\|x+h\|_1 &= \|(x+h)_{T_0}\|_1 + \|(x+h)_{T_0^C}\|_1 \\ &\geq \|x_{T_0}\|_1 - \|h_{T_0}\|_1 + \|h_{T_0^C}\|_1 - \|x_{T_0^C}\|_1.\end{aligned}$$

Combining the above two inequalities yield

$$\|h_{T_0^C}\|_1 \leq \|h_{T_0}\|_1 + 2\|x_{T_0^C}\|_1 + \|h\|_2. \quad (10)$$

Applying (10) and the Cauchy–Buniakowsky–Schwarz inequality to bound $\|h_{T_0}\|_1$ by $\sqrt{s}\|h_{T_0}\|_2$, (9) yields

$$\|h_{(T_0\cup T_1)^C}\|_2 \leq \|h_{T_0}\|_2 + s^{-\frac{1}{2}}(2\|x_{T_0^C}\|_1 + \|h\|_2). \quad (11)$$

We observe that $Ah_{T_0\cup T_1} = Ah - \sum_{j\geq 2} Ah_{T_j}$, therefore

$$\|Ah_{T_0\cup T_1}\|_2^2 = \langle Ah_{T_0\cup T_1}, Ah\rangle - \langle Ah_{T_0\cup T_1}, \sum_{j\geq 2} Ah_{T_j}\rangle,$$

$$\|Ah\|_2 = \|A(\overline{x} - x)\|_2 \leq \|A\overline{x} - y\|_2 + \|Ax - y\|_2 \leq 2\epsilon.$$

It follows from the above inequality and Lemma 1 that

$$\begin{aligned}|\langle Ah_{T_0\cup T_1}, Ah\rangle| &\leq \|Ah_{T_0\cup T_1}\|_2\|Ah\|_2 \\ &\leq 2\epsilon\sqrt{1+\mu_1(2s-1)}\|h_{T_0\cup T_1}\|_2.\end{aligned}$$

Moreover, it follows from Lemma 3 that $|\langle Ah_{T_0}, Ah_{T_j}\rangle| \leq \mu_1(2s-1)\|h_{T_0}\|_2\|h_{T_j}\|_2$, and similarly, for T_1 instead of T_0. Consequntly, $\|h_{T_0}\|_2 + \|h_{T_1}\|_2 \leq \sqrt{2}\|h_{T_0\cup T_1}\|_2$ for T_0 and T_1 are disjoint.

$$\begin{aligned}&(1-\mu_1(2s-1))\|h_{T_0\cup T_1}\|_2^2 \leq \|Ah_{T_0\cup T_1}\|_2^2 \\ &\leq \|h_{T_0\cup T_1}\|_2(2\epsilon\sqrt{1+\mu_1(2s-1)} + \sqrt{2}\mu_1(2s-1)\sum_{j\geq 2}\|h_{T_j}\|_2).\end{aligned}$$

Therefore, (8) and Lemma 4 give

$$\|h_{T_0\cup T_1}\|_2 \leq \frac{2\epsilon\sqrt{1+\mu_1(2s-1)}}{1-\mu_1(2s-1)} + \frac{\sqrt{\frac{2}{s}}\mu_1(2s-1)\|h_{T_0^C}\|_1}{1-\mu_1(2s-1)}.$$

It follows from this last inequality and (10) that

$$\begin{aligned}&\|h_{T_0\cup T_1}\|_2 \leq \frac{2\epsilon\sqrt{1+\mu_1(2s-1)}}{1-\mu_1(2s-1)} + \\ &\frac{\sqrt{\frac{2}{s}}\mu_1(2s-1)}{1-\mu_1(2s-1)}(\sqrt{s}\|h_{T_0}\|_2 + 2\|x_{T_0^C}\|_1 + \|h\|_2) \\ &\leq \frac{2\epsilon\sqrt{1+\mu_1(2s-1)}}{1-\mu_1(2s-1)} + \frac{\sqrt{2}\mu_1(2s-1)}{1-\mu_1(2s-1)}\|h_{T_0\cup T_1}\|_2 \\ &+ \frac{\sqrt{\frac{2}{s}}\mu_1(2s-1)}{1-\mu_1(2s-1)}(2\|x_{T_0^C}\|_1 + \|h\|_2).\end{aligned}$$

So from Lemma 4, we have

$$\begin{aligned}\|h_{T_0\cup T_1}\|_2 &\leq \frac{2\epsilon\sqrt{1+\mu_1(2s-1)}}{1-(1+\sqrt{2})\mu_1(2s-1)} \\ &+ \frac{\sqrt{\frac{2}{s}}\mu_1(2s-1)}{1-(1+\sqrt{2})\mu_1(2s-1)}(2\|x_{T_0^C}\|_1 + \|h\|_2).\end{aligned}$$

It follows from the above inequality and (11) that

$$\begin{aligned}&\|h\|_2 \leq \|h_{T_0\cup T_1}\|_2 + \|h_{(T_0\cup T_1)^C}\|_2 \\ &\leq 2\|h_{T_0\cup T_1}\|_2 + \frac{2}{\sqrt{s}}\|x_{T_0^C}\|_1 + \frac{1}{\sqrt{s}}\|h\|_2 \\ &\leq \frac{4\epsilon\sqrt{1+\mu_1(2s-1)}}{1-(1+\sqrt{2})\mu_1(2s-1)} + (\frac{4\sqrt{\frac{2}{s}}\mu_1(2s-1)}{1-(1+\sqrt{2})\mu_1(2s-1)} + \frac{2}{\sqrt{s}})\|x_{T_0^C}\|_1 + \\ &(\frac{2\sqrt{\frac{2}{s}}\mu_1(2s-1)}{1-(1+\sqrt{2})\mu_1(2s-1)} + \frac{1}{\sqrt{s}})\|h\|_2.\end{aligned}$$

So from Lemma 4, we have

$$\begin{aligned}\|h\|_2 \leq &\frac{4\sqrt{s(1+\mu_1(2s-1))}\epsilon}{\sqrt{s}-1+(1-\sqrt{2})\mu_1(2s-1)-(1+\sqrt{2})\sqrt{s}\mu_1(2s-1)} + \\ &\frac{2+(2\sqrt{2}-2)\mu_1(2s-1)}{\sqrt{s}-1+(1-\sqrt{2})\mu_1(2s-1)-(1+\sqrt{2})\sqrt{s}\mu_1(2s-1)}\|x_{T_0^C}\|_1.\end{aligned}$$

□

From the conclusion of Theorem 3, we can easily get the following result.

Theorem 4 *Assume $\epsilon = 0$ in model* (1)*, if the cumulative coherence of the measurement matrix A satisfies* (7)*, then ℓ_{1-2}-minimization problem can accurately recover any $s-$sparse vector.*

4 Conclusion

From this paper, we find that based on some condition of cumulative coherence, the ℓ_{1-2}-minimization problem can exactly recover s-sparse signals in noiseless cases and stably recover s-sparse signals in the noise cases.

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