Event triggered control of an islanded network of microgrids via hybrid dynamical systems

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Abstract

In this paper a distributed event-triggered control of an islanded network of microgrids is proposed. Microgrids are connected to their neighbors from the point of view of energy exchange and data communication level. The control objective consists of driving the state of charge of batteries to a desired attractor, managing the connections, and disconnections of energy transfer. The control design and asymptotic convergence analysis is based on hybrid dynamical system theory, considering the state of charge of batteries and powers as continuous-time variables and the connections and disconnections among microgrids as discrete-time variables. The stability properties of a given attractor are guaranteed even when an unintentional connection/disconnection of any microgrid element occurs or when any battery is saturated in any of its bounds. This attractor is selected as a trade-off between reaching a small size of a consensus neighbourhood among the state of charge of the batteries and reducing the energy losses associated with the interconnections between microgrids.

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Summary

In this paper a distributed event-triggered control of an islanded network of microgrids is proposed. Microgrids are connected to their neighbors from the point of view of energy exchange and data communication level. The control objective consists of driving the state of charge of batteries to a desired attractor, managing the connections, and disconnections of energy transfer. The control design and asymptotic convergence analysis is based on hybrid dynamical system theory, considering the state of charge of batteries and powers as continuous-time variables and the connections and disconnections among microgrids as discrete-time variables. The stability properties of a given attractor are guaranteed even when an unintentional connection/disconnection of any microgrid element occurs or when any battery is saturated in any of its bounds. This attractor is selected as a trade-off between reaching a small size of a consensus neighbourhood among the state of charge of the batteries and reducing the energy losses associated with the interconnections between microgrids.

KEYWORDS:
Event triggered control, hybrid dynamical systems, consensus algorithms, network of microgrids.

1 | INTRODUCTION

In today's society, renewable energy is playing an increasingly important role in the electrical system. Furthermore, the trend is beginning to lead to the division of the electrical network into distributed energy systems, where the concept of a microgrid is considered a key element. A microgrid is defined as a set of generation units, loads, and storage devices controlled in a coordinated way. The use of renewable sources in microgrids causes intermittency and uncertainty problems, and the mismatch between local generation and loads. These problems can be reduced with Energy Storage Systems (ESS) being able to operate in islanded mode or operating connected to the utility grid. In this paper, an islanded network of microgrids will be considered, that is, there is no connection to the utility grid but energy exchange between two connected microgrids is allowed.

The increasing interest in microgrids has driven to develop more complex microgrids, as a network of microgrids that exchange energy. Consequently, Control research to make these systems more efficient has emerged more and

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more, especially using centralized and distributed approaches. The last one follows the principle *"thinking globally, acting locally"* and allows to get more scalable and robust microgrids.

The control of a microgrid typically requires a hierarchical control system with three levels, primary, secondary, and tertiary levels [1]. Focusing on the higher level, known as the Energy Management System (EMS), different distributed control techniques have been applied to networks of microgrids [1,2]. A review of several techniques for all hierarchical levels can be found in [3].

There are several research works that address EMS for a network of microgrids using optimal control techniques [4] and model predictive control methods [5,6]. The authors in [7] propose a solution that minimizes costs while managing the State Of Charge (SOC) of the different ESSs. A result based on Pontryagin's minimum principle is given in [8], where the minimization of power flows among microgrids is performed, maintaining the ESS around a given reference value. The economic dispatch problem of a network of microgrids connected to the utility grid is studied in [9], where a leader-follower consensus algorithm is applied, ensuring that the incremental costs of the generation units tend to the electricity price of the utility grid. The work presented in [10] proposes another optimization algorithm in order to reach an agreement on the dispatch with a central microgrid. Most of the papers consider grid-connected networks, and optimization problems include selling and buying energy to the utility grid. Furthermore, [11] considers the consensus theory for the design of EMS of a microgrid using demand-side management.

Some technological solutions are committed to considering a distributed ESS in a microgrid network to compensate the lack of a connection to the utility grid in islanded networks. Furthermore, depending on the operational conditions, it can be necessary for the energy of any storage system to be shared with other microgrids through the connections between them and therefore can be considered as a Distributed Energy Storage Unit (DESU) [12]. Focusing on the control problem of a distributed ESS in a microgrid network based on balancing SOCs, some works are found in the literature [13,14,15]. The idea is to achieve consensus on the SOCs and ESS charging/discharging powers. The main aspects associated with battery degradation are the charge/discharge power, the depth of discharge, and the cycle times [16,17]. SOC consensus helps to avoid individual batteries exceeding their operational capacity limits and also to maintain battery powers at safe values, increasing the lifespan of the storage system [18]. SOC consensus can be used in both on-grid and off-grid systems, but it is more interesting in islanded applications where the need for cooperation between microgrids is more crucial.

There are several works that are based on achieving consensus on the SOCs of an ESS network in islanded and/or connected microgrids. In [12], the authors try to achieve consensus on the SOCs and powers of the batteries, with a simple design, treating the control input as a double integrator system. As stated in this paper, a consensus among SOC and power among energy storage systems is desirable, since such a condition maintains high efficiency and longer lifespan. A consensus-based SOC balancing droop control for an islanded microgrid with multiple storage units is proposed in [19]. The authors in [17] base their work on the leader-following consensus theory of multi-agent systems to realize the power and capacity consensus tracking of distributed battery storage in isolated microgrids. The work in [20] presents a dynamic consensus control algorithm for managing the SOC of the ESSs applied at the secondary control level. In [21], multiple ESSs composed of batteries and supercapacitors are considered, and a distributed control scheme is presented based on a leaderless consensus protocol to split the power between batteries and supercapacitors. The authors in [22] show the advantages of multiple interconnected microgrids from the point of view of optimal power flow. These works resolve a consensus problem without managing the connections and disconnections between microgrids to reduce energy transfer. Moreover, there are no asymptotic convergence guarantees for large-signal systems.

This paper is devoted to control the power flow of the different devices in each microgrid and between the connected microgrids, taking into account the management of the energy stored in the ESSs to reach a consensus on the SOCs. Satisfaction with this objective will imply an increase in the lifespan and efficiency of batteries and a reduction in energy losses between microgrids.

In terms of theoretical solutions, the novel idea of this work is to use the Hybrid Dynamical System (HDS) theory [23] to design a distributed event-triggered controller [24] for a network of microgrids. This theory is suitable because this system is composed of continuous-time dynamics (SOC of batteries and power flows) and discrete-time dynamics (connections/disconnections between microgrids). Moreover, it allows to model nonlinear behaviors such as saturation, plug-and-play events [1], and any connection failure. Each microgrid in the network is connected

[1] A plug-and-play event is defined here as a voluntary/involuntary connection or disconnection of a microgrid element: source, load or energy storage system.

to a set of other neighboring microgrids, allowing energy exchange among them. The control of a microgrid is local, but it receives information from its neighbors. This distributed character allows to increase the robustness and scalability of the complete microgrid. The control objective is to drive the SOCs to to the neighborhood of a consensus, managing the interconnections with their neighbors. Then, a trade-off between consensus neighborhood size and energy losses associated with the interconnections appears. The control goal is accomplished from HDS ensuring asymptotic convergence to a desired attractor, even when any plug-and-play event or connection failure of any microgrid element occurs.

In summary, in this paper we propose a novel controller for a set of islanded microgrids such that

- the SOCs converge to a neighborhood of a consensus without the need of an aggregator, reducing the degradation of the ESS.
- A trade-off between a reduced size of a consensus neighborhood and energy transfer is given.
- Nonlinearities as hybrid dynamics, saturations, plug-and-play events, and connection failures are integrated in a compact control-oriented model.
- Stability properties of the non-linear closed loop are given.

In conclusion, the potential of this work is to provide a more reliable, efficient, robust, and scalable islanded network of microgrids by applying HDS theory.

The rest of the paper is organized as follows. Section 2 presents the considered islanded network of microgrids and states the control problem. The proposed control and hybrid closed-loop model is given in Section 3, which is validated in simulation in Section 4. The paper ends with a conclusion section.

Notation: Throughout the paper, $\mathbb{N}$ denotes the set of natural numbers and $\mathbb{R}$ the set of real numbers, $\mathbb{R}^n$ the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ the set of all real $n \times m$ matrices. The set of nonnegative real numbers is denoted by $\mathbb{R}_{\geq 0}$. $\boldsymbol{I}_n$ is the identity matrix of dimension $\boldsymbol{n}$. $x^\top$ means the transpose vector of x, x^+ represents the value of x after an instantaneous change. $\mathrm{tr}(A)$ is the trace of the matrix A. Finally, $\mathrm{diag}(v) \in \mathbb{R}^n$ is the diagonal matrix composed of the elements of the vector $v \in \mathbb{R}^{n\times 1}$ such that $v(i) = \mathrm{diag}(v)(i,i)$ for all $i \in [1,n]$. $\mathrm{dim}_r(M)$ represents the number of rows of matrix $\boldsymbol{M}$.

2 | SYSTEM DESCRIPTION AND PROBLEM FORMULATION

Generally, an islanded microgrid is a system composed of sources, loads, and ESSs interconnected among them. Microgrids can have several renewable sources as well as different ways of ESSs. In this work, we consider an islanded microgrid composed of a set of microgrids, MG_i, $i \in \mathcal{N} = \{1,2,..,N\}$, which are interconnected through energy exchange and data communication. Moreover, ESSs are considered to be batteries.

Regarding the power flows, the power balanced in an MG_i is given by

$$P_{n,i} = P_{d,i} - P_{gen,i} \tag{1}$$

$$P_{b,i} = P_{n,i} + \Delta P_i. \tag{2}$$

being $P_{b,i}$, the battery power, $P_{n,i}$, the net power, $P_{d,i}$, the demanded power, $P_{gen,i}$ the generated power and ΔP_i, the power of the total energy exchange with the neighbour microgrids (see Fig. 1).

The dynamic in the SOC_i, which provides the level of energy in the storage unit in the MG_i, is considered to be

$$\dot{x}_i = -\eta(P_{b,i})\frac{P_{b,i}}{C_{max,i}} = -\eta_{C,i}P_{b,i} \tag{3}$$

$$\eta(P_{b,i}) = \begin{cases} 1/\eta_i & \text{if} \quad P_{b,i} \geq 0 \\ \eta_i & \text{if} \quad P_{b,i} < 0 \end{cases} \tag{4}$$

where for readability, we adopt the notation $x_i = \mathrm{SOC}_i$ and $\eta_{C,i} = \frac{\eta_i(P_{b,i})}{C_{max}}$. Likewise, the maximum capacity of the battery in energy units is $C_{max,i}$ and the storage efficiency coefficient is $\eta(P_{b,i})$, which takes two different values, if the battery is discharging $(1/\eta_i)$ or charging (η_i).

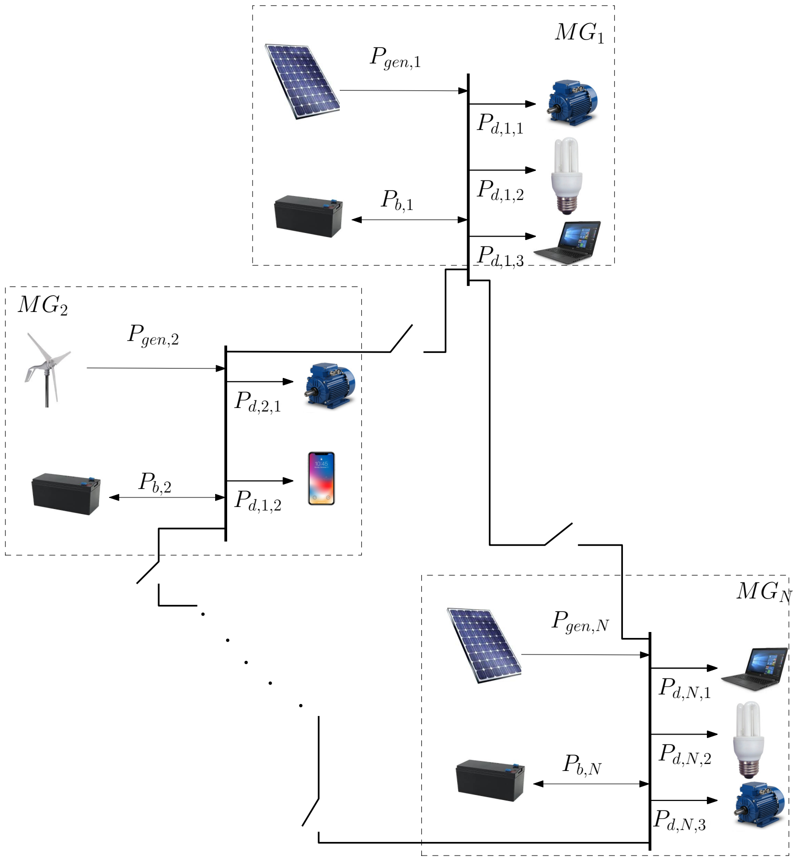

Figure 1 A network of microgrids

Assumption 1. Each battery power, $P_{b,i}$, and the total exchange power, ΔP_i, for MG_i are constrained as follows:

$$|P_{b,i}| \leq P_b^M \tag{5}$$

$$|\Delta P_i| \leq \Delta P^M. \tag{6}$$

Both $P_b^M > 0$ and $\Delta P^M > 0$ are given parameters for the MG network.

Finally, the batteries are designed with the following constraints

$$\mathrm{SOC}^m \leq x_i \leq \mathrm{SOC}^M, \tag{7}$$

being SOC^m and SOC^M the lower and upper bounds of the SOC.

Each MG_i can be connected / disconnected to its neighbors from an energy level, which is translated by a SOC_i, with the aim of driving all SOC_i to a desired set. This goal is motivated by the fact that the battery will extend its lifespan. However, these connections/disconnections should be intelligently managed from a point of view of reducing energy losses.

Highlighting that the network is composed of $i \in \mathcal{N}$ microgrids which can be connected/disconnected to their neighbours from an energy level, the idea is to *control locally* every one of these MGs, in such a way that the *network of MGs globally* enjoys of suited convergence and robustness properties.

The next definition of the Laplacian matrix is introduced to define a compact hybrid system that models all dynamics of the network of MGs.

Definition 1. The Laplacian matrix represents the undirected graph $\mathcal{G}(\mathcal{X}, \mathcal{E})$, where $\mathcal{X} := \{x_1, x_2, .., x_N\}$ collects the network of MGs and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ represents the set of interconnections between MGs.

It is important to note that not all MGs necessarily connect with all MGs.

The connected/disconnected functioning modes are modeled by the variable α_{ij}, which is $\alpha_{ij} = 0$ when x_i is not connected to their neighbor x_j and $\alpha_{ij} = 1$ otherwise. Then defining e_i as a reduced identity matrix obtained by removing the rows j corresponding to $(i,j) \notin \mathcal{E}$, the control variable vector is

$$\alpha_i = e_i[\alpha_{i1}, \alpha_{i2}, .., \alpha_{iN}]^\top,$$

which contains the connection/disconnection variable α_{ij}'s, being its dimension $E_i = \dim_r(e_i) \leq N$. $E_i < N$ is given when MG$_i$ is not fully connected.

The global control variable vector is here

$$\alpha = [\alpha_1^\top, \alpha_2^\top, ..., \alpha_N^\top]^\top.$$

From this definition the adjacency matrix $A(\alpha) = [a_{ij}(\alpha)]$ is

$$a_{ij}(\alpha) : \begin{cases} \alpha_{ij} & \text{if } i \neq j \text{ and } \forall (i,j) \in \mathcal{E} \\ 0 & \text{otherwise} \end{cases}$$

and the diagonal matrix $\Delta(\alpha) = [\delta_{ij}]$ is

$$\delta_{ij}(\alpha) : \begin{cases} \sum_{k=1}^{N} a_{ij}(\alpha) & \text{if } i \neq j \text{ and } \forall (i,j) \in \mathcal{E} \\ 0 & \text{otherwise.} \end{cases}$$

Then, the well-known Laplacian matrix which represents the undirected graph of the interconnections between MGs is

$$L(\alpha) = \Delta(\alpha) - A(\alpha).$$

Note that the Laplacian matrix $L(\alpha)$ is a symmetric positive semi-definite matrix i.e., $L(\alpha)L(\alpha) = L(\alpha)^\top L(\alpha) \geq 0$[25].

The control objective is to avoid running out of charge of any MG. To do so, the main idea is to interconnect the MGs from an energy level in order to drive the SOCs to a neighbourhood of a consensus among the SOCs and to manage the interconnections to reduce the energy losses associated with energy exchange and connections/interconnections switching.

The objective of a first problem is to ensure that all $|x_i - x_j|$ converge to a neighborhood of zero (i.e. $|x_i - x_j| < \varepsilon_M$, being ε_M a given positive parameter). The consensus objective is global, that means it depends on all the couples $(P_{d,i}, P_{gen,i})$, but it is highlighted that the communication is distributed, i.e., each MG i has only access to its local information (x_i) and to the information of their neighbors (x_j). The second objective of the problem is to reduce the interconnections given by $\alpha_{i,j} = 1$ and the switching of each $\alpha_{i,j}$, mainly when $|x_i - x_j| < \varepsilon_M$. The switchings are necessary to reduce the energy losses due to the energy transfer; otherwise, they would always be transferring energy. In the last particular case, all batteries would achieve $|x_i - x_j| = 0$ and not a neighborhood. Thus, a trade-off must be made between the consensus neighbourhood size and energy transfer. Moreover, the convergence properties can be extended to a system with any unintentional connection/disconnection of any element of the MGs, achieving the same conclusions but for the different generated clusters. Then, the control problem of a network of islanded MGs is stated.

Problem 1. Modelling a network of islanded MGs through a hybrid formulation, i.e., considering the continuous-time dynamics, (x_i and power flows), discrete-time dynamics (α_i) and discontinuities as unintentional connection/disconnection of any element of the MGs and saturation of the SOCs in its boundaries and considering a distributed control that manages the energy exchange between neighbour MGs, the control objectives are

i) to ensure the convergence of all $|x_i - x_j|$ to a neighbourhood of zero.

ii) To reduce the interconnections, that means the number of $\alpha_{i,j} = 1$.

3 | HYBRID CONTROL FOR ISLANDED MICROGRID

We adopt here the paradigm given in[23], since the system is composed of continuous-time dynamics, which are SOCs and power flows, and discrete-time dynamics, corresponding to the connections/disconnections among MGs. Moreover, this method allows us to consider discontinuities due to saturations, plug-and-play events, and connection failures.

First, ignoring the unintentional connection/disconnection of any element of the MGs (we shall study these events later), a solution to Problem 1 is provided, i.e., all $|x_i - x_j|$ converge to a neighborhood of zero reducing the connections with their neighbors, MG_j.

Let us denote $x = [x_1, x_2, ..., x_N]^\top$. Now, it is possible to define an hybrid model scheme of MG_i according to the framework given in[23],

$$\mathcal{H}_i : \begin{cases} \begin{bmatrix} \dot{x}_i \\ \dot{\alpha}_i \\ \dot{q}_i \end{bmatrix} = f_i(x, \xi_i), & \xi_i \in \mathcal{C}_i \\ \begin{bmatrix} x_i^+ \\ \alpha_i^+ \\ q_i^+ \end{bmatrix} = g_i(x, \xi_i), & \xi_i \in \mathcal{D}_i, \end{cases} \tag{8}$$

where $\xi_i = [x_i \ \alpha_i \ q_i]^\top \in \mathbb{H}_i$ such that $\mathbb{H}_i := [\mathrm{SOC}^m, \mathrm{SOC}^M] \times \{0,1\}^{E_i \times 1} \times \{0,1\}$. Recall that α_i is the control input.

The maps f_i and g_i capture both the continuous-time and discrete-time dynamics and are defined as follows:

$$\begin{aligned} f_i(x, \xi_i) &= \begin{bmatrix} -q_i \left(\eta_{C,i} P_{n,i} + K \sum_{j=1}^N a_{ij}(\alpha)(x_i - x_j) \right) \\ 0_{E_i,1} \\ 0 \end{bmatrix}, \\ g_i(x, \xi_i) &= \begin{bmatrix} x_i \\ u_i(x)\varsigma(x_i) + \alpha_i(1 - \varsigma(x_i)) \\ (1 - q_i)(1 - \varsigma(x_i)) + q_i \varsigma(x_i) \end{bmatrix}. \end{aligned} \tag{9}$$

From definition of the battery power given in Section 2 the power of the total exchanged energy with the neighbour MGs is defined here as

$$\eta_{C,i} \Delta P_i := K \sum_{j=1}^N a_{ij}(\alpha)(x_i - x_j). \tag{10}$$

This is an input variable, where the scalar $K/\eta_{C,i}$ defines the maximum net exchanged power that MG_i can receive from the other ones. Hence, from condition (6), it is obtained the following constraint for parameter K in order to satisfy (10),

$$K \leq \frac{\eta_{C,i} \Delta P^M}{\sum_{j=1}^N a_{ij}(\alpha)(x_i - x_j)} \leq \frac{\eta_C^m \Delta P^M}{a^M(\mathrm{SOC}^M - \mathrm{SOC}^m)}, \tag{11}$$

where $\eta_C^m := \min_i \eta_{C,i}$ and $a^M := \max_\alpha \sum_{j=1}^N a_{ij}(\alpha)$.

As K is higher, the convergence speed between all x_i to a consensus is higher.

$P_{n,i}$ is an exogenous variable defined in (1). Moreover, function $\varsigma(s)$ is defined as follows

$$\varsigma(s) := \begin{cases} 0 \text{ if } s \geq \mathrm{SOC}^M \text{ or } s \leq \mathrm{SOC}^m \\ 1 \quad \text{otherwise.} \end{cases}$$

The artificial discrete-time variable $q_i \in \{0,1\}$ describes the functioning mode related to the saturation of x_i given in condition (7). Finally, vector

$$u_i(x) = e_i[u_{i,j}(x_i, x_j)]^\top$$

is a input variable whose elements are defined by the following function

$$u_{ij}(x_i, x_j) := \begin{cases} 1 \text{ if } |x_i - x_j| \geq \varepsilon \text{ and } \forall (i,j) \in \mathcal{E} \\ 0 \quad \text{otherwise} \end{cases} \tag{12}$$

being ε a positive parameter.

Then, the so-called flow and jump sets are selected,

$$
\begin{aligned}
\mathcal{C}_{i,1} &:= \{\xi_i \in \mathbb{H}_i : \alpha_{i,j} = 1, |x_i - x_j| \geq \varepsilon\} \\
\mathcal{C}_{i,2} &:= \{\xi_i \in \mathbb{H}_i : \alpha_{i,j} = 0, |x_i - x_j| \leq \varepsilon_M\} \\
\mathcal{C}_{i,3} &:= \{\xi_i \in \mathbb{H}_i : q_i = 0,\ x_i = \mathrm{SOC}^M, \phi_i(x,\alpha) \geq 0\} \\
\mathcal{C}_{i,4} &:= \{\xi_i \in \mathbb{H}_i : q_i = 0,\ x_i = \mathrm{SOC}^m, \phi_i(x,\alpha) \leq 0\} \\
\mathcal{C}_{i,5} &:= \{\xi_i \in \mathbb{H}_i : q_i = 1,\ x_i \in [\mathrm{SOC}^m, \mathrm{SOC}^M]\} \\
\mathcal{D}_{i,1} &:= \{\xi_i \in \mathbb{H}_i : \alpha_{i,j} = 1, |x_i - x_j| \leq \varepsilon\} \\
\mathcal{D}_{i,2} &:= \{\xi_i \in \mathbb{H}_i : \alpha_{i,j} = 0, |x_i - x_j| \geq \varepsilon_M\} \\
\mathcal{D}_{i,3} &:= \{\xi_i \in \mathbb{H}_i : q_i = 1,\ x_i = \mathrm{SOC}^M, \phi_i(x,\alpha) \geq 0\} \\
\mathcal{D}_{i,4} &:= \{\xi_i \in \mathbb{H}_i : q_i = 1,\ x_i = \mathrm{SOC}^m, \phi_i(x,\alpha) \leq 0\} \\
\mathcal{D}_{i,5} &:= \{\xi_i \in \mathbb{H}_i : q_i = 0,\ x_i = \mathrm{SOC}^M, \phi_i(x,\alpha) \leq 0\} \\
\mathcal{D}_{i,6} &:= \{\xi_i \in \mathbb{H}_i : q_i = 0,\ x_i = \mathrm{SOC}^m, \phi_i(x,\alpha) \geq 0\} \\
\mathcal{C}_i &:= (\mathcal{C}_{i,1} \cup \mathcal{C}_{i,2}) \cap (\mathcal{C}_{i,3} \cup \mathcal{C}_{i,4} \cup \mathcal{C}_{i,5}) \\
\mathcal{D}_i &:= \mathcal{D}_{i,1} \cup \mathcal{D}_{i,2} \cup \mathcal{D}_{i,3} \cup \mathcal{D}_{i,4} \cup \mathcal{D}_{i,5} \cup \mathcal{D}_{i,6}
\end{aligned}
$$

where

$$\phi_i(x,\alpha) = -\eta_{C,i} P_{n,i} - K \sum_{j=1}^{N} a_{ij}(\alpha)(x_i - x_j)$$

and $\varepsilon_M \in (\varepsilon, SOC^M - SOC^m]$.

This regularization comes from the fact that x_i only can evolve in $[\mathrm{SOC}^m, \mathrm{SOC}^M]$. The jumps are allowed if

- two connected MGs present their SOCs relatively close, $|x_i - x_j| \leq \varepsilon$.
- two disconnected MGs present their SOCs relatively far, $|x_i - x_j| \geq \varepsilon_M$.
- the SOC is saturated by (7).

The motivation of this hybrid scheme comes addressed by the fact of reducing the energy transfer, which can infer in energy losses. For this purpose, the exchanges between couple of MGs are avoided if their SOCs are relatively close. Moreover, a Zeno solution is avoided if $\varepsilon_M > \varepsilon$. Note that if ε and ε_M are closer, then more jumping can occur. Moreover, it is not necessary that all SOCs are in a consensus such that $\mathrm{SOC}_1 = \mathrm{SOC}_2 = ... = \mathrm{SOC}_N$, it is sufficient if they are close enough. Hence, that parameter ε ad ε_M defines a trade-off, between the energy transfer reduction (associated to the interconnected MGs) and the reduced size of the consensus neighbourhood among MGs.

$\varsigma(x_i)$ avoids that the discrete variables α_i and q_i are updated when the solutions are in $\mathcal{D}_{i,3} \cup \mathcal{D}_{i,4}$ or $\mathcal{D}_{i,1} \cup \mathcal{D}_{i,2}$, respectively. It is emphasized that the reason to use hybrid systems is the presence of the nonlinearities: resets and saturations, captured by the jump map.

Now, we can formalize a hybrid model of the overall microgrid in the following compact form.

$$
\mathcal{H}: \begin{cases} \begin{bmatrix} \dot{x} \\ \dot{\alpha} \\ \dot{q} \end{bmatrix} = f(\xi), & \xi \in \mathcal{C} \\ \begin{bmatrix} x^+ \\ \alpha^+ \\ q^+ \end{bmatrix} = g(\xi), & \xi \in \mathcal{D}, \end{cases} \tag{13}
$$

where $q = [q_1, q_2, ..., q_N]^\top$ and $\xi = [x\ \alpha\ q]^\top \in \mathbb{H}$ such that $\mathbb{H} := [\mathrm{SOC}^m, \mathrm{SOC}^M]^N \times \{0,1\}^{\sum_{i=1}^{N} E_i \times 1} \times \{0,1\}^N$.

The maps f and g are

$$f(\xi) := \begin{bmatrix} -\operatorname{diag}(q)\left(KL(\alpha)x + P_{\eta_C}\right) \\ 0_{\sum_{i=1}^{N} E_i,1} \\ \mathbf{0} \end{bmatrix}, \qquad (14)$$
$$g(\xi) := \begin{bmatrix} x \\ u(x)\varsigma(x) + \alpha(1-\varsigma(x)) \\ (\mathbf{1}-q)(1-\varsigma(x)) + q\varsigma(x) \end{bmatrix}.$$

being $P_{\eta_C} = [\eta_{C,1}P_{n,1}, \eta_{C,2}P_{n,2}, ..., \eta_{C,N}P_{n,N}]^\top$ and $u(x) = [u_1(x), u_2(x), ..., u_N(x)]^\top$ defined from (12).

The flow and complete sets are

$$\mathcal{C} := \prod_{i\in\mathcal{N}} (\mathcal{C}_{i,1} \cup \mathcal{C}_{i,2}) \cap (\mathcal{C}_{i,3} \cup \mathcal{C}_{i,4} \cup \mathcal{C}_{i,5}) \qquad (15)$$

$$\mathcal{D} := \prod_{i\in\mathcal{N}} (\mathcal{D}_{i,1} \cup \mathcal{D}_{i,2} \cup \mathcal{D}_{i,3} \cup \mathcal{D}_{i,4}) \cup \mathcal{D}_{i,5} \cup \mathcal{D}_{i,6}). \qquad (16)$$

The controlled system implementation is depicted in Fig. 2.

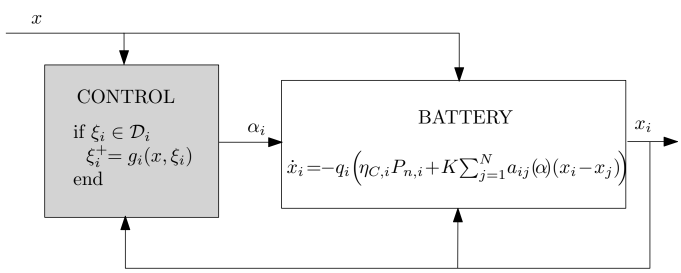

Figure 2 Block diagram of the control mechanism.

Before presenting the main result, next proposition is given.

Proposition 1. Hybrid sytem (13)–(16) satisfies the Basic Hybrid Conditions[23].

Proof. Hybrid system $\mathcal{H}(f, g, \mathcal{C}, \mathcal{D})$ with function (12) is well-posed because it verifies:

- $\mathcal{C}$ and $\mathcal{D}$ are closed sets in $\mathbb{H}$.
- f is a continuous function, hence it is locally bounded and outer semi-continuous. Moreover, it is convex for each $\xi \in \mathcal{C}$.
- g is outer semi-continuous and locally bounded.

□

Before to prove that hybrid scheme (13)–(16) includes a solution to Problem 1, a result about stability properties of the flows is stated in the following lemma.

Lemma 1. Consider the overall microgrid whose dynamics are governed by (13)–(16) and any ΔP^M and P_b^M such that Assumption 1 is satisfied. Consider also any given parameter $K \leq \frac{\eta_C^m \Delta P^M}{a^M(\mathrm{SOC}^M - \mathrm{SOC}^m)}$, being $\eta_C^m := \min_i \eta_{C,i}$ and $a^M := \max_\alpha \sum_{j=1}^{N} a_{ij}(\alpha)$, and any given $\mu_0, \mu_1 > 0$ such that $\varepsilon > 0$ are solutions to the following optimization problem

$$\min_{\varepsilon} \quad \varepsilon \qquad (17)$$

$$\varepsilon \geq \frac{N\eta_C^M(P_b^M + \Delta P^M)}{n(\alpha)\sqrt{\mu_0 + \mu_1}} \quad \forall \alpha \in \mathcal{E} \qquad (18)$$

$$2KL^2(\alpha) \geq \mu_0 L(\alpha) + \mu_1 L^2(\alpha) \quad \forall \alpha \in \mathcal{E}, \qquad (19)$$

where $\eta_C^M := \max_i \eta_{C,i}$ and $n(\alpha) := \frac{\text{tr}(\Delta(\alpha))}{2}$. If all $\xi_i \in C_{i,1} \cap C_{i,5}$, then the flows of the interconnected MGs converge to the set

$$\mathcal{A}_\alpha := \left\{ \xi \in \mathbb{H} : x^\top L(\alpha)x \le n(\alpha)\varepsilon^2 \right\}.$$

Proof. Consider the following function:

$$V_\alpha(\xi) = \max\left\{ x^\top L(\alpha)x - n(\alpha)\varepsilon^2, 0 \right\}, \tag{20}$$

which is definite positive according to $\mathcal{A}_\alpha$ and radially unbounded. The objective is to prove that

$$\langle \nabla V_\alpha(x,\alpha), f(\xi)\rangle = -2x^\top \operatorname{diag}(q)\left(L(\alpha)KL(\alpha)x + L(\alpha)P_{\eta_C}\right) \le 0$$

is satisfied for all $\xi_i \in C_{i,1} \cap C_{i,5}$ such that $x^\top L(\alpha)x \ge n\varepsilon^2$, i.e., outside of $\mathcal{A}_\alpha$.

To do so we apply a S-procedure, such that

$$-2x^\top\left(L(\alpha)KL(\alpha)x + L(\alpha)P_{\eta_C}\right) + \mu_0(x^\top L(\alpha)x - n(\alpha)\varepsilon^2) \le 0, \tag{21}$$

for any parameter $\mu_0 > 0$. Note that the fact that all $\xi_i \in C_{i,1} \cap C_{i,5}$ implies that $\operatorname{diag}(q) = I$.

Now, from applying the next property

$$(a+b)^2 \ge 0 \Leftrightarrow \mu a^2 + \frac{1}{\mu}b^2 \ge \pm 2ab\frac{\mu^{\frac{1}{2}}}{\mu^{\frac{1}{2}}}$$

the following holds

$$\begin{aligned}
&-2x^\top\left(L(\alpha)KL(\alpha)x + L(\alpha)P_{\eta_C}\right) + \mu_0(x^\top L(\alpha)x - n(\alpha)\varepsilon^2)\\
&\le -x^\top\left(2L(\alpha)KL(\alpha) - \mu_0 L(\alpha)\right)x - \mu_0 n(\alpha)\varepsilon^2 + \mu_1 x^\top L(\alpha)^2 x + \frac{1}{\mu_1}P_{\eta_C}{}^\top P_{\eta_C},\\
&\le -x^\top\left((2K-\mu_1)L^2(\alpha) - \mu_0 L(\alpha)\right)x - \mu_0 n(\alpha)\varepsilon^2 + \frac{1}{\mu_1}P_{\eta_C}{}^\top P_{\eta_C} \le 0.
\end{aligned}$$

The last condition comes from the satisfaction of conditions (18) and (19). Then, Lemma 1 statement is proven. □

Remark 1. Lemma 1 is given for the particular case that the solutions to hybrid system (13)–(16) are such that $\xi \in \prod_{i\in\mathcal{N}}(C_{i,1} \cap C_{i,5})$. However, if any $\xi_i \in C\backslash(C_{i,1} \cap C_{i,5})$ and the solution is flowing means that

- any $x_i \in C_{i,1} \cap (C_{i,3} \cup C_{i,4})$, i.e. any couple of (x_i, x_j) satisfies $|x_i - x_j| \ge \varepsilon$ being $x_i = \text{SOC}^M$ or $x_i = \text{SOC}^m$.
- or/and that any $x_i \in C_{i,2} \cap (C_{i,3} \cup C_{i,4} \cup C_{i,5})$, i.e., $\alpha_{i,j} = 0$ and $|x_i - x_j| \le \varepsilon_M$, independently if x_i is saturated or not.

Now, the next theorem consists of establishing Uniform Global Asymptotic Stability (UGAS) property of a distributed event-triggered control for a network of microgrids.

Theorem 1. Consider the overall microgrid whose dynamics are governed by (13)–(16) and any ΔP^M and P_b^M such that Assumption 1 is satisfied. Consider also any given parameter $K \le \frac{\eta_C^m \Delta P^M}{a^M(\text{SOC}^M - \text{SOC}^m)}$, being $\eta_C^m := \min_i \eta_{C,i}$ and $a^M := \max_\alpha \sum_{j=1}^N a_{ij}(\alpha)$, and any given $\mu_0, \mu_1 > 0$ such that $\varepsilon > 0$ are solutions to the optimization problem (17)–(19). Then, for $n(\alpha) := \frac{\text{tr}(\Delta(\alpha))}{2}$ and any given $\varepsilon_M \in (\varepsilon, \text{SOC}^m - \text{SOC}^M]$ set

$$\mathcal{A} := \left\{ \xi \in \mathbb{H} : \sum_{i,j=1}^N \max_{i,j}\left\{ a_{i,j}(\alpha)((x_i - x_j)^2 - \varepsilon_M^2), 0\right\} \right\} \tag{22}$$

is UGAS.

Proof. Due to the properties of f and g, system $\mathcal{H}$ satisfies the hybrid basic conditions of[23, As. 6.5] and several useful results pertaining to well-posed hybrid systems can be applied.

Let us consider the following Lyapunov function candidate,

$$V(x,\alpha) = \sum_{i,j=1}^{N} \max_{i,j} \left\{ a_{i,j}(\alpha)((x_i - x_j)^2 - \varepsilon_M^2), 0 \right\}. \tag{23}$$

This function is clearly definite positive with respect to $\mathcal{A}$ and radially unbounded.

In order to prove that

$$\langle \nabla V(x,\alpha), f(\xi) \rangle \leq 0,$$

we consider that

- $\xi \in \prod_{i\in\mathcal{N}}(C_{i,1} \cap C_{i,5})$: From Lemma 1, it is proven that set $\mathcal{A}_\alpha$ is attractive during flows by taking (20). Likewise, note that $\mathcal{A}_\alpha \subset \mathcal{A}$, because

$$\sum_{i,j=1}^{N} \max_{i,j} \left\{ a_{i,j}(\alpha)((x_i - x_j)^2 - \varepsilon^2), 0 \right\} \leq x^\top L(\alpha) x \leq \varepsilon^2 < \varepsilon_M^2,$$

  noting that $n(\alpha) = \frac{\mathrm{tr}(\Delta(\alpha))}{2} = \sum_{i,j=1}^{N} \alpha_{i,j}$. Then, it is easy to see that

$$\langle \nabla V(x,\alpha), f(\xi) \rangle \leq 0.$$

- Any $x_i \in C_{i,2} \cap (C_{i,3} \cup C_{i,4} \cup C_{i,5})$: Any couple (x_i, x_j) satisfies $|x_i - x_j| \leq \varepsilon_M$ and $\alpha_{i,j} = 0$, because (x_i, x_j) is in $C_{i,2}$. Then (x_i, x_j) evolves in the interior of $\mathcal{A}$, without modifying $V(x,\alpha)$ because

$$\left\{ a_{i,j}(\alpha)((x_i - x_j)^2 - \varepsilon_M^2), 0 \right\} = 0.$$

- Any $x_i \in C_{i,1} \cap (C_{i,3} \cup C_{i,4})$: If any $x_i \in C_{i,1} \cap (C_{i,3} \cup C_{i,4})$ means that any couple of (x_i, x_j) satisfies $|x_i - x_j| \geq \varepsilon$ being $x_i = \mathrm{SOC}^M$ or $x_i = \mathrm{SOC}^m$. Then, ξ_i can flow for a while until reach $\mathcal{D}_{i,2}$, i.e., until to get $|x_i - x_j| \geq \varepsilon_M$ or remain forever here, satisfying $\varepsilon_M > |x_i - x_j| \geq \varepsilon$, i.e., satisfying

$$\left\{ a_{i,j}(\alpha)((x_i - x_j)^2 - \varepsilon_M^2), 0 \right\} = 0.$$

From control law (12) and jump set definitions, it is got

$$V(x,\alpha^+) - V(x,\alpha) = 0. \tag{24}$$

This statement is deduced analyzing each term of

$$D(x,\alpha_{i,j}) = \max \left\{ a_{i,j}(\alpha)((x_i - x_j)^2 - \varepsilon_M^2), 0 \right\} = \max \left\{ \alpha_{i,j}((x_i - x_j)^2 - \varepsilon_M^2), 0 \right\}. \tag{25}$$

- If before and after the jump $\alpha_{i,j}^+ = \alpha_{i,j}$, then $D(x,\alpha_{i,j}^+) - D(x,\alpha_{i,j}) = 0$.
- If after the jump $\alpha_{i,j}^+ = 1$ and before the jump $\alpha_{i,j} = 0$, it is because $\xi_i \in \mathcal{D}_{i,2}$ and the control law (12) are applied. For continuity, the solution is $(x_i - x_j)^2 = \varepsilon_M^2$ in the jump. Then, $D(x,\alpha_{i,j}^+) = D(x,\alpha_{i,j}) = 0$.
- If after the jump $\alpha_{i,j}^+ = 0$ and before the jump $\alpha_{i,j} = 1$, it is because $\xi_i \in \mathcal{D}_{i,1}$. Again, for continuity, in the jump $(x_i - x_j)^2 = \varepsilon^2$. As $\varepsilon < \varepsilon_M$ we have $D(x,\alpha_{i,j}^+) = D(x,\alpha_{i,j}) = 0$.

Finally, note that if any jump occurs due to the fact that $\xi_i \in \mathcal{D}_{i,3} \cup \mathcal{D}_{i,4} \cup \mathcal{D}_{i,5} \cup \mathcal{D}_{i,6}$, then (24) does not change because the discrete vector α_i is not modified.

□

Remark 2. The tuned parameters ε and ε_M play an important role in a trade-off between the reduced size of the consensus neighbourhood and the energy losses associated with the interconnections. Indeed, if ε is smaller, then $\mathcal{A}_\alpha$ is also smaller and the consensus neighbourhood size between x_i's are susceptible to diminish. Likewise, if ε_M is smaller, then the consensus neighbourhood size also decreases, but the interconnections can increase. Moreover, if ε is close to ε_M, it can generate more jumping, because $\mathcal{A}$ and $\mathcal{A}_\alpha$ are closer.

The theorem given above holds for the system dynamics described by (13)–(16), i.e., by $P_{b,i}$ and x_i defined by continuous-time dynamics and the discrete-time variable α governed by $u(x)$. However, the system maintains nice properties regardless of any unintentional connection/disconnection of any element of the MGs is given. Let us define $\gamma_{ij} \in \{0,1\}$ to define any unintentional connection/disconnection from any battery or communication failure (if $\gamma_{ij}=0$ then the control input α^+_{ij} is forced to be 0). Then, $\gamma = [\gamma_1^\top, \gamma_2^\top, ..., \gamma_N^\top]\top$, where $\gamma_i = e_i[\gamma_{i1}, \gamma_{i2}, .., \gamma_{iN}]^\top$ and e_i were defined in Section 2. Finally, $\Gamma(\gamma) = [v_{ij}(\gamma)]$ is

$$v_{ij}(\gamma) : \begin{cases} 1-\gamma_{ij} & \text{if } i \neq j \text{ and } \forall (i,j) \in \mathcal{E} \\ 0 & \text{otherwise.} \end{cases}$$

Let us rewrite (13)–(16) as follows $\mathcal{H}_p(f, C, g_p, D)$ with

$$f_p(\xi) := \begin{bmatrix} -\operatorname{diag}(q)\left(KL(\alpha)x + \eta_C(P_n + \Delta P_n + \Gamma(\gamma)h(P_n))\right) \\ 0_{\sum_{i=1}^N E_i, 1} \\ \mathbf{0} \end{bmatrix}$$

$$g_p(\xi) := \begin{bmatrix} x \\ \gamma\left(u(x)\varsigma(x) + \alpha(1-\varsigma(x))\right) \\ (\mathbf{1}-q)(1-\varsigma(x)) + q\varsigma(x) \end{bmatrix}.$$

being $h(P_n) = [h(P_{n,1}), h(P_{n,2}), ...h(P_{n,N})]$ with $h(P_{n,i}) \in \eta_{C,i}[-P^M_{b,i}, P^M_{b,i}]$ and $\gamma \in \{0,1\}^{\sum_{i=1}^N E_i \times 1}$ exogenous discrete-time variables. Moreover, ΔP_n represents an exogenous variation in P_n due to any load, source, or battery connection/disconnection in MG_i, respectively.

We introduce the next result on the convergence of state x when any unintentional connection/disconnection of any element of the MGs occurs.

Corollary 1. Consider the overall microgrid whose dynamics are governed by $\mathcal{H}_p(f, C, g_p, D)$ and any ΔP^M and P^M_b such that Assumption 1 is satisfied. Consider also any given parameter $K \leq \frac{\eta^m_C \Delta P^M}{a^M(\mathrm{SOC}^M - \mathrm{SOC}^m)}$, being $\eta^m_C := \min_i \eta_{C,i}$ and $a^M := \max_\alpha \sum_{j=1}^N a_{ij}(\alpha)$, and any given $\mu_0, \mu_1 > 0$ such that $\varepsilon > 0$ are solutions to the optimization problem (17)–(19). Then, for $n(\alpha) := \frac{\mathrm{tr}(\Delta(\alpha))}{2}$ and any given $\varepsilon_M \in (\varepsilon, \mathrm{SOC}^m - \mathrm{SOC}^M]$, x_i of MG_i converges in finite time to $\mathcal{A}$ (22).

Proof. The proof follows Theorem 1 proof and the fact that the UGAS property for $\mathcal{A}$ is no longer valid, because the convergence of x_i is non-uniform due to the exogenous variables γ, $h(P_n)$ and ΔP_n. Indeed, they can take an arbitrarily long time to reach $\mathcal{A}$. □

4 | SIMULATION RESULTS

This section is devoted to validate in simulation the results given along the paper. To do so, we use Matlab/Simulink and the Toolbox given in[26].

We consider a network composed of 4 MGs, assuming that the batteries are homogeneous. They will be connected both in communication and energy in a ring configuration, as given in Fig. 3. The microgrid parameters are given in Table 1. Note that the maximum of the total exchange power of each MG is $\Delta P^M_i = 20\% P^M_{b,i}$. From these values, we have $\frac{\eta^m_C \Delta P^M}{a^M(\mathrm{SOC}^M - \mathrm{SOC}^m)} = 0.2386$. Then, we select $K = 0.23$. The optimization problem (17)–(19) for $\mu_0 = 0.02$ and $\mu_1 = 0.0008$ gives $\varepsilon = 0.0012$. The simulations are performed for one day.

Figure 4 shows a scenario in which there is no unintentional connection/disconnection of any element of the MGs. The x_i and $\alpha_{i,j}$ evolutions are shown for different values of ε and ε_M, noting the trade-off between connected MGs and the consensus neighbourhood size of all x_i given in Remark 2. The smaller selected ε is 0.0012 which is the solution of the optimization problem (17)–(19) for $\mu_0 = 0.02$ and $\mu_1 = 0.0008$. Note that as ε is larger the MGs consensus neighbourhood size between the x_i's increase and the switching also increase because ε is closer to ε_M. Moreover, as ε_M is larger, consensus neighbourhood size also increase, but switching connections between MGs decrease. A particular attention is done to the case $\varepsilon = \varepsilon_M = 0.01$. Note that the switching increase considerably because $\mathcal{A} = \mathcal{A}_\alpha$, enabling a Zeno solution, which must be avoided. That is why in the hybrid scheme (13)–(16) is defined $\varepsilon_M \in (\varepsilon, SOC^M - SOC^m]$.

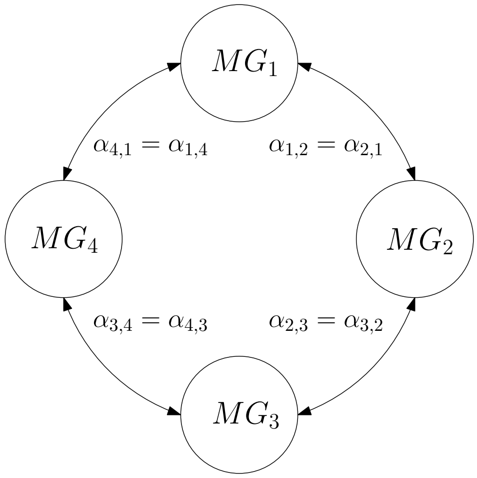

Figure 3 The microgrid configuration.

Table 1 Microgrid parameters

Component	Value
$C_{max,i}$, $i = \{1, 2, 3, 4\}$	17600Wh
η_i, $i = \{1, 2, 3, 4\}$	0.8
$P_{b,i}^{M}$, $i = \{1, 2, 3.4\}$	7000W
ΔP_i^M, $i = \{1, 2, 3.4\}$	1400W
SOC^M, $i = \{1, 2, 3.4\}$	0.9
SOC^m, $i = \{1, 2, 3.4\}$	0.1
K	0.23

Figure 5 shows the evolutions of $V(x, \alpha)$ and $V_\alpha(x, \alpha)$ for the particular case of $\varepsilon = 0.005$ and $\varepsilon_M = 0.03$. Note that as $V(x, \alpha)$ is a Lyapunov function, i.e., it converges to zero and, once it reaches zero, it evolves invariant in this value. However, note that the evolution of $V_\alpha(x, \alpha)$ is quite different. Indeed, $V_\alpha(x, \alpha)$ converges to zero, but it is not invariant in this value, because on the same occasions there is any $\xi_i \notin C_{i,1} \cap C_{i,5}$.

Now, we show in Fig. 6 the result given in Corollary 1. Here, we have some unintentional connection/disconnection, being $\varepsilon = 0.005$ and $\varepsilon_M = 0.03$. More precisely, in time 2h there is a disconnection in MG_1 of a load of 100W. Between 5 and 14 hours, MG_2 is disconnected because the battery has failed, therefore x_2 has not information about its neighbors ($\gamma_{1,2} = \gamma_{2,3} = 0$) and MG_3 receives $\Delta P_{n,2} = P_{n,2}$. Lastly, between 17 and 21 hours a load of $300W$ is connected. In this figure we see, as the microgrids without failure collaborate among them, such that SOCs converge to a neighborhood of the set $|x_i - x_j| < \varepsilon_M$ assuming more or less instantaneous demanded power, as well as, MG interconnections. Note that through few connections, we get to lead the SOCS to a neighborhood of a consensus. It is also shown that the convergence is not uniform due to the unintentional connection/disconnection of any element of the MGs. However, the time elapsed to reach the states $\mathcal{A}$ is quite acceptable.

5 | CONCLUSION

We have proposed a control law for an islanded network of microgrids with the objective that the SOCs of the batteries converge close to a consensus, exchanging energy between the neighboring microgrids. The disconnections are motivated to reduce energy-transfer losses. Then, a trade-off between consensus neighbourhood size and reduction of energy transfer is obtained by selecting parameters ε and ε_M. The goal is obtained through a distributed event-triggered control that only communicates with the neighbors, providing robustness and scalability to the microgrid as

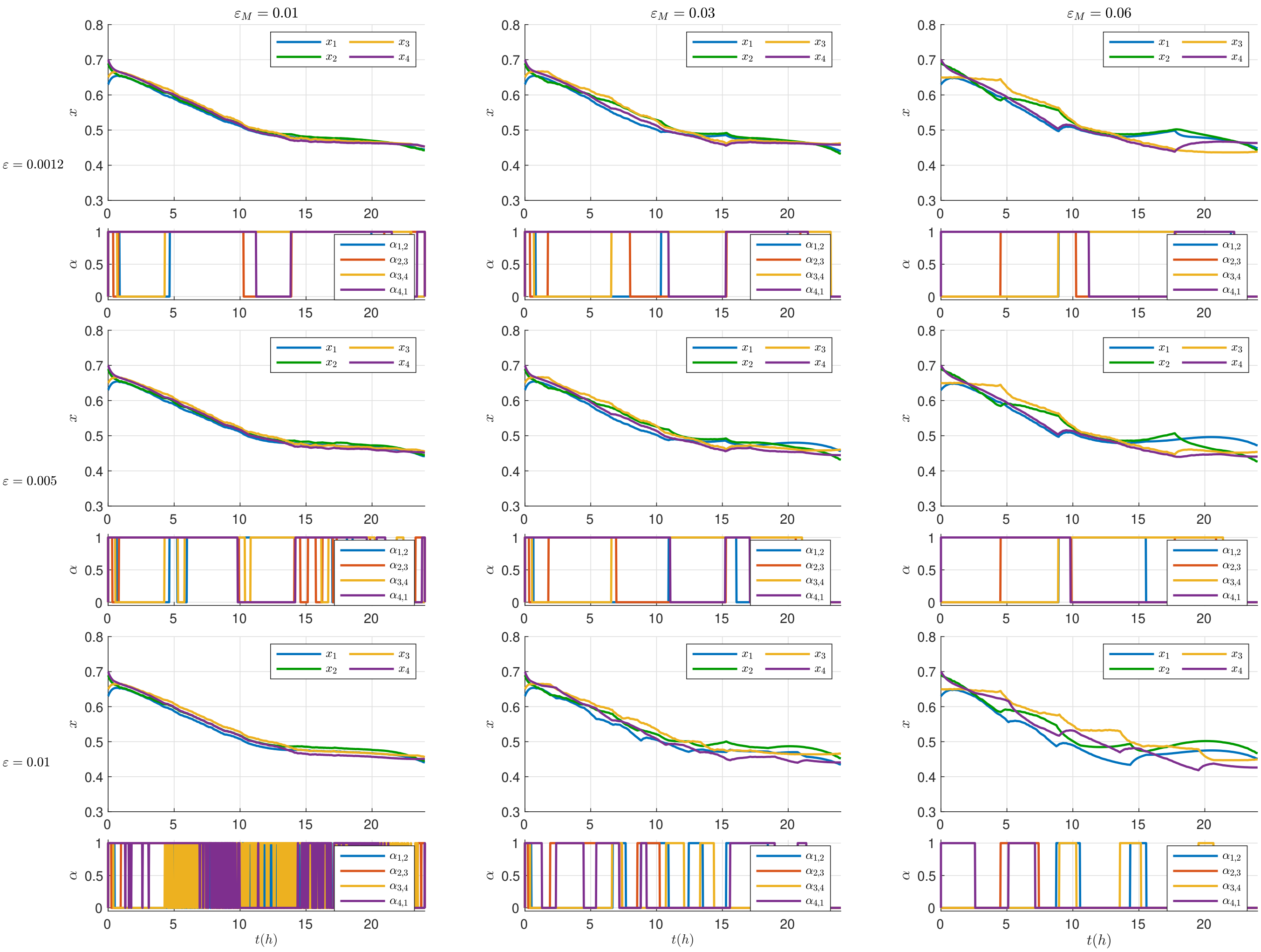

Figure 4 The evolutions of x_i and the connection and disconnection of $\alpha_{i,j}$ microgrid configuration for different values of ε and ε_M.

a whole. The closed loop is modeled and analyzed using HDS theory, providing asymptotically convergence guarantees, even when plug-and-play events occur or any battery is saturated in any of its bounds. Simulations performed for different parameters validate the main results.

In future work, we will validate the controller in experiments as well as consider a network of microgrids connected to the utility grid.

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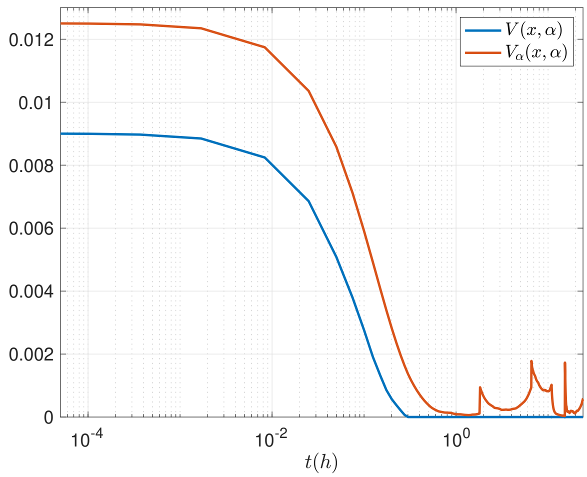

Figure 5 Evolutions of $V(x, \alpha)$ and $V_\alpha(x, \alpha)$, being $\varepsilon = 0.005$ and $\varepsilon_M = 0.03$.

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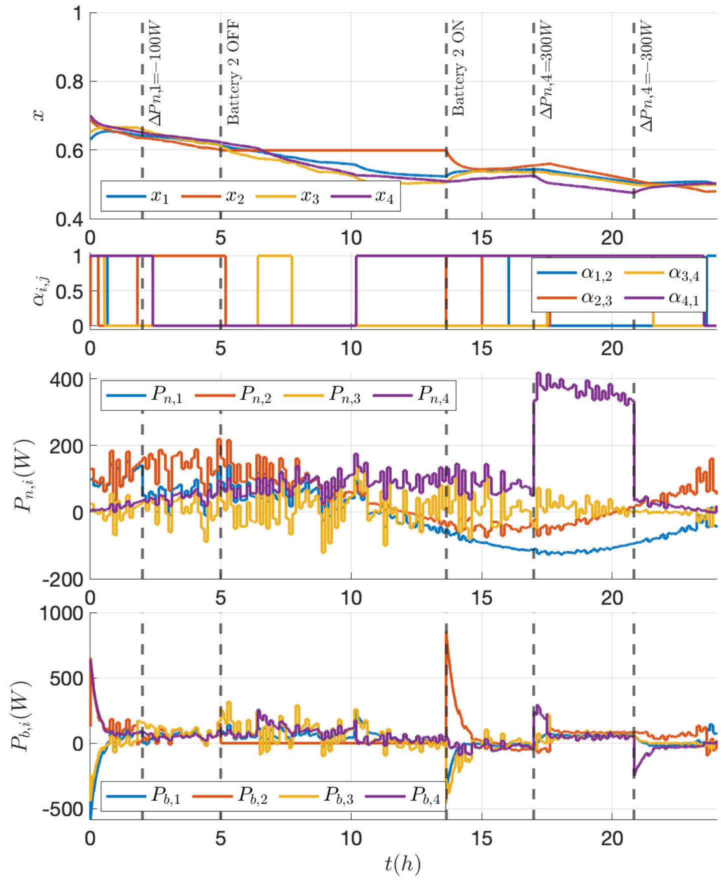

Figure 6 For $\varepsilon = 0.005$ and $\varepsilon_M = 0.03$, we have from the top to the bottom: the x_i, $\alpha_{i,j}$, $\boldsymbol{P}_{n,i}$ and $\boldsymbol{P}_{b,i}$ evolutions, respectively. There are unintentional connection/disconnection of any element of the MGs marked by the discontinuous lines.

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