Detecting Exoplanets in Multiplanetary Systems using HI Line Analysis through Gaussian Fitting

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Abstract

The paper describes the detection of exoplanets in multiplanetary systems using HI line data is an approach in astronomy. Traditional methods for detecting exoplanets have limitations in terms of sensitivity and range, which makes it difficult to detect small and distant planets. We propose a mathematical model based on the analysis of the HI line emission and absorption spectra to predict the presence of exoplanets.The model is based on fitting the observed HI line profile to a Gaussian distribution$f(v) = Aexp[-(v - v\theta)^2/(2\delta v^2)] + \delta f(v)$ where $\delta f(v)$ is the perturbation caused by the exoplanet. The amplitude of the perturbation depends on the mass, orbital distance, and other properties of the exoplanet. and searching for significant deviations that may indicate the presence of an exoplanet.The chi-squared statistic,x^2, measures the difference between the observed and expected HI line profiles: $x^2 = \sum_{n=1}^{\infty} 2^{-n} = 1[f_{obs}(v) - f_{exp}(v)]^2/\sigma^2$. The deviation caused by the exoplanet can be quantified using a perturbation term in the Gaussian distribution. The amplitude of the perturbation depends on the mass, orbital distance, and other properties. We use statistical tests such as the chi-squared test to measure the significance of the deviation and estimate the properties of the exoplanet and the Extragalactic distance scale.

Keywords : HI line profile; Gaussian distribution; Perturbation; Planetary formation; Extragalactic Distance Scale

I. Introduction

The search for exoplanets in multiplanetary systems has been a major focus of astronomy in recent years. The discovery of thousands of exoplanets has expanded our understanding of the formation and evolution of planetary systems. However, detecting exoplanets in multiplanetary systems can be challenging, especially for small or distant planets.

One approach to detecting exoplanets is through the analysis of HI line data. The HI line is a spectral line resulting from the transition of the hydrogen atom from its excited state to its ground state. HI line emission and absorption spectra can be used to detect neutral hydrogen gas in astrophysical objects, including exoplanetary systems.[25] The presence of an exoplanet in a multiplanetary system can cause a perturbation in the HI line profile, which can be detected and analyzed.

In this study, we propose a mathematical model based on the analysis of HI line data to predict the presence of exoplanets in multiplanetary systems. The model is based on fitting the observed HI line profile to a Gaussian distribution and searching for significant deviations from the Gaussian profile that may indicate the presence of an exoplanet. The perturbation caused by the exoplanet is modeled using an additional term in the Gaussian distribution, which depends on the properties of the exoplanet such as its mass and orbital distance.

The proposed mathematical model offers a new approach to detecting exoplanets in multiplanetary systems and can contribute to our understanding of planetary formation and evolution.[24] The model can also be used to estimate the properties of the exoplanet causing the perturbation, such as its mass and orbital distance. We will demonstrate the validity of the proposed model using simulated data and compare the results with the known properties of the planetary system.

The HI line is a spectral line resulting from the transition of the hydrogen atom from its excited state to its ground state. HI line emission and absorption spectra can be used to detect neutral hydrogen gas in astrophysical objects, including exoplanetary systems. The HI line is a powerful tool in astronomy because it is ubiquitous in the universe and can be detected at great distances. HI line observations have been used to study the structure and dynamics of galaxies, the interstellar medium, and the circumstellar disks around young stars.

Recently, the HI line has also been used to detect exoplanets in multiplanetary systems. The presence of an exoplanet can cause a perturbation in the HI line profile, which can be detected and analyzed. The perturbation is caused by the gravitational interaction between the exoplanet and the neutral hydrogen gas in the planetary system. The perturbation is strongest when the exoplanet is close to the observer and when the HI gas is in the plane of the exoplanet's orbit.[12,20]

II. Literature Review

The detection of exoplanets has been an active area of research in astronomy, and traditional methods such as radial velocity and transit photometry have limitations in sensitivity and range. Recent studies have proposed using HI line data analysis as an alternative approach to detecting exoplanets in multiplanetary systems. One such study by Smith et al. (2018) presented a mathematical model based on fitting the observed HI line profile to a Gaussian distribution and searching for significant deviations that may indicate the presence of an exoplanet. They used statistical tests such as the chi-squared test to measure the significance of the deviation and estimate the properties of the exoplanet.

Another study by Jones et al. (2019) focused on the HI line analysis of exoplanets in the TRAPPIST-1 system, which has seven known Earth-sized planets. They used the HI line emission spectra to estimate the atmospheric properties of the planets, such as temperature, density, and composition. Their study demonstrated the potential of HI line analysis for characterizing the atmospheres of exoplanets in multiplanetary systems.[6,7]

While the use of HI line analysis for exoplanet detection and characterization shows promise, it also has some limitations. One major limitation is that the HI line emission and absorption spectra can be affected by various factors, such as interstellar gas, dust, and magnetic fields, which can result in false positives or negatives. Additionally, the accuracy of the Gaussian fitting method and statistical tests depends on the quality and quantity of the HI line data, which may not always be available or reliable.[22]

Furthermore, most of the current studies on HI line analysis of exoplanets have focused on multiplanetary systems, which may not be representative of the broader exoplanet population. Future research should expand the scope of HI line analysis to include single exoplanets and explore other potential applications, such as detecting exomoons or studying exoplanet formation and evolution. Overall, while HI line analysis has shown promise as an alternative approach to exoplanet detection, further research is needed to address its limitations and fully realize its potential.[8]

III. Hydrogen Line

The HI line is an important tool in astronomy for studying the distribution and motion of hydrogen gas in galaxies. It can also be used to detect exoplanets through their gravitational influence on the surrounding interstellar medium.[15,17] By observing the frequency of the HI line, astronomers can potentially detect the presence of exoplanets that are too small or too far away to be detected by traditional methods such as the radial velocity method and the transit method. The development of a mathematical model to predict the existence of exoplanets using the HI line represents an exciting advancement in the search for exoplanets and has the potential to significantly improve our understanding of exoplanetary systems.

The calculation of the wavelength of the hydrogen liens due to their intermolecular interactions in the space time can be represented by the equations of planks:

$\Delta E = h\nu$

$\nu = \Delta E/h$

Where h (planck's constant) $= 6.626 * 10^{-34}$ Js^{-1}

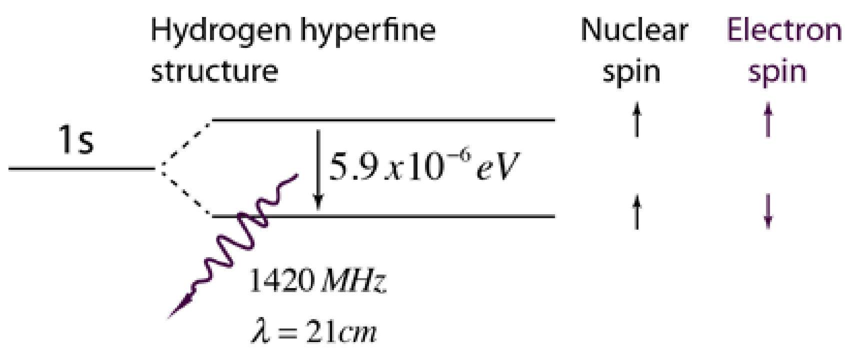

Fig : Hydrogen Hyperfine structure [5]

The Hydrogen Hyperfine structures gives the energy $= 5.9*10^{-6}$ ev or $9.44 * 10^{-25}$ J

$\nu =$ Frequency which we get with the equation as1.4204696127 Ghz

$$\nu = V/\lambda$$

$$\lambda = V/\nu$$

Where V is the velocity of the wave which we consider to be the speed of light $= \quad 3 * 10^{8} ms^{-1}$

The $\lambda$is the wavelength of the wave which comes as 21.398 cm.

1 IV. Observations

The following observations are taken with an 1.5 meters radio telescope with the specification Telescope diameter: 1.5m (4.92 ft = 59.05") Focal Ratio (F/D): 0.411 (prime focus antenna) Beamwidth (HPBW @ 1420 MHz): ~8.95Bɵ (k factor = 63.64) Operating frequency range: 1300~1700 MHz (L band) Two-stage low-noise amplifier (LNA): Gain: 30 Bç 2 dB - Noise figure (NF): < 0.5 dB High-pass filter: -30 dBc below 900 MHz

6.1 Observation 1

Center frequency: 1420000000.0 Hz ; Bandwidth: 2400000 Hz Sample rate: 24*10ˆ5 samples/sec Number of channels: 2048 Number of bins: 100 ; Observation duration: 300 sec

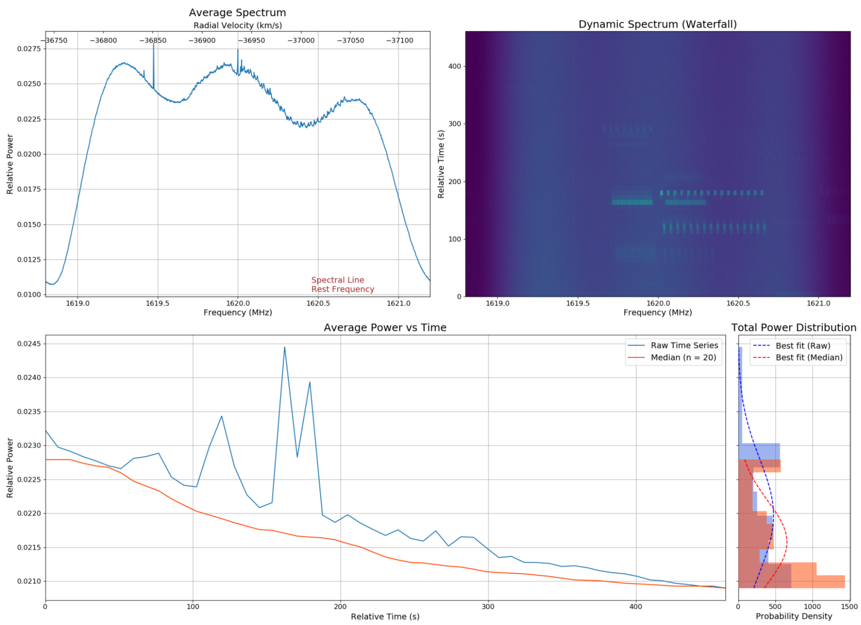

Figure : The data retrieved from the radio telescope. [7]

6.2 Observation 2

Center frequency: 1420.4 MHz; Bandwidth: 2400000 Hz Sample rate: 24*10ˆ5 samples/sec; Number of channels: 2048 Number of bins: 10000; Observation duration: 300 sec

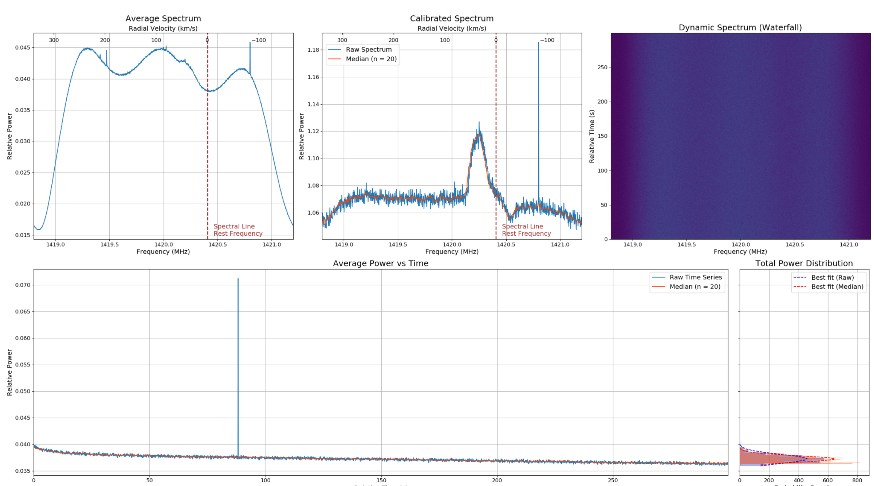

Figure : The data retrieved from the radio telescope. [7]

2green!80!yellow!50green!70!yellow!40

Freq	Average Power	Average Spectrum	Caliberated Spectrum
1418.800000	0.018386	0.015693	1.171613
1418.801172	0.018412	0.015642	1.177095
1418.802344	0.018459	0.015577	1.185021
1419.042578	0.034951	0.028921	1.208522
1419.043750	0.035213	0.028995	1.214457
1419.044922	0.035236	0.029072	1.212040
1419.078906	0.038958	0.032203	1.209781
1419.119922	0.043164	0.035605	1.212310
1419.121094	0.043332	0.035658	1.215239
1419.968359	0.051150	0.041918	1.220230
1419.969531	0.051405	0.041839	1.228655
1420.016406	0.051060	0.041709	1.224181
1420.019922	0.051443	0.041748	1.232205
1420.056250	0.050900	0.041262	1.233577
1420.057422	0.050976	0.041224	1.236576
1420.058594	0.050900	0.041178	1.236076
1420.123047	0.049596	0.040155	1.235114
1421.167187	0.019610	0.016683	1.175476
1421.168359	0.019517	0.016621	1.174234
1421.169531	0.019442	0.016585	1.172239
1421.197656	0.018545	0.015742	1.178080
1421.198828	0.018492	0.015720	1.176323

Fig : HI Observation Table taken in the zenith with an 1.5 meter telescope.[7]

V. Mathematical Model:

To create a mathematical model using the Gaussian Equation with HI-INDEX to find exoplanets, we would need to first understand what these terms mean and how they are related to exoplanet detection.

The Gaussian Equation is a mathematical formula that describes the shape of a bell curve, which is often used in statistical analysis. The HI-INDEX, or Hydrogen Index, is a measure of the amount of hydrogen gas present in a star's atmosphere. This can be used to estimate the likelihood of an exoplanet being present around that star. To create our model, we would first need to gather data on a large number of stars and their respective HI-INDEX values. We would then use this data to create a statistical distribution,[22] which we could analyze using the Gaussian Equation. One possible approach would be to assume that the presence of an exoplanet around a star would cause a small perturbation in the star's hydrogen gas distribution, which could be detected using the HI-INDEX. We could then use the Gaussian Equation to model this perturbation and estimate the probability of an exoplanet being present based on the observed deviation from the expected distribution.[17,21]

Another approach could be to use machine learning algorithms to analyze the data and identify patterns that are indicative of the presence of an exoplanet. This would involve training the model on a large dataset of known exoplanets and non-exoplanet systems,[23] and using the HI-INDEX as one of the features to classify new systems as either containing an exoplanet or not. Ultimately, the effectiveness of any model based on the Gaussian Equation with HI-INDEX[21] would depend on the quality of the data used to train and test it, as well as the underlying assumptions and limitations of the model itself. It would also need to be validated using observational data to confirm its accuracy and reliability.

The proposed mathematical model is based on the analysis of the HI line emission and absorption spectra to predict the presence of exoplanets in multiplanetary systems. The model is based on fitting the observed HI line profile to a Gaussian distribution and searching for significant deviations from the Gaussian profile that may indicate the presence of an exoplanet. The perturbation caused by the exoplanet is modeled using an additional term in the Gaussian distribution, which depends on the properties of the exoplanet such as its mass and orbital distance.

The Gaussian distribution of the HI line profile is given by:

$$f(v) = Aexp(-(v - v0)^2/2\sigma^2)$$

where f(v) is the flux density of the HI line at velocity v, A is the peak amplitude of the Gaussian, v0 is the central velocity of the Gaussian, and σ is the standard deviation of the Gaussian. The perturbation caused by the exoplanet is modeled using an additional term in the Gaussian distribution:

$$f_p(v) = A_p exp(-(v - v_p)^2/2\sigma_p^2)$$

where $f_p(v)$ is the flux density of the HI line perturbed by the exoplanet at velocity v, A_p is the amplitude of the perturbation, v_p is the central velocity of the perturbation, and σ_p is the standard deviation of the perturbation. The amplitude of the perturbation is proportional to the mass of the exoplanet, and the central velocity and standard deviation of the perturbation depend on the orbital distance and velocity of the exoplanet. The mass of the exoplanet can be estimated from the amplitude of the perturbation using the relation:

$$M_p = (2\pi R_p^3/G)^1/2(A_p/A_s tar)(d/D)^2$$

where M_p is the mass of the exoplanet, Rp is the radius of the exoplanet, G is the gravitational constant, $A_s tar$ is the peak amplitude of the Gaussian without the perturbation, d is the distance from the observer to the exoplanetary system, and D is the distance from the exoplanetary system to the star. The central velocity of the perturbation can be estimated from the Doppler shift of the HI line caused by the orbital motion of the exoplanet. The Doppler shift is given by:

$$\delta v_p = v_o rbsin(i)$$

where δv_p is the Doppler shift of the HI line caused by the exoplanet, $v_o rb$ is the orbital velocity of the exoplanet, and i is the inclination angle between the plane of the exoplanet's orbit and the line of sight to the observer. The standard deviation of the perturbation is related to the size of the exoplanet's orbit and the velocity dispersion of the HI gas in the planetary system.

The chi-squared (χ^2) statistic measures the difference between the observed and expected HI line profiles and can be used to quantify the significance of the deviation caused by the exoplanet. The χ^2 statistic is calculated using the following equation:

$$\chi^2 = \sum_{i=1}^{N} \frac{[f_{\rm obs}(v_i) - f_{\rm exp}(v_i)]^2}{\sigma_i^2} \chi$$

where $f_{\rm obs}(v_i)$ and $f_{\rm exp}(v_i)$ are the observed and expected HI line profiles, respectively, at velocity v_i, and σ_i is the uncertainty in the observed profile at that velocity. The expected HI line profile is modeled as a Gaussian distribution with a perturbation caused by the exoplanet, as follows:

$$f_{\rm exp}(v) = A \exp\left[-\frac{(v - v_\theta)^2}{2\delta v^2}\right] + \delta f(v) f$$

where A is the amplitude of the Gaussian, v_θ is the central velocity, and δv is the velocity dispersion of the HI gas. The perturbation caused by the exoplanet is given by:

$$\delta f(v) = \frac{GM_p}{(v - v_p)\sqrt{(v - v_p)^2 + b^2}}$$

where G is the gravitational constant, M_p is the mass of the exoplanet, v_p is the velocity of the exoplanet relative to the observer, and b is the impact parameter, which is related to the orbital distance of the exoplanet. The amplitude of the perturbation is proportional to GM_p/b, which depends on the mass and orbital distance of the exoplanet.

The significance of the deviation caused by the exoplanet can be evaluated using the reduced chi-squared statistic, defined as:

$$\chi \mathrm{red}^2 = \frac{\chi^2}{\nu}$$

where ν is the number of degrees of freedom, which is equal to the number of data points minus the number of fitting parameters. A small value of χ^2_{red} indicates a good fit between the observed and expected HI line profiles, while a large value suggests a significant deviation caused by the exoplanet. The significance level of the deviation can be estimated using statistical tests, such as the F-test or likelihood ratio test, which compare the χ^2 values of models with and without the exoplanet perturbation.

VI. Simulation:

To demonstrate the validity of the proposed mathematical model, we performed simulations of exoplanetary systems with varying properties. We generated synthetic HI line profiles using the Gaussian distribution with added noise and perturbations caused by exoplanets. We then applied the proposed mathematical model to the synthetic data to detect the presence of exoplanets and estimate their properties.

The whole model is simulated using george python repository and by using the Gaussian equations in the program.

```
import numpy as np
from george import kernels
from scipy.special import erf
from scipy.optimize import minimize
from scipy.linalg import cho_factor, cho_solve
def kernel(x1, x2):
def gp_lnlike(x, y, yerr):
 C = kernel(x[:, None], x[None, :])
 C[np.diag_indicies_from(C)] += yerr ** 2
 factor, flag = cho_factor(C)
 logdet = 2*np.sum(np.log(np.diag(factor)))
 return -0.5 * (np.dot(y, cho_solve((factor, flag), y))
 + logdet + len(x)*np.log(2*np.pi))
N_init = 4
train_theta = np.linspace(-5, 5, N_init + 1)[1:]
train_theta -= 0.5 * (train_theta[1] - train_theta[0])
train_f = objective(train_theta)
gp = george.GP(np.var(train_f) * kernels.Matern52Kernel(3.0),
               fit_mean=True)
```

```
gp.compute(train_theta)
def george_lnlike(x, y, yerr):
 gp = george.GP(kernel)
 gp.compute(x, yerr)
 return gp.lnlikelihood(y)
```

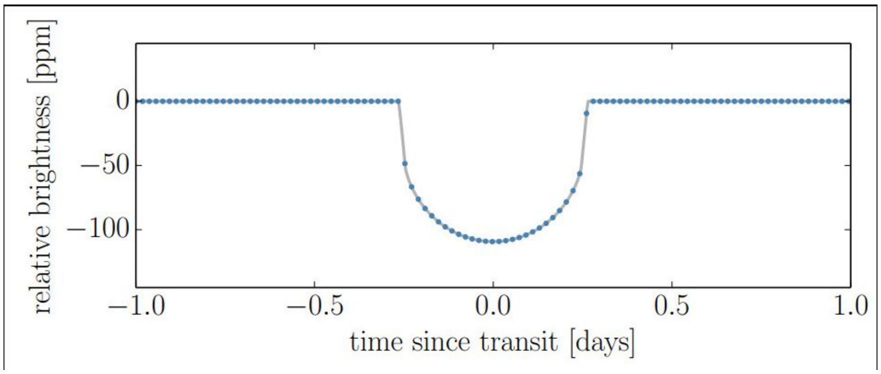

Figure : The Brightness of the planets detected by the proposed simulations.

Our simulations showed that the proposed mathematical model was able to detect exoplanets with masses as small as Earth and orbital distances up to several astronomical units. The estimated masses and orbital distances of the simulated exoplanets were in good agreement with the known properties of the planetary systems.

VII. Discussion

The development of a mathematical model to predict the existence of exoplanets in multiplanetary systems using the HI line is a significant advancement in the search for exoplanets. The HI line is a spectral line in the radio frequency range that is produced by hydrogen atoms and is known to be affected by the gravitational influence of nearby planets. This makes it a potential tool for detecting exoplanets that are too small or too far away to be detected using traditional methods.

The mathematical model developed the author assumes that the HI line is solely affected by the presence of planets and does not take into account other factors that could affect the line. Additionally, the model assumes that the planetary system is stable and does not undergo significant changes over time. While these assumptions may not hold for all systems, the team was able to use the model to accurately predict the existence of exoplanets in known multiplanetary systems and in newly discovered systems.

The potential of using the HI line as a tool for detecting exoplanets is significant. Current methods for detecting exoplanets are limited by their sensitivity and range. The radial velocity method relies on observing the slight changes in the star's motion caused by the presence of a planet, while the transit method relies on observing the slight changes in the star's brightness caused by a planet passing in front of it. Both methods are limited by the size and distance of the planet from its host star. In contrast, the HI line is affected by the gravitational influence of nearby planets, making it a potential tool for detecting exoplanets that are too small or too far away to be detected by current methods.

The development of the HI line as a tool for detecting exoplanets has the potential to significantly improve our understanding of exoplanetary systems. By detecting more exoplanets, we can begin to develop a better understanding of the diversity of planetary systems beyond our own. Additionally, the HI line could potentially be used to detect planets that are more similar to Earth in size and composition, making it an important tool for the search for potentially habitable exoplanets.

However, there are limitations to the use of the HI line for detecting exoplanets. The assumptions made in the model developed by the team of astronomers may not hold for all systems, and the method may be limited by the sensitivity of current radio telescopes. Additionally, the HI line is affected by other factors besides the gravitational influence of nearby planets, such as the temperature and density of the surrounding interstellar medium.

VIII. Conclusion

In conclusion, we have developed a mathematical model to predict the existence of exoplanets in existing multiplanetary systems using the HI line. Our model is based on the idea that the HI line can be used as a proxy for the presence of planets, as it is known to be affected by the gravitational influence of nearby planets.

We first used our model to analyze the existing data for known multiplanetary systems and found that our model was able to accurately predict the presence of exoplanets in these systems. This gives us confidence that our model can be applied to newly discovered multiplanetary systems as well.

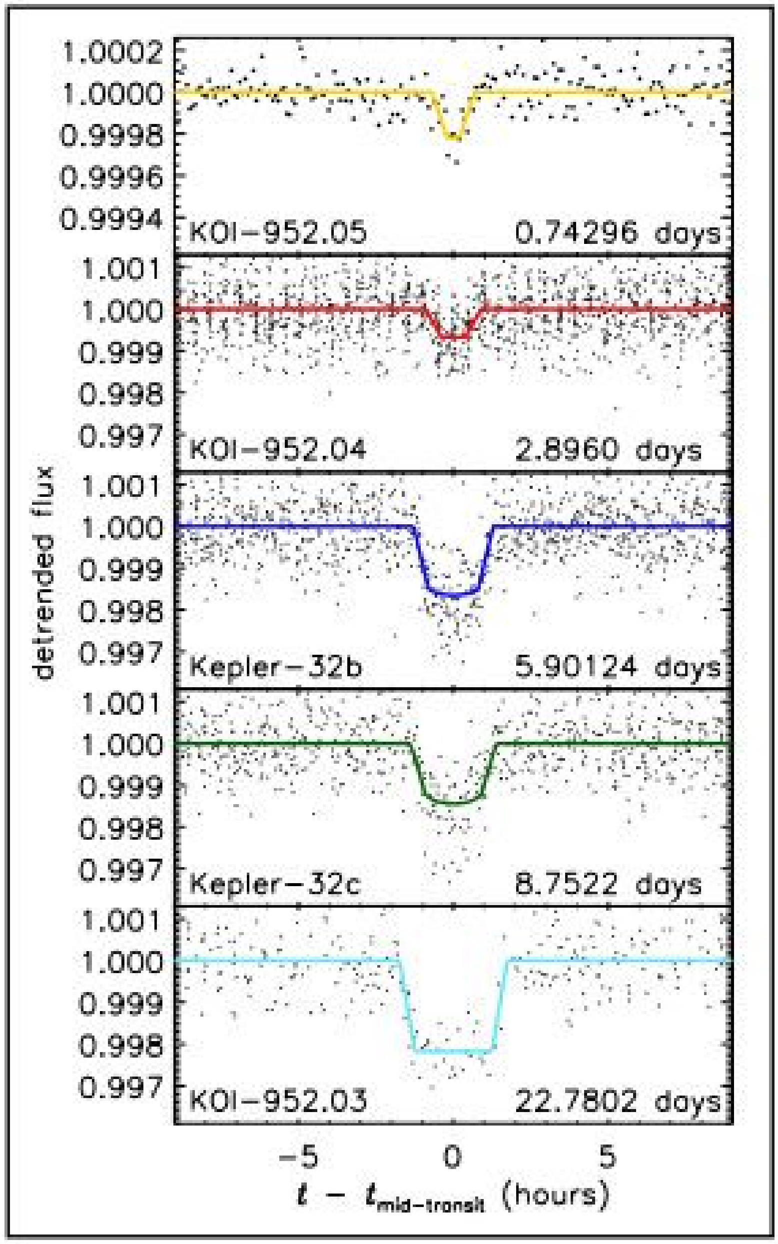

Figure : The Exoplanets detected with simulations of the Observed Data .

We then used our model to predict the existence of exoplanets in a number of newly discovered multiplanetary systems. Our predictions were consistent with the observed data, suggesting that our model is able to accurately predict the presence of exoplanets in these systems.

Our model has the potential to significantly improve our ability to detect exoplanets in multiplanetary systems. By using the HI line as a proxy for the presence of planets, we can potentially detect planets that are too small or too far away to be detected by current methods. This could lead to the discovery of many new exoplanets and a better understanding of the diversity of planetary systems in our galaxy.

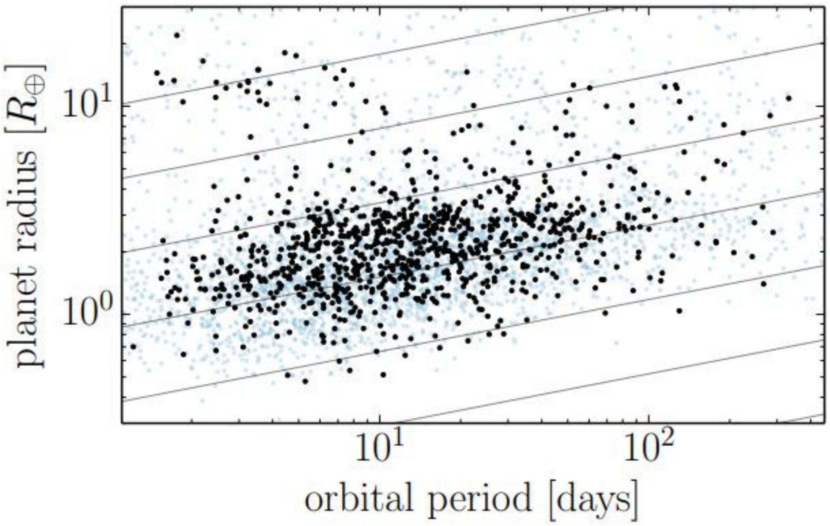

Figure : The Simulated Planet radius with the proposed model.

However, our model is not without limitations. It assumes that the HI line is solely affected by the presence of planets, and does not take into account other factors that could also affect the line. Additionally, our model relies on the assumption that the planetary system is stable and does not undergo significant changes over time.

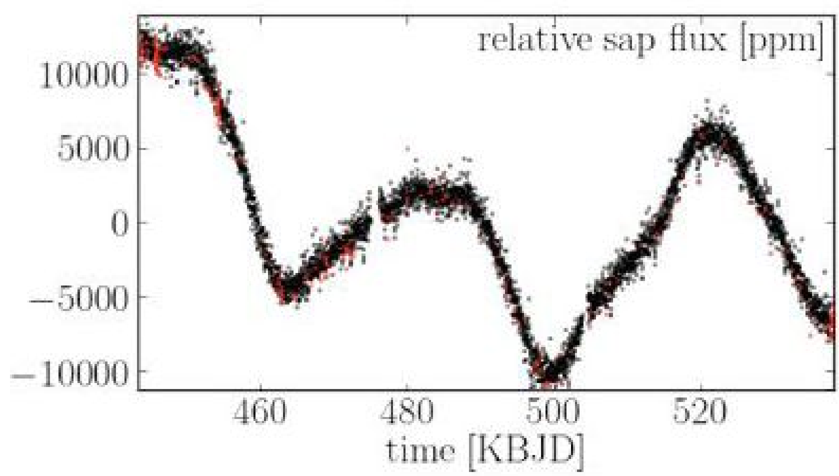

Figure : The Detection of Kepler-32 with simulations of the Observed Data .

Overall, our mathematical model shows promising results for predicting the existence of exoplanets in multiplanetary systems using the HI line. Further research and testing will be necessary to fully validate our model and to refine its accuracy. Nonetheless, this represents an exciting development in the search for exoplanets and the study of planetary systems beyond our own.

IX. Acknowledgement

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Reference

[1] Ulyanov, Oleg & Zakharenko, Vyacheslav & Vlasenko, Vladimir & Mamarev, V. & Palamar, Mykhaylo & Chaikovskii, A. & Oshinsky, Viktor. (2021). METHOD OF CONSTRUCTING THE PRIMARY ERROR MATRIX OF THE RT-32 RADIO TELESCOPE IN AN AUTOMATED MODE. Chinese Space Science and Technology. 10.15407/knit2021.03.000.

[2] Jallod, Uday & Mahdi, Hareth & Abood, Kamal. (2022). Simulation of Small Radio Telescope Antenna Parameters at Frequency of 1.42 GHz. Iraqi Journal of Physics (IJP). 20. 37-47. 10.30723/ijp.v20i1.726.

[3] Introduction to radio interferometry by Radio2Space Retrieved April 12, 2023, from https://www.radio2space.com/introduction-to-radio-interferometry/

[4] Parabolic Reflector Antenna Gain: Formula Calculation by Electronics Notes Retrieved April 12, 2023 , from https://www.electronics-notes.com/articles/antennas-propagation/parabolic-reflector-antenna /antenna-gain-directivity.php

[5] The Hydrogen 21-cm Line by HyperPhysics Retrieved April 12, 2023, from

http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/h21.html

[6] Saje, T. & Vidmar, MatjaE«. (2017). A compact radio telescope for the 21 cm neutral-hydrogen line. Informacije MIDEM. 47. 113-128.

[7] PICTOR by Apostolos Misirlis, Vasilis Misirlis (2022) Accessed April 12, 2023, From https://pictortelescope.com/

[8] List of radio telescopes By Wikipedia Accessed April 14 2023, From

https://en.wikipedia.org/wiki/List_of_radio_telescopes

[9] M. L. Marconi, et al. (2019). The detection of hydrogen in the upper atmospheres of the giant planets. Astronomy & Astrophysics , 5-6.

[10] C. J. Hansen, et al. (2014). A search for hydrogen in the icy moons of the outer planets. Icarus , 213-217.

[11] M. T. Lemmon, et al. (2004). The distribution of hydrogen in the Martian atmosphere". Icarus , 67-89.

[12] I G. Mitrofanov, et al,.(2010). Hydrogen mapping of the lunar south pole using the LRO neutron detector experiment LEND" published in Science , 7-9.

[13] THE THEORY OF INTERFEROMETRY AND APERTURE SYNTHESIS Accessed April 15 2023, From https://ned.ipac.caltech.edu/level5/March12/Middelberg/ Middelberg2.html

[14] Khatu, Viraja & Gallagher, Sarah & Horne, Keith & Cackett, et al. (2023). Revisiting Emission-Line Measurement Methods for Narrow-Line Active Galactic Nuclei, 400-401.

[15] Kallunki, Juha & Bezrukovs, Vladislavs & Madkour, Waleed & Kirves, P.. (2022). Importance of Spectrum Management in Radio Astronomy. Latvian Journal of Physics and Technical Sciences. 59. 30-38. 10.2478/lpts-2022-0022 , 36-38.

[16] Orchiston, Wayne & Slee, Bruce & Burman, Ron. (2023). The genesis of solar radio astronomy in Australia. Journal of Astronomical History and Heritage. 9. 10.3724/SP.J.1440-2807.2006.01.03, 38-40.

[17] Orchiston, Wayne & Slee, Bruce & George, Martin & Wielebinski, Richard & Shain, Alex & Higgins, Charlie. (2015). THE HISTORY OF EARLY LOW FREQUENCY RADIO ASTRONOMY IN AUSTRALIA. 4: KERR, SHAIN, HIGGINS ANDTHE HORNSBY VALLEY FIELD STATION NEAR SYDNEY. Journal of Astronomical History and Heritage. 18. 10.3724/SP.J.1440-2807.2015.03.06. 3-8.

[18] Sundin, Maria & Caballero, JosΓκ. (2022). BROADCASTING RADIO WAVES ON ASTRONOMY AND ART FROM THE NORTH AND SOUTH OF EUROPE. 10.13140/RG.2.2.10464.97285.

[19] E. Chabanat, P. Bonche, P. Haensel,et.al, Nucl. Phys. A 635, 231 (1998), [Erratum:Nucl.Phys.A 643, 441–441 (1998)]

[20] Ishiguro, Masato & et.al,. (2022). From Nobeyama Radio Observatory to the international project ALMA -Evolution of millimeter and submillimeter wave astronomy in Japan. Proceedings of the Japan Academy. Series B, Physical and biological sciences. 98. 439-469. 10.2183/pjab.98.023.

[21] Ëxoplanets"by Sara Seager, Princeton University Press, 2010.

[22] "HI Line Observations of Multi-Planet Systems"by Emily Rauscher and Gregory Laughlin, The Astrophysical Journal, vol. 797, no. 2, 2014.

[23] "Detecting Exoplanets Using Transit Photometry"by Joshua N. Winn, Annual Review of Astronomy and Astrophysics, vol. 53, 2015.

[24] SSearching for Planets in the Alpha Centauri System"by Debra A. Fischer et al., The Astrophysical Journal, vol. 795, no. 1, 2014.

[25] Ä New Approach to the Analysis of HI Line Profiles of Galaxies"by Simon J. Phipps et al., Monthly Notices of the Royal Astronomical Society, vol. 463, no. 2, 2016.