A novel analytical solution for ponded infiltration with consideration of a developing saturated zone

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Abstract

Ponding at the soil surface exerts profound impacts on infiltration. However, the effects of ponding depth on infiltration, especially the development of a saturated zone below the soil surface, have not been considered in present infiltration models. A new general Green-Ampt model solution (GAMS) was derived for a one-dimensional vertical infiltration into soils under a uniform initial moisture distribution with ponding on its surface. An expression was included in the new solution for simulating the saturated layer developed below the soil surface as long as the pressure head at the surface is greater than the water-entry suction. The GAMS simulates the infiltration processes closer to the numerical solution by HYDRUS-1D than the traditional and a recently improved Green-Ampt model. Moreover, an inversion method to improve the estimates of soil hydraulic parameters from one-dimensional vertical infiltration experiments that is based on the GAMS was suggested. The effect of ponding depth (hp), initial soil moisture content, soil texture, and hydraulic soil properties (Ks, hd and n) in the saturated zone was also evaluated. The results indicate that the saturated zone developed at a much faster rate than the unsaturated zone during infiltration. Generally, a larger saturated zone was found for soils with higher initial soil moisture content, coarser texture, higher Ks values and lower hd and n. Our findings reveal that including the saturated zone in the infiltration model yields a better estimate for the soil hydraulic parameters. The proposed GAMS model can improve irrigation design and rainfall-runoff simulations.

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A novel analytical solution for ponded infiltration with consideration of a developing saturated zone

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Key Points:

- The saturated zone is longer than the unsaturated wetted zone during ponded infiltration.
- The new proposed infiltration equation includes an expression of the saturation zone versus time.
- The new solution simulates infiltration and estimates soil hydraulic properties more accurately.

Abstract

Ponding at the soil surface exerts profound impacts on infiltration. However, the effects of ponding depth on infiltration, especially the development of a saturated zone below the soil surface, have not been considered in present infiltration models. A new general Green-Ampt model solution (GAMS) was derived for a one-dimensional vertical infiltration into soils under a uniform initial moisture distribution with ponding on its surface. An expression was included in the new solution for simulating the saturated layer developed below the soil surface as long as the pressure head at the surface is greater than the water-entry suction. The GAMS simulates the infiltration processes closer to the numerical solution by HYDRUS-1D than the traditional and a recently improved Green-Ampt model. Moreover, an inversion method to improve the estimates of soil hydraulic parameters from one-dimensional vertical infiltration experiments that is based on the GAMS was suggested. The effect of ponding depth (h_p), initial soil moisture content, soil texture, and hydraulic soil properties (K_s, h_d and n) in the saturated zone was also evaluated. The results indicate that the saturated zone developed at a much faster rate than the unsaturated zone during infiltration. Generally, a larger saturated zone was found for soils with higher initial soil moisture content, coarser texture, higher K_s values and lower h_d and n. Our findings reveal that including the saturated zone in the infiltration model yields a better estimate for the soil hydraulic parameters. The proposed GAMS model can improve irrigation design and rainfall-runoff simulations.

1 Introduction

Infiltration is one of the most important components of land surface water cycles. Accurate simulation of infiltration rate is crucial in hydrological forecast, biogeochemical process simulation, agricultural water management, and soil and water conversation (Assouline, 2013).

However, infiltration is a complex process affected by many factors such as (1) soil structure and its spatial heterogeneity influenced by soil mineral particles and organic matter in physical, chemical and biological cycles (Bonetti et al., 2021; Fatichi et al., 2020; Vereecken et al., 2022); (2) chemical compositions of the soil water and infiltrating water (Klopp and Daigh, 2020); (3) initial soil moisture distribution across soil profile (Stewart et al., 2013; Wu et al., 2021); (4) type and rate of water supply on the soil surface (Assouline et al., 2007; Corradini et al., 1994; 1997). During the past one century, a vast amount of attentions have been attracted to the studies on developing mathematical infiltration models under various conditions (Green and Ampt, 1911; Haverkamp et al., 1994; Hogarth et al., 2013; Morbidelli et al., 2018; Moret-Fernández et al., 2020; Parlange, 1971; Parlange et al., 1982; Philip, 1969; Selker and Assouline, 2017; Stewart, 2019; Talsma and Parlange, 1972), as well as establishing methods to determine infiltration model parameters (Angulo-Jaramillo et al., 2019; Ma et al., 2017; Neuman, 1976; Parlange, 1975; Touma et al., 2007; Valiantzas, 2010; Vauclin and Haverkamp, 1985). Generally, the model expression and its deriving method for one-dimentional infiltration into homogeneous soils with uniform initial soil moisture distributions under a saturated or ponded upper boundary condition were taken as a base for the development of one, two or three-dimentional infiltration models under more complex conditions (Kargas and Londra, 2021; Selker and Assouline, 2017; Wu et al., 2022).

The effects of ponding on infiltration can be profound (Philip, 1958a; 1958b), especially in initially wet soils, and it acts through not only added surface water pressure but also water redistribution in soils (Parlange, 1972). The ponding can increase water pressure at soil surface and thus improve infiltration. Soil moisture profile would change accordingly to transmit and redistribute soil water potential gradient across the whole profile. Consequently, a saturated zone forms below the soil surface and increases during ponded infiltration. In addition, for soils with non-zero water-entry suction ($-h_d$), the saturated zone, defined as tension-saturated zone by (Philip, 1958a), would still develop even if the surface water pressure is zero or a negative value greater than water-entry suction (Haverkamp et al., 1990). Given that soils with non-zero water-entry suction are common in nature, a saturated zone composed of the two types above was found in most soil moisture profiles during infiltration under a surface pressure greater than water-entry suction (Philip, 1958a).

However, the effects of ponded water on infiltration have not been fully considered in the two types of widely used infiltration models under ponded conditions. One type includes the empirical and semi-empirical models which neglect the effect of ponding depth, such as Horton's infiltration model (Horton, 1941) in hydrology, Kostiakov model (Kostiakov, 1932) and Lewis-Kostiakov model (Mezencev, 1948) in surface irrigation. The other type includes analytical and semi-analytical infiltration models, such as Philip's Two-Term model (Philip, 1957a), Brutsaert's model (Brutsaert, 1977), Parlange's three-parameter model (Parlange et al., 1982), Swartzendruber model (Swartzendruber, 1987), the traditional Green-Ampt model (TGAM) (Green and Ampt, 1911) and the recently improved Green-Ampt model (GAME) (Ma et al., 2015). In these analytical and semi-analytical models, the effects of ponding depth on infiltration were only expressed in the form of an added surface pressure head.

Actually in 1958, Philip (1958a) has proposed an analytical method which includes a series expression of the time-dependent saturated zone by assuming a negligible effect of ponding depth on infiltration rate. His solution showed a good simulation accuracy in a short time and revealed a time-dependent soil moisture profile shape (Philip, 1958b). In 1972, Parlange (1972) built a general iterative solution in integral forms addressing the infiltration under ponded conditions, which can achieve accurate simulation for a longer time. A more concise solution was derived by Haverkamp et al. (1990) to depict the effects of ponding depth on infiltration, based on the first-order approximation of the above Parlange's solution (Parlange, 1972) and a flux-saturation relation. These solutions substantially improved the infiltration simulation by taking into account the profound effects of ponding depth on infiltration. Unfortunately, few of the results has been adopted in subsequent infiltration simulations. One of the most important reasons is that the solutions in integral forms are too complex to use in practice.

The main applications of infiltration formulas are to simulate infiltration and to estimate its parameters. During the past two decades, some analytical or semi-analytical solutions to infiltration problems have found their exciting function of rapidly inverting soil hydraulic properties from infiltration experiments (Jaiswal et al., 2022; Ma et al., 2016, 2017; Rahmati et al., 2021) and overcoming the problem of non-convergence and unitability in numerical inversion. However, the accuracy of the inverted parameters of soil hydraulic property model were found sensitive to the accuracy of the forward analytical or semi-analytical solutions (Ma et al., 2009). A small difference in an infiltration formula from a real infiltration process may result in time-dependent estimated parameters in its applications. Our previous studies (Ma et al., 2015, 2017) exhibited that to build the quantitative relationship between the Green-Ampt model simple in form and Richards equation accurate in simulation was an effective approach to achieve soil hydraulic properties from infiltration experiments. The traditional Green-Ampt model (TGAM) is just a special solution to Richards equation for soils with delta-type diffusivity (Philip, 1957b) and thus too simplified to accurately estimate soil hydraulic properties. Recently, a more sophisticated Green-Ampt model (GAME) found its deterministic relevance to Richards equation for general soils (Ma et al., 2015). Based on the new approximate analytical solution, a compatible method was derived to determine soil hydraulic properties (Ma et al., 2015, 2017). However, the effects of ponding depth on infiltration were still not fully considered in that solution, especially the changes in the shape of soil moisture profile.

Therefore, the objectives of this research are (1) to develop a new solution to one-dimensional infiltration under ponded conditions, including a simple infiltration equation with ponding depth effects and explicit expressions of saturated and unsaturated zone length varying with time; (2) to evaluate the effects of ponding depth on infiltration simulation and soil hydraulic parameter estimation.

2 Theory

2.1 A general solution to infiltration with a constant water head

According to Ma et al. (2015), a general relationship for a vertical infiltration into soils with initially uniform soil moisture distribution under ponding with a constant water depth is given as:

$$\int_0^{z_\mathrm{f}} \left(\left(J_{\mathrm{w}0} - K_\mathrm{i} \right) F + K_\mathrm{i} \right) dz = \int_0^{z_\mathrm{f}} K dz - \int_0^{h_\mathrm{i}} K dh - \int_{h_\mathrm{p}}^{0} K dh \qquad (1)$$

where K is the soil hydraulic conductivity (cm min^{-1}); K_i is the soil hydraulic conductivity at the initial soil water content (cm min^{-1}); $J_{\mathrm{w}0}$ is the infiltration rate or surface soil water flux (cm min^{-1}); F is the soil water flux-saturation function; h is the soil water pressure head (cm); h_i is the initial soil water pressure head (cm); h_p is the water depth on soil surface (cm); z is the soil depth (cm) with zero point at soil surface and downward coordinate axis; and z_f is the wetting front advance or the length of wetted zone (cm).

Since $J_{\mathrm{w}0}$ is a variable independent of z, after rearranging Equation (1), it is expressed as:

$$J_{\mathrm{w0}} = \frac{\int_0^{z_\mathrm{f}} \left(K - \left(1 - F\right) K_\mathrm{i} \right) dz - \int_0^{h_\mathrm{i}} K dh - \int_{h_\mathrm{p}}^0 K dh}{\int_0^{z_\mathrm{f}} F dz} \tag{2}$$

Since the saturated soil drains only when water head drops below air entry suction $-h_\mathrm{d}$ (cm), there must be a saturated zone on the upper part of the wetted zone where $h > -h_\mathrm{d}$. Assuming that soil water content profile in the unsaturated wetted zone can be described with a simple function (e.g. in Ma et al., 2015) and its relative shape does not change with time, then the soil water content profile (Figure 1) can be described as:

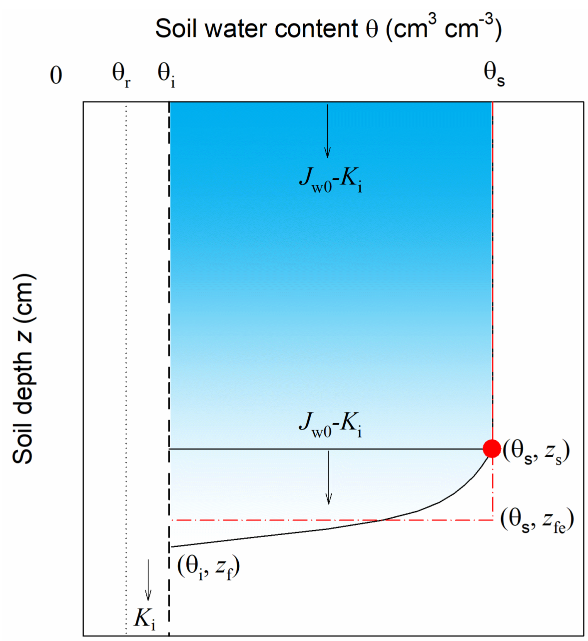

Figure 1. Schematic diagram of soil moisture profile including saturated and unsaturated wetted zones. θ_r and θ_i are the residual and initial soil water content, respectively; θ_s is the soil water content at the water inlet; J_w0 is the surface water flux; K_i is the soil hydraulic conductivity at θ_i; z_s is the length of saturated zone; z_f and z_fe are actual and equivalent lengths of wetted zone, respectively.

$$S=\frac{\theta-\theta_{\mathrm{r}}}{\theta_{\mathrm{s}}-\theta_{\mathrm{r}}}=\begin{cases}1 & z<z_{\mathrm{s}}\\ S\left(\dfrac{z-z_{\mathrm{s}}}{z_{\mathrm{f}}-z_{\mathrm{s}}}\right) & z_{\mathrm{f}}\geq z\geq z_{\mathrm{s}}\\ S_{\mathrm{i}} & z\geq z_{\mathrm{f}}\end{cases} \tag{3}$$

where θ is the soil water content (cm^3 cm^{-3}); θ_s and θ_r are the saturated and residual soil water content (cm^3 cm^{-3}), respectively; S is the relative saturation; S_i is the relative saturation at, θ_i, the initial soil water content (cm^3 cm^{-3}); z_s is the length of saturated zone (cm).

Correspondingly, the soil hydraulic conductivity in the profile is written as

$$K=\begin{cases}K_{\mathrm{s}} & 0\leq z<z_{\mathrm{s}} & h>-h_{\mathrm{d}}\\ K(h) & z_{\mathrm{f}}\geq z\geq z_{\mathrm{s}} & h_{\mathrm{i}}\leq h\leq -h_{\mathrm{d}}\\ K_{\mathrm{i}} & z\geq z_{\mathrm{f}} & h=h_{\mathrm{i}}\end{cases} \tag{4}$$

where K_s is the saturated soil hydraulic conductivity (cm min^{-1}).

Then, with Equation (4), the second and third terms on the right side of Equation (1) can be transformed to

$$\int_0^{h_{\mathrm{i}}} K dh=\int_{-h_{\mathrm{d}}}^{h_{\mathrm{i}}} K dh+\int_0^{-h_{\mathrm{d}}} K dh=\int_{-h_{\mathrm{d}}}^{h_{\mathrm{i}}} K dh-K_{\mathrm{s}} h_{\mathrm{d}} \tag{5}$$

$$\int_{h_{\mathrm{p}}}^{0} K dh=-K_{\mathrm{s}} h_{\mathrm{p}} \tag{6}$$

According to Philip (1973), the relationship between F and soil water content can be described with a simple function:

$$F(\theta)=\frac{J_{\mathrm{w}}-K_{\mathrm{i}}}{J_{\mathrm{w0}}-K_{\mathrm{i}}}=\frac{\theta-\theta_{\mathrm{i}}}{\theta_{\mathrm{s}}-\theta_{\mathrm{i}}} \tag{7}$$

where J_w is the soil water flux (cm min^{-1}). More accurate functions can be found in Ma et al. (2017a). No matter what the specific function of F is, F should be 1 in the saturated zone and gradually decreases from 1 to 0 in the unsaturated wetted zone.

Defining the equivalent wetting front length of the unsaturated wetted zone (Figure 1) as z_ufe by using the piston-type assumption of water flow in Green & Ampt (1911), after considering Equation (3) and Equation (7), we get

$$z_\mathrm{ufe}=\frac{\int_{z_\mathrm{s}}^{z_\mathrm{f}}(\theta-\theta_\mathrm{i})dz}{\theta_\mathrm{s}-\theta_\mathrm{i}}=\left(z_\mathrm{f}-z_\mathrm{s}\right)B_0 \tag{8}$$

where

$$B_0=\int_0^1\frac{\theta-\theta_\mathrm{i}}{\theta_\mathrm{s}-\theta_\mathrm{i}}d\frac{z-z_\mathrm{s}}{z_\mathrm{f}-z_\mathrm{s}} \tag{9}$$

Combining Equation with Equation (8) yields

$$\int_0^{z_\mathrm{f}}Fdz=z_\mathrm{s}+\int_{z_\mathrm{s}}^{z_\mathrm{f}}Fdz=z_\mathrm{s}+z_\mathrm{ufe} \tag{10}$$

$$\int_0^{z_\mathrm{f}}\left(K\text{-}\left(1-F\right)K_\mathrm{i}\right)dz=K_s z_\mathrm{s}+\frac{z_\mathrm{ufe}}{B_0}\int_0^1\left(K-\left(1-F\right)K_\mathrm{i}\right)d\frac{z-z_\mathrm{s}}{z_\mathrm{f}-z_\mathrm{s}} \tag{11}$$

Substituting Equation (5), Equation (6), Equation (10) and Equation (11) in Equation (2) and rearranging the equation, the expression of J_w0 related to the length of saturated zone and the equivalent one of unsaturated wetted zone can be expressed as

$$J_\mathrm{w0}=K_\mathrm{s}\left(1+\frac{B_1+B_3-B_2 z_\mathrm{ufe}}{z_\mathrm{s}+z_\mathrm{ufe}}\right) \tag{12}$$

where

$$B_1 = -\int_{-h_d}^{h_i} \frac{K}{K_s} dh \tag{13}$$

$$B_2 = 1 - \frac{\int_0^1 \left(\frac{K}{K_s} - (1-F)\frac{K_i}{K_s} \right) d\frac{z - z_s}{z_f - z_s}}{\int_0^1 F d\frac{z - z_s}{z_f - z_s}} \tag{14}$$

$$B_3 = h_p + h_d \tag{15}$$

Actually, the expression of J_{w0} can be also derived by applying Darcy's law to the saturated zone. Soil water flux is evenly distributed with saturated hydraulic conductivity in the saturated zone. The pressure head should be h_p at the upper boundary of the saturated zone and $-h_d$ at its bottom.

Then,

$$J_{w0} = K_s\left(1 + \frac{h_p + h_d}{z_s}\right) = K_s\left(1 + \frac{B_3}{z_s}\right) \tag{16}$$

Combining Equation (12) with Equation (16) yields

$$z_{ufe} = \frac{B_1 z_s}{B_2 z_s + B_3} \tag{17}$$

According to the definition by Green and Ampt (1911), the length of equivalent wetted zone, z_{fe}, should be the sum of the saturated zone length, z_s, and the equivalent wetting front length of the unsaturated zone, z_{ufe}, that is,

$$z_{fe} = z_s + z_{ufe} = z_s + \frac{B_1 z_s}{B_2 z_s + B_3} \tag{18}$$

Then,

$$I=\left(\theta_{\mathrm{s}}-\theta_{\mathrm{i}}\right)z_{\mathrm{fe}}+K_{\mathrm{i}}t=\left(\theta_{\mathrm{s}}-\theta_{\mathrm{i}}\right)\left(z_{\mathrm{s}}+\frac{B_1 z_{\mathrm{s}}}{B_2 z_{\mathrm{s}}+B_3}\right)+K_{\mathrm{i}}t \tag{19}$$

where I is the cumulative infiltration or cumulative surface water flux (cm); K_{i} is normally negligible in most cases.

Since the surface water flux is the derivative of the cumulative infiltration, we obtain other expression of J_{w0} by neglecting K_{i},

$$J_{\mathrm{w0}}=\frac{dI}{dt}=\left(\theta_{\mathrm{s}}-\theta_{\mathrm{i}}\right)\frac{dz_{\mathrm{fe}}}{dt}=\left(\theta_{\mathrm{s}}-\theta_{\mathrm{i}}\right)\left(1+\frac{B_1 B_3}{\left(B_2 z_{\mathrm{s}}+B_3\right)^2}\right)\frac{dz_{\mathrm{s}}}{dt} \tag{20}$$

Combining Equation (16) and Equation (20) to eliminate J_{w0} and conducting definite integration of z_{s} from 0 to z_{s} and t from 0 to t, we derive the implicit expression of the saturated zone length with time,

$$z_{\mathrm{s}}-\left(B_3+B_5\right)\ln\left(1+\frac{z_{\mathrm{s}}}{B_3}\right)+B_5\ln\left(1+\frac{B_2 z_{\mathrm{s}}}{B_3}\right)+\frac{B_1}{\left(1-B_2\right)B_2}\left(1-\frac{B_3}{\left(B_2 z_{\mathrm{s}}+B_3\right)}\right)=B_4 t \tag{21}$$

where

$$B_4=\frac{K_s}{\theta_{\mathrm{s}}-\theta_{\mathrm{i}}} \tag{22}$$

$$B_5=\frac{B_1}{\left(1-B_2\right)^2} \tag{23}$$

With Equation (8) and Equation (17), z_{f} can be derived from z_{s}

$$z_{\mathrm{f}}=z_{\mathrm{s}}+\frac{1}{B_0}z_{\mathrm{ufe}}=z_{\mathrm{s}}+\frac{B_1 z_{\mathrm{s}}}{B_0\left(B_2 z_{\mathrm{s}}+B_3\right)} \tag{24}$$

Given flux–saturation relationship F, specific soil moisture profile function (S~z), soil hydraulic properties (K~h and h~θ), initial condition (θ_i) and boundary condition (h_p), the length of saturated zone can be calculated by solving Equation (21). Then, the cumulative infiltration can be calculated from z_s by using Equation (19). Accordingly, the surface water flux and the length of the wetted zone can be calculated using Equation (16) and Equation (24), respectively. Finally, Equation (16), Equation (19), Equation (21) and Equation (24) constitute a new solution to one-dimensional vertical infiltration with the upper boundary of a constant pressure head. In order to differentiate the new solution from TGAM and GAME, the new solution is named as Green-Ampt Model Solution, the GAMS.

2.2 A special solution for infiltration with Brooks-Corey model

Soil water retention curves and unsaturated hydraulic conductivities can be described using the model proposed by Brooks and Corey (1964), denoted as the BC model:

$$S(h)=\begin{cases}\dfrac{\theta-\theta_{\mathrm{r}}}{\theta_{\mathrm{s}}-\theta_{\mathrm{r}}}=\left|\dfrac{h_{\mathrm{d}}}{h}\right|^{n} & h<-h_{\mathrm{d}}\\ 1 & h\geq -h_{\mathrm{d}}\end{cases} \tag{25}$$

$$K(h)=\begin{cases}K_s\left|\dfrac{h_d}{h}\right|^{m}=K_sS^{m/n} & h<-h_d\\ K_s & h\geq -h_d\end{cases} \tag{26}$$

where $m = (l + 1)\,n + 2$ with Burdine's method (Burdine, 1953); l is the soil pore tortuosity factor and normally $l = 2$ in the BC model.

With the upper boundary condition of saturation (i.e. $h = -h_d$), Ma et al. (2015) derived an expression of soil moisture profile for infiltration into soils with initially uniform soil water content. Similar expression can be obtained with exactly the same deriving steps in Ma et al.

(2015) for the soil water content profile of the unsaturated wetted zone in the current study. Substituting the length of unsaturated zone here for the length of wetted zone in Ma et al. (2015) yields,

$$S=\left(1-b\frac{z-z_{\mathrm{s}}}{z_{\mathrm{f}}-z_{\mathrm{s}}}\right)^{a} \tag{27}$$

where

$$a=\frac{n}{2n+2} \tag{28}$$

$$b=1-S_{\mathrm{i}}^{1/a} \tag{29}$$

Substituting Equation (25)-(27) to Equation (13) and Equation (14), we get

$$B_{0}=\left(\theta_{\mathrm{s}}-\theta_{\mathrm{r}}\right)\frac{1-\left(1+ab\right)S_{\mathrm{i}}}{b\left(a+1\right)} \tag{30}$$

$$B_{1}=\frac{1-S_{\mathrm{i}}^{3+1/n}}{3n+1} \tag{31}$$

$$B_{2}=1-\frac{\left(a+1\right)\left(1-S_{\mathrm{i}}\right)-\left(a\left(a+2\right)b-\left(1-b\right)\left(1-S_{\mathrm{i}}\right)\right)S_{\mathrm{i}}^{3+2/n}}{\left(1-\left(1+ab\right)S_{\mathrm{i}}\right)\left(a+2\right)} \tag{32}$$

The other parameter B_3, B_4 and B_5 can be calculated by Equation (15), Equation (22) and Equation (23), respectively. Finally, Equation (16), Equation (19), Equation (21) and Equation (24) constitute a special solution with the BC model compared to the general solution above. The GAMS in the next part refers to this special solution.

3 Materials and methods

3.1 Model validation and evaluation

As an example, a loam soil with BC model parameters (θ_s = 0.434 cm cm^{-3}, θ_r = 0.027 cm cm^{-3}, n = 0.22, h_d = 11.15 cm, K_s = 0.022 cm min^{-1}) was used to validate the performance of the GAMS model. Additionally, the relations with the TGAM and GAME models were investigated regarding their different ways of treating the saturated zone. In order to avoid the disturbance of uncertain errors in real experiments on the theoretical evaluation, the numerical solution of HYDRUS-1D (Šimůnek et al., 2005) was used as the exact solution to produce the infiltration data needed to validate the new solution. The simulations were also conducted by both analytical and numerical methods to evaluate the influence of the developing saturated zone on infiltration.

The GAMS calculation was made following the procedures provided in section 2.2. The calculation of TGAM and GAME followed the same procedures descried in Ma et al. (2015). The numerical simulations of the Richards equation were conducted by the HYDRUS-1D software package (version 3.0) for the constant-head 1D vertical infiltration problem (Šimůnek et al., 2005). The soil column in the simulation was 200 cm in length with a discrete interval of 0.25 cm, and a uniform initial soil moisture distribution (θ_i = 0.04 cm cm^{-3}). The upper boundary of a constant water head (h_p = 0) and free drainage lower boundary were defined for the simulation. The infiltration time was 2800 min. The soil column was considered as semi-infinite, since the simulation was set to stop before the wetting front reaches the bottom. The simulated surface water flux, cumulative surface water flux, soil water flux and soil water content profiles at time steps of 10 min, 30 min, 60 min, 100 min, 500 min, 1000 min, 1500 min and 2000 min, were directly extracted from the simulated data by HYDRUS-1D. The lengths of the saturated zone (z_s) and wetted zone (z_f) were determined by checking the soil moisture profiles.

Specifically, on a soil moisture profile, the depth where soil water content began to be lower than θ_s was taken as z_s, and the depth where soil water content drop to θ_i was taken as z_f.

Relative error (RE) was employed to evaluate the deviation of the three models (GAMS, GAME and TGAM) from the numerical solution.

$$\mathrm{RE_i} = \frac{Y_\mathrm{i} - O_\mathrm{i}}{O_\mathrm{i}} \times 100\% \tag{33}$$

where Y_i is the simulated value by analytical solutions (GAMS, GAME and TGAM); O_i is the observed value produced by HYDRUS-1D.

3.2 Estimation of model parameters

The new GAMS was numerically inverted to obtain the three parameters of soil hydraulic properties (n, h_d and K_s) from infiltration data (i.e. cumulative infiltration and wetted zone length versus time). Notably, the method based on the GAMS considered not only the unsaturated zone, which was ignored in the TGAM based methods (Ma et al., 2017), but also the developing saturated zone, which was neglected in the GAME method in the first stage of infiltration (Ma et al., 2017). The estimated soil hydraulic parameters by the methods based on GAMS, GAME and TGAM were compared to check the influence of the developing saturated zone on the estimation of soil hydraulic properties.

Furthermore, the time-dependent accuracy of analytical solution was investigated. The same loam soil (θ_s = 0.434 cm cm^{-3}, θ_r = 0.027 cm cm^{-3}, θ_i = 0.04 cm cm^{-3}, n = 0.22, h_d = 11.15 cm, K_s = 0.022 cm min^{-1}) was used as the tested soil. The observed time-lapse data (OBS) of cumulative infiltration and the length of wetted zone were produced by Hydrus-1D for the estimation of soil hydraulic parameters. Given the known parameters (θ_s = 0.434 cm cm^{-3}, θ_r = 0.027 cm cm^{-3}, θ_i =

0.04 cm cm^{-3}) and the unknown parameters (n, h_d and K_s), the Levenberg-Marquardt algorithm was employed to minimize the objective function to estimate the unknown parameters (n, h_d and K_s)

$$Q = \sum_{i=1}^{N}\left(\hat{I}_i\left(n, h_d, K_s\right) - I_i\right)^2 + \sum_{i=1}^{N}\left(\hat{z}_{f,i}\left(n, h_d, K_s\right) - z_{f,i}\right)^2 \tag{34}$$

where Q is the objective function; $\hat{I}_i$ and I_i are the simulated cumulative infiltration by the GAMS and the observed data produced by HYDRUS-1D, respectively; $z_{f,i}$ and $\hat{z}_{f,i}$ are the length of wetted zone simulated by the GAMS and the observed data produced by HYDRUS-1D, respectively. Details of the methods based on the GAME and TGAM for estimating soil hydraulic parameters from one-dimensional vertical infiltration can be found in Ma et al. (2017).

Moreover, the ratio of the saturated zone length to the effective wetted zone length (denoted as $LR_{S/EW} = z_s/z_{fe}$ hereafter) in the GAMS was calculated based on Equation (18) and Equation (21), and its sensitivities to soil texture (listed in Table 1), initial condition (θ_i) and boundary condition (h_p) were investigated to theoretically analyze their influences on the development of the saturated zone. The initial and boundary conditions were defined based on the loam soil (θ_s = 0.434 cm cm^{-3}, θ_r = 0.027 cm cm^{-3}, n = 0.22, h_d = 11.15 cm, K_s = 0.022 cm min^{-1}) with six values of θ_i (i.e. 0.04 cm cm^{-3}, 0.08 cm cm^{-3}, 0.12 cm cm^{-3}, 0.16 cm cm^{-3}, 0.2 cm cm^{-3} and 0.25 cm cm^{-3}) for h_p = 0, and with six values of h_p (i.e. 0 cm, 2 cm, 5 cm, 10 cm, 15 cm and 20 cm) for θ_i = 0.04 cm cm^{-3}.

Table 1. Soil hydraulic properties of different textured soils (Hydrus-1D) and initial conditions for numerical simulations.

Soil texture	θ_i cm cm^{-3}	θ_r cm cm^{-3}	θ_s cm cm^{-3}	n	h_d cm	K_s cm min^{-1}
Loamy Sand	0.05	0.035	0.401	0.474	8.70	0.1018
Sandy Loam	0.05	0.041	0.412	0.322	14.66	0.0432
Loam	0.04	0.027	0.434	0.220	11.15	0.0220
Silt	0.03	0.015	0.486	0.211	20.75	0.0113
Sandy Clay Loam	0.08	0.068	0.330	0.250	28.09	0.0072
Clay Loam	0.09	0.075	0.390	0.194	25.91	0.0038
Silty Clay Loam	0.08	0.040	0.432	0.151	32.57	0.0025
Sandy Clay	0.12	0.109	0.321	0.168	29.15	0.0020
Silty Clay	0.10	0.056	0.423	0.127	34.25	0.0015
Clay	0.12	0.090	0.385	0.131	37.31	0.0010

4 Results

4.1 The forward solution for infiltration simulation

As depicted in Figure 2 and Figure 3a, the surface water flux, cumulative surface water flux and wetted zone length simulated by the GAMS with the parameters independently obtained from soil hydraulic properties agree well with those simulated by HYDRUS-1D for the tested loam soil. The relative errors of the simulated surface water flux, cumulative surface water flux and wetted zone length are less than 8% for all time steps and less than 5% for most of the time steps (Figure 4). The relative errors of these three infiltration variables are the highest at the primary stage of the infiltration and gradually drops with the lapsed time. The simulated surface water flux is very close to the exact solution with relative errors close to 0 after 500 min (Figure 4a). The relative errors of the simulated cumulative surface water flux and wetted zone length decrease to about 2% after 500 min (Figure 4b and 4c). As shown in Figure 3b, the GAMS give accurate estimates of the saturated zone length all along the time. However, the GAMS seems to slightly overestimate the length of the unsaturated zone but with no increasing deviation (Figure 3c). This should be responsible for the 2% relative errors of the simulated cumulative surface water flux and wetted zone length in the later stage of the infiltration. Generally, the soil moisture profiles simulated by the GAMS agree well with the those simulated by HYDRUS-1D from a long time perspective but slight deviation exists in the initial short time of infiltration (Figure 5).

In general, the novel GAMS model enables a more accurate simulation of infiltration process than the GAME and TGAM models for a loam soil (Figures 2-5). Obviously, the TGAM overestimated the surface water flux, cumulative surface water flux, and the length of the wetted zone with REs of about 10%-18%. The GAME underestimated the surface water flux, cumulative surface water flux and the length of wetted zone with REs of about 10%. The GAMS gave the best simulations with the smallest and decreasing REs among the three models. Only in the first stage of infiltration, the simulation accuracy of the GAME (Figure 4) is comparable to and even higher than the GAMS (Figure 4c, Figure 5). However, in the first stage of infiltration, the GAME shows increasing errors (Figure 3c, Figure 4). In the second stage of infiltration, the surface water flux was still underestimated by the GAME with REs of about -10%. It should be noted that the GAME simulated the soil moisture profiles better than the GAMS in the first stage but the REs of the GAME substantially increased after the critical time (Figure 5).

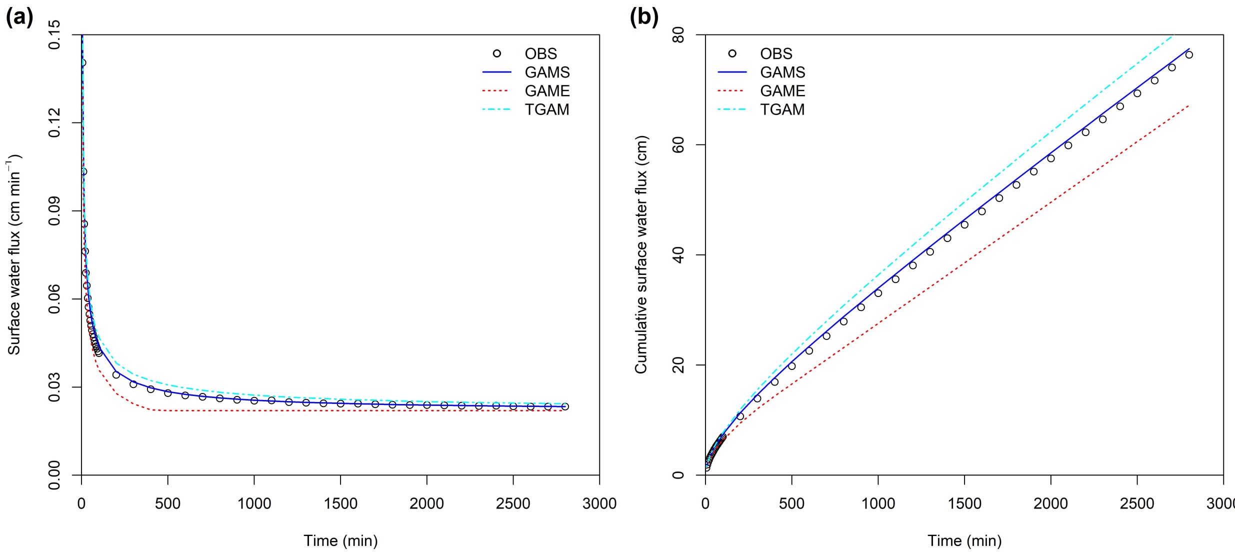

Figure 2. Simulated (a) surface water flux, and (b) cumulative surface water flux by the GAMS, GAME and TGAM, respectively, compared with the observed data (OBS) produced by the numerical solution (HYDRUS-1D) for a loam soil.

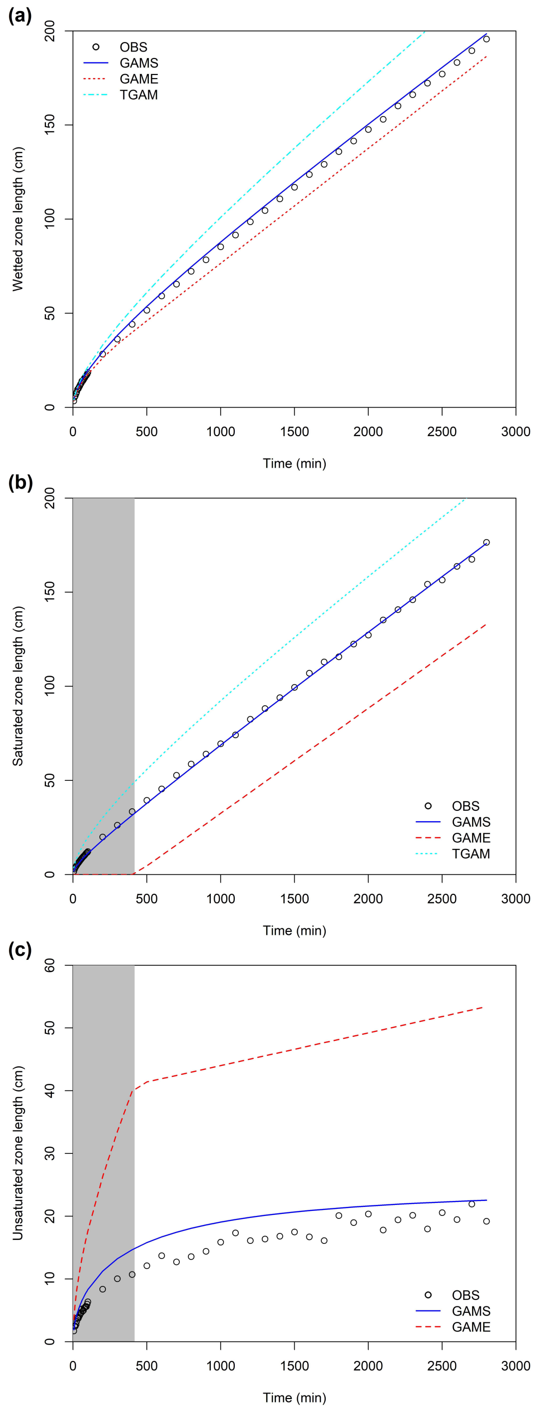

Figure 3. Simulated lengths of (a) wetted zone, (b) saturated zone, and (c) unsaturated zone by the GAMS, GAME and TGAM, respectively, compared with the observed data (OBS) produced by the numerical solution (HYDRUS-1D) for a loam soil.

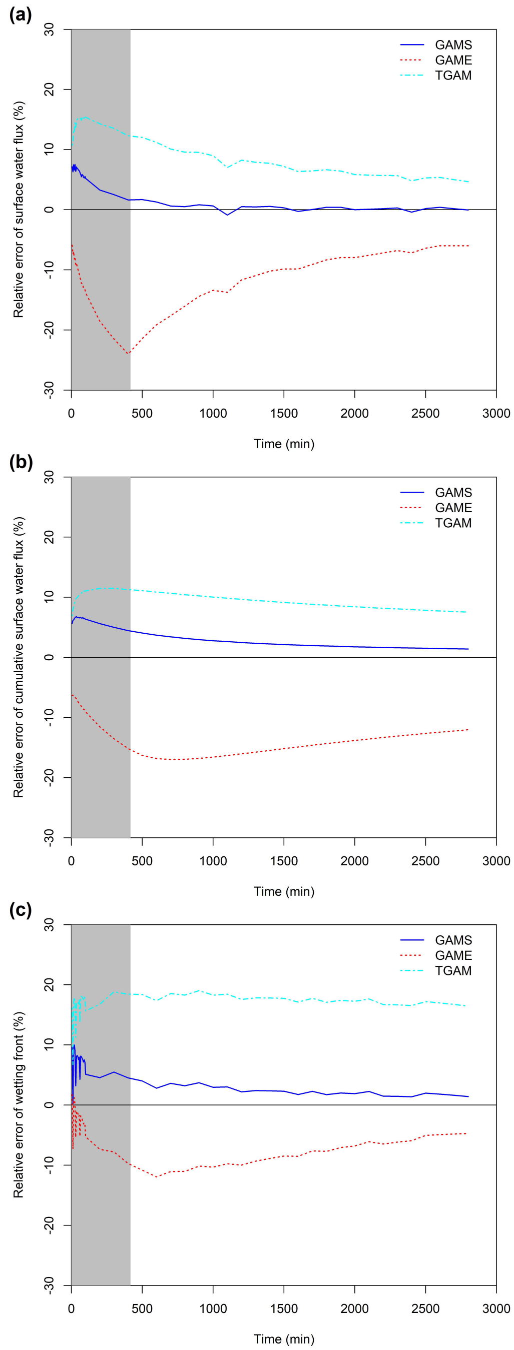

Figure 4. Relative error (RE) of (a) surface water flux, (b) cumulative surface water flux and (c) length of wetted zone simulated by the GAMS, GAME and TGAM, respectively, to that by the numerical solution (HYDRUS-1D) for a loam soil. The gray zone represents the infiltration before the critical time calculated by equation (29) in Ma et al. (2015).

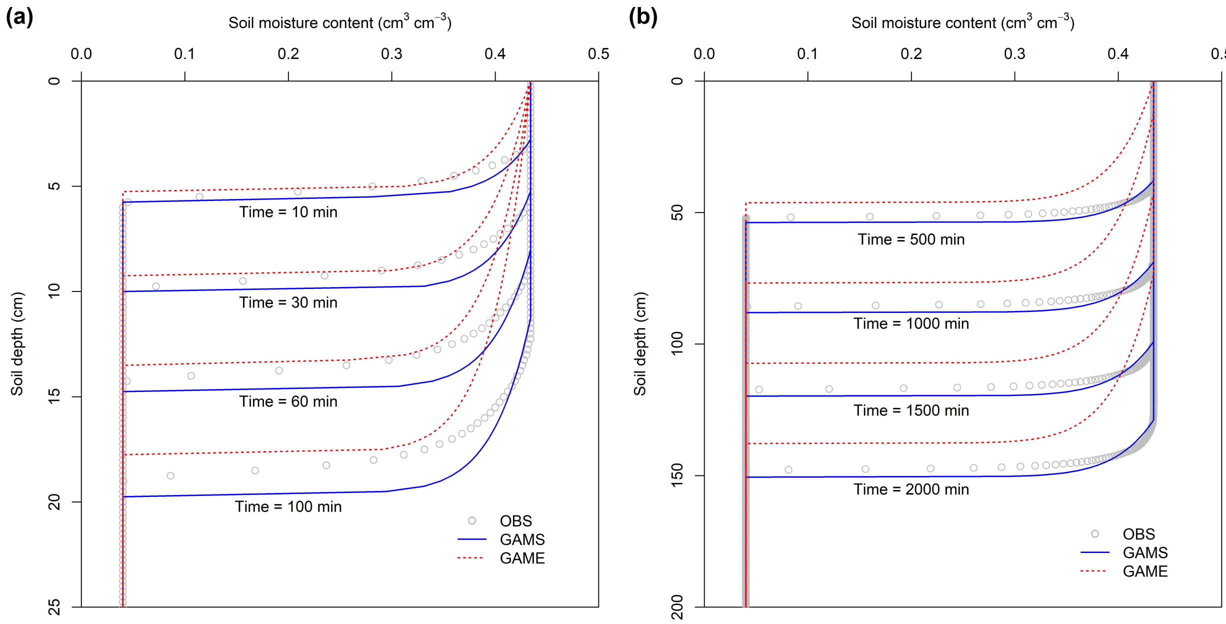

Figure 5. Simulated soil moisture profiles in (a) short infiltration time and (b) long infiltration time by the GAMS and GAME, respectively, compared with the observed data (OBS) produced by the numerical solution (HYDRUS-1D) for a loam soil.

4.2 The influence of saturated zone on the estimation of soil hydraulic properties

Figure 6 shows the estimated parameters of soil hydraulic properties for a loam soil by inverting the GAMS, GAME and TGAM models. The estimated values of n and h_d by these three models exhibit a similar decline tendency along the infiltration time, while the estimated values of K_s increase with the infiltration time. Generally, the estimated parameters by the GAMS are closer to the real values compared with those by the GAME. The values of n estimated by the TGAM is equal to those by the GAME, while the estimated values of h_d and K_s by the TGAM are close to those by the GAMS. The results indicate that the critical time is important for the estimation accuracy of the hydraulic parameters. Before the critical time, especially in a short time, the estimated parameters by the GAME show relatively lower errors than that by the GAMS. After the critical time, however, the estimation by the GAMS exhibits a higher accuracy than that by the GAME. Whether in a short or long time, the inverting of the TGAM cannot simultaneously give accurate estimates of n, h_d and K_s. Generally, the GAMS improved the estimate accuracy of soil hydraulic properties by the infiltration models from the infiltration process after the critical time.

4.3 The sensitivity analysis of saturated zone length

As shown in Figure 7, $LR_{S/EW}$ can be affected by soil properties (i.e. soil texture), boundary conditions (i.e. surface water depth), and initial conditions (i.e. initial soil water content). Without ponding water over soil surface, the values of $LR_{S/EW}$ were initially equal to 0.5, increased with infiltration time and approached to 1 at infinity. Increased ponding depth can promote the proportion of the saturated zone especially at the initial stage of infiltration but its effects attenuate with time (Figure 7a). The $LR_{S/EW}$ increased with elevating initial soil water content while little influence can be found at the initial stage of infiltration (Figure 7b). Obviously, soil texture shows the greatest effect on the development of saturated zone (Figure 7c). The $LR_{S/EW}$ for a clay soil increased slowly with time and was close to 0.5 for most of time. The results indicate that the coarser the soil texture, the greater the $LR_{S/EW}$ at the same infiltration time. The $LR_{S/EW}$ for a loamy sandy soil increased rapidly and approached to 1 as infiltration continued.

As shown in Figure 8a, $LR_{S/EW}$ can also be affected by soil hydraulic properties, that is, the shape coefficient n of soil water retention curve, water-entry suction h_d, and saturated hydraulic conductivity K_s. A higher n value caused a lower fraction of saturated zone (Figure 8a) during infiltration. A greater value of h_d resulted in a lower $LR_{S/EW}$ (Figure 8b). A higher value of K_s accelerated the development of z_s (Figure 8c). Compared to h_d and K_s, the influence of the shape coefficient n on $LR_{S/EW}$ seems to be negligible.

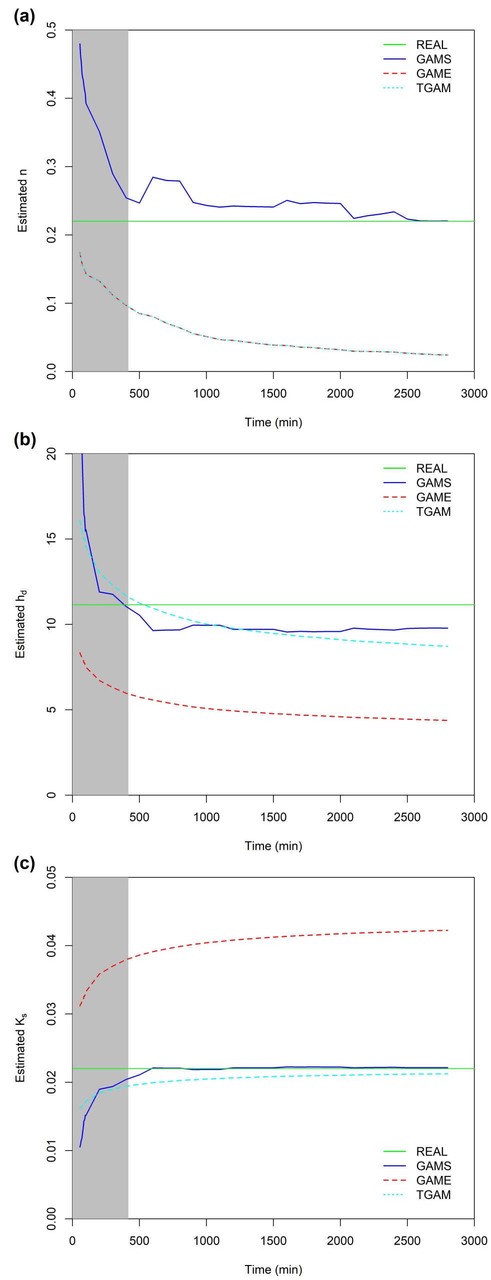
(a)
Estimated n
Time (min)
REAL
GAMS
GAME
TGAM
(b)
Estimated h_d
Time (min)
REAL
GAMS
GAME
TGAM
(c)
Estimated K_s
Time (min)
REAL
GAMS
GAME
TGAM

Figure 6. Time-dependent (a) shape coefficient n, (b) water-entry suction h_d, and (c) saturated hydraulic conductivity K_s estimated by model inversion of GAMS, GAME and TGAM, respectively, with the observed data of different infiltration time, compared with the real parameter values (REAL) of a loam soil. The gray zone represents the infiltration before the critical time calculated by equation (29) in Ma et al. (2015).

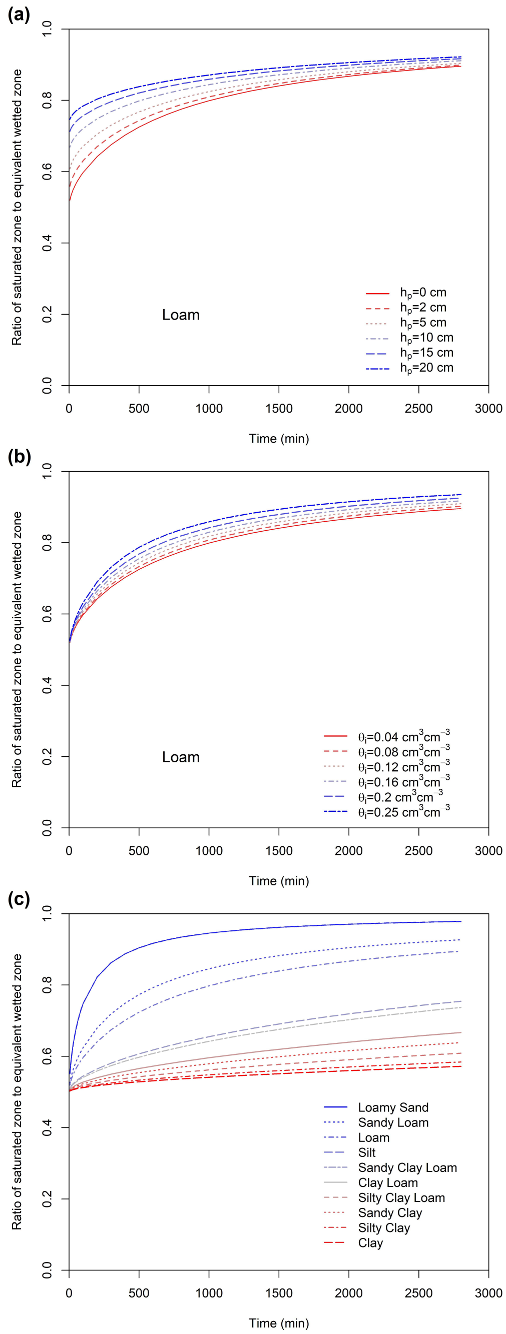
(a)
Ratio of saturated zone to equivalent wetted zone
Time (min)
Loam
h_p=0 cm
h_p=2 cm
h_p=5 cm
h_p=10 cm
h_p=15 cm
h_p=20 cm
(b)
Ratio of saturated zone to equivalent wetted zone
Time (min)
Loam
θ_i=0.04 cm^{3}cm^{-3}
θ_i=0.08 cm^{3}cm^{-3}
θ_i=0.12 cm^{3}cm^{-3}
θ_i=0.16 cm^{3}cm^{-3}
θ_i=0.2 cm^{3}cm^{-3}
θ_i=0.25 cm^{3}cm^{-3}
(c)
Ratio of saturated zone to equivalent wetted zone
Time (min)
Loamy Sand
Sandy Loam
Loam
Silt
Sandy Clay Loam
Clay Loam
Silty Clay Loam
Sandy Clay
Silty Clay
Clay

Figure 7. Sensitive analysis of the length ratio of saturated zone to equivalent wetted zone $LR_{S/EW}$ to (a) surface water depth (h_p= 0 cm, 2 cm, 5 cm, 10 cm, 15 cm and 20 cm) for a loam soil, (b) initial soil water content (θ_i = 0.04 cm cm^{-3}, 0.08 cm cm^{-3}, 0.12 cm cm^{-3}, 0.16 cm cm^{-3}, 0.2 cm cm^{-3} and 0.25 cm cm^{-3}) for a loam soil, and (c) soil texture (see in Table 1) by using the GAMS.

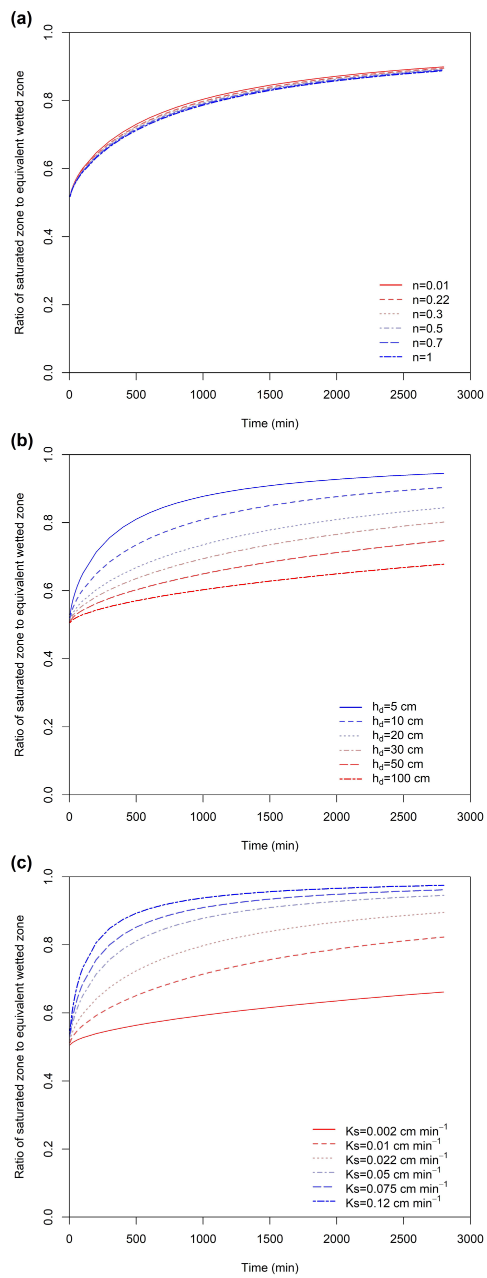

Figure 8. Sensitive analysis of the length ratio of saturated zone to equivalent wetted zone $LR_{S/EW}$ to (a) the shape coefficient n (n = 0.01, 0.22, 0.3, 0.5, 0.7 and 1), (b) water-entry suction h_d (h_d = 5 cm, 10 cm, 20 cm, 30 cm, 50 cm and 100 cm), and (c) saturated hydraulic conductivity K_s (K_s = 0.002 cm min^{-1}, 0.01 cm min^{-1}, 0.022 cm min^{-1}, 0.05 cm min^{-1}, 0.075 cm min^{-1} and 0.12 cm min^{-1}) for a loam soil by using the GAMS.

5 Discussion

5.1 The factors influencing the development of saturated zone

The results in Figure 7a agree well with the early research by Philip (1958b). A higher ponding depth on the surface can promote the development of saturated zone, which can be deduced by Equation (12) and Equation (15). Initially wetter soils have smaller space for further water storage and narrower range of soil moisture in the wetted zone. Thus, the saturated zone developed more quickly in a wet soil than in a dry soil (Philip, 1958a), which was confirmed by the results in Figure 7b.

Soil texture could affect the development of saturated zone mainly from three aspects: (a) the shape coefficient n, which reflects soil pore size distribution; (b) water-entry suction h_d, which is related to the maximum equivalent capillary pore size of a soil, and (c) saturated hydraulic conductivity K_s. A higher n value represents a steeper pore size distribution which is closer to the delta-type soil water diffusivity and could cause a lower fraction of saturated zone during infiltration as shown in Figure 8a. Since soil is tension-saturated when $0 > h > -h_d$ (Philip, 1958a), the water-entry suction shall have contrary impacts on the saturated zone to h_p from Equation (12) and Equation (15). A greater value of h_d resulted in a lower $LR_{S/EW}$. The parameter

B_4 is the average velocity of pore water under gravity gradient which corresponds to the cases in a large infiltration time. For a given soil porosity, B_4 is positively correlated to K_s. According to Equation (21), a higher value of K_s will accelerate the development of z_s as shown in Figure 8c. In contrast to a fine-textured soil, a coarse-textured soil normally has greater K_s and n but lower h_d. Obviously, the positive effects of K_s and h_d on $LR_{S/EW}$ overwhelmed the negative effect of n, which can explain the results shown in Figure 7c.

5.2 The influence of saturated zone on infiltration

The key character differentiating the GAMS from the TGAM and GAME models is its consideration of the wetting zone composed of both saturated and unsaturated zones. The TGAM was derived by assuming a piston-type soil moisture profile, that is, a fully saturated wetting zone (Green & Ampt, 1911). While, in the GAME, the wetting zone was considered unsaturated before a critical time, when the surface water flux dropped to K_s, and after which the saturated zone developed linearly with time (Ma et al., 2015). Insight into the internal relationships of the GAMS with the TGAM and GAME could provide an improved understanding on the influence of the saturated zone on infiltration.

With the definition of Equation (18), the Equation (12) can be rewritten as

$$J_{w0} = K_s\left(1 + \frac{B_1 + B_3}{z_{fe}}\right) - K_s B_2\left(1 - \frac{z_s}{z_{fe}}\right) \tag{35}$$

The first term on the right side of Equation (35) is equal to the TGAM which is characterized by piston-type water profile and average pressure head at the wetting front (i.e. Equation (13)) with the form of Neuman (1976). The second term on the right side of Equation (35) represents the influence of the saturated zone on surface soil water flux. For a soil with delta-type water diffusivity, a piston-type water profile is expected and z_s is close to z_{fe}. Then, the GAMS (Equation (35)) can be transformed to the TGAM, given:

$$J_{w0} = K_s \left(1 + \frac{B_1 + B_3}{z_{fe}} \right) \tag{36}$$

For a soil with water diffusivity far from delta-type, a non-piston-type water profile and a low length ratio of the saturated zone to wetted zone are expected. If neglecting the development of saturated zone (i.e. $z_s = 0$), the GAMS (Equation (35)) can be transformed to the GAME, that is,

$$J_{w0} = K_s \left(1 + \frac{B_1 + B_3}{z_{fe}} \right) - K_s B_2 = (1 - B_2) K_s \left(1 + \frac{B_1 + B_3}{(1 - B_2) z_{fe}} \right) \tag{37}$$

As depicted in Equation (35), it is $LR_{S/EW}$ rather than the saturated zone length that leads the transformations of the GAMS model to GAME and TGAM models. Obviously, the TGAM overestimated the surface water flux and thus cumulative surface water flux, and the length of the wetted zone because the wetting zone was considered overall saturated as shown in Equation (36). Neglecting the development of the saturated zone as shown in Equation (37) resulted in the underestimation of the surface water flux by the GAME. In contrast, a developing saturated zone was accurately characterized in the GAMS as shown in Equation (35), which contributed to the best simulations among the three models.

However, in the first stage of infiltration, $LR_{S/EW}$ was relatively small (Figure 7) and same to the effects of saturated zone on the surface water flux. In addition, the soil moisture profile shape in GAME was closer to the real one in the first stage than that in the GAMS (Figure 9), which will be discussed in details in the section 5.3. Consequently, the simulation or estimation accuracy of the GAME (Figure 4) is comparable to and even higher than the GAMS (Figure 4c, Figure 5) in the first stage of infiltration. Nevertheless, $LR_{S/EW}$ and the effects of saturated zone on the surface water flux increased with time (Figure 7). Then, the simulation errors of the GAME rose up (Figure 3c, Figure 4) as the saturated zone was completely ignored (Figure 3b). Although a linearly increasing saturated zone length was considered in the second stage of infiltration (Figure 3b), it is not enough to fully characterize the developing saturated zone and thus the surface water flux was still underestimated.

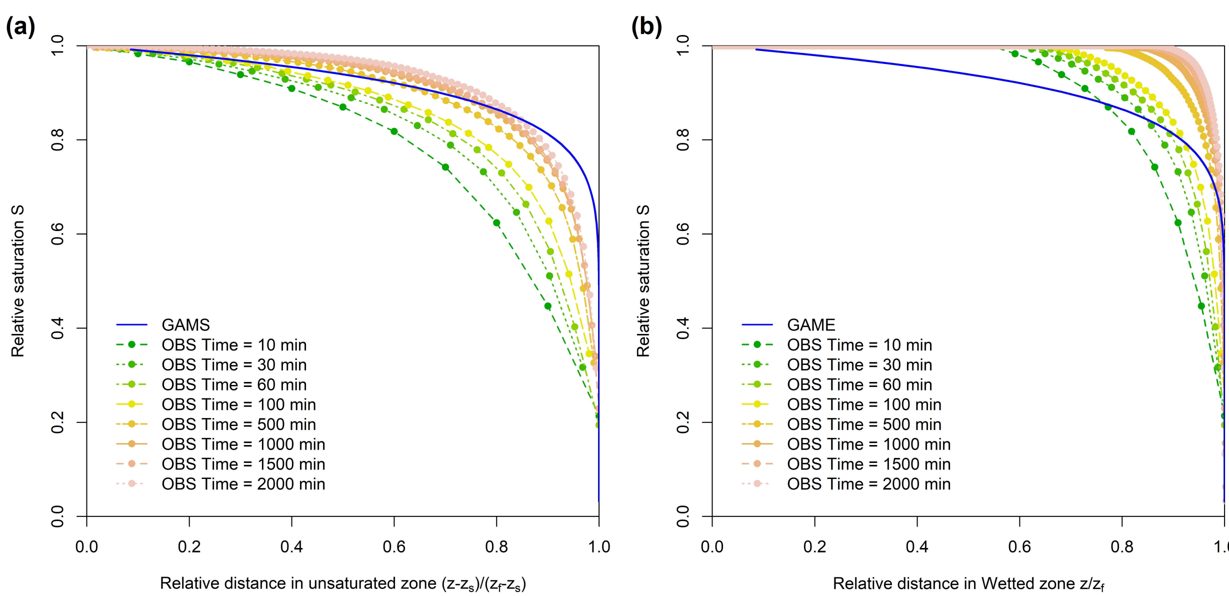

Figure 9. Comparison of the simulated relative soil moisture profile by (a) the GAMS, and (b) the GAME to the observed data (OBS) produced by the numerical solution (HYDRUS-1D) for a loam soil.

5.3 The factors causing the deviation of the GAMS

That the GAMS exhibits relatively larger errors in the simulated water fluxes in short infiltration times than those in long infiltration times could be induced by the two assumptions of time-independency in soil water flux-saturation relationship (Equation (7)) and relative soil moisture profile (Equation (3)) in the unsaturated zone for the derivation of the GAMS.

According to the theoretical analysis by Philip (1973), the soil water flux-saturation relationship in the form of Equation (7) represents the case of coarse-textured soils, that are, linear soils or "delta-function" soils. For natural soils, it may vary with soil texture, infiltration time and boundary conditions but will converge to the curve of Equation (7) in the long infiltration time (Philip, 1973). The texture-dependency of soil water flux-saturation relationship has been confirmed by the observed data in White (1979) and Ma et al. (2017). More accurate expressions for the soil water flux-saturation relationship can be found in literatures (Evangelides et al., 2005; Kargas et al., 2019; Ma et al., 2017; Vauclin and Haverkamp, 1985). Actually, the soil water flux-saturation relationship depended little on time especially in a short infiltration period (Ma et al., 2017) and did not exert obvious influences on infiltration simulations (Haverkamp et al., 1990; Hogarth et al., 2011). Moreover, the expression of Equation (7) has been successfully used for deriving accurate approximate analytical solutions of infiltration problems from Richards equation (Assouline, 2013; Haverkamp et al., 1990; Hogarth et al., 2011; Ma et al., 2015). Although more accurate expression than Equation (7) can be adapted to improve the infiltration simulation (Hayek, 2018), the corresponded solutions are more complex for practical application. For the loam soil in this research, no obvious time-dependency can be found in the soil water flux-saturation relationship and the Equation (7) in the GAMS exhibits enough accuracy as shown in Figure 10. It seems impossible that the relatively great errors of the surface water flux simulated by the GAMS (Figure 4) was caused by the assumption of soil water flux-saturation relationship inherent in Equation (7).

The assumption of time-dependent soil moisture profile shape simplified the derivation of the GAMS. Actually, the relative soil moisture profile in the unsaturated zone for the loam soil exhibits time-dependency in short infiltration time and approaches to a steady shape only in long infiltration time (Figure 9). Consequently, the deviation of the simulated soil moisture profiles by the GAMS from those by HYDRUS-1D is mainly in the early infiltration stage and exhibits in two aspects (Figure 5): (1) the overestimated length of unsaturated zone, (2) the twisted shape of soil moisture profiles in the unsaturated zone. Including the developing saturated zone makes the simulated relative soil moisture profiles by the GAMS closer to the steady shape in the long infiltration time (Figure 9a) while the GAME yielded a relative soil moisture profile closer to the unsteady shape in the short time for no consideration of saturated zone (Figure 9b). In addition, the expression for calculating the length of unsaturated zone was derived based on the soil moisture profile of Equation (3) in the unsaturated zone. Therefore, it could be concluded that the simulation errors of the GAMS in the short time is mainly induced by the errors in the function of relative soil moisture profile in the unsaturated zone.

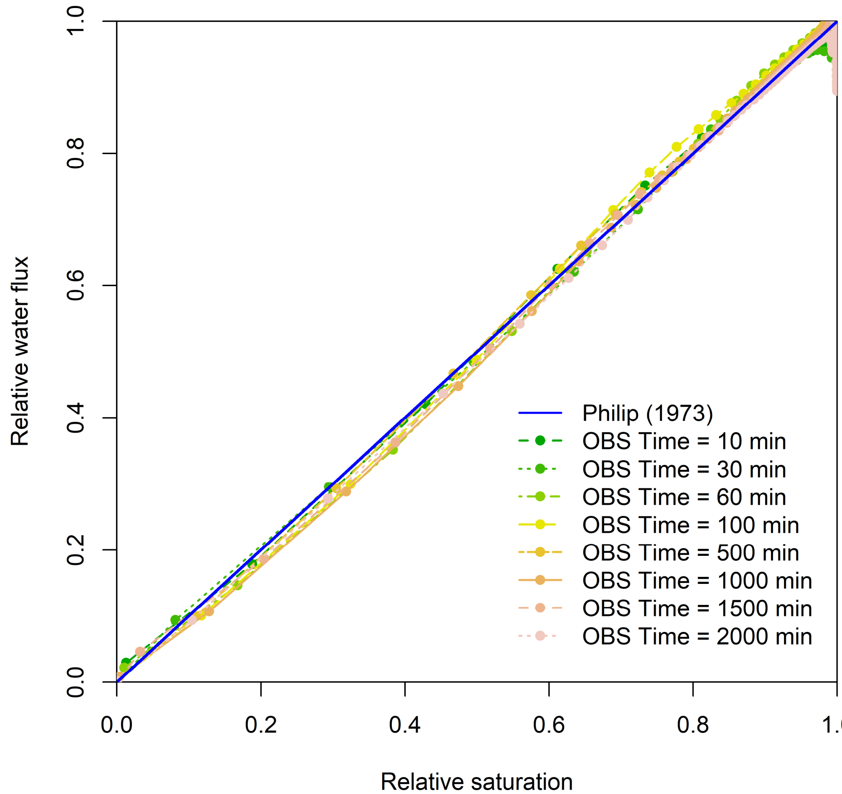

Figure 10. Comparison of the simulated soil water flux-saturation relationship by the numerical solution (HYDRUS-1D) and the expression proposed by Philip (1973) for a loam soil.

Actually, the expression of the relative soil moisture profile (i.e. Equation (3)) in both the GAME and GAMS was derived based on Equation (7) and an approximation of time-independent soil potential profile shape (Equation (A6) in Ma *et al.* (2015). A more accurate description of soil water flux-saturation relationship and the consideration of the time-dependency of soil moisture profile shape are expected to further improve the accuracy of soil moisture profile simulation (Hayek, 2018; Hogarth et al., 2013; Hogarth et al., 2011). Unfortunately, it is difficult to derive such a time-dependent expression of soil moisture profile shape in the current study, given the complex relationship between soil moisture profile and soil water flux (Haverkamp et al., 1990; Hogarth et al., 2011). A recent work by Su et al. (2018) proposed an expression of time-dependent soil moisture profile shape. However, it makes the derivation of infiltration model difficult and fails at a large infiltration time.

5.4 The general use of the GAMS

Based on the assumptions of time-independency of soil water flux-saturation relationship and relative soil moisture profile, the GAMS was derived with no limitation of the specific form of relative soil moisture profile and soil hydraulic properties. According to Wang et al. (2013) and Ma et al. (2015, 2017b), the form of relative soil moisture profile depends on the specific function of soil hydraulic properties and soil water flux-saturation relationship, and the shape coefficient of soil moisture profile is only related with the shape parameter of the soil water retention curve. Given the soil moisture profile function derived for a specific soil hydraulic property model (e.g. BC model, VG model), it is easy to obtain the parameters of the GAMS by Equation (9), Equations (13)-(15) and Equations (22)-(23) from soil hydraulic properties. Furthermore, other forms of soil water flux-saturation relationship (Evangelides et al., 2005; Ma et al., 2017) can be also included in the novel GAMS by substituting the soil water flux-saturation relationship in Philip (1973).

In the future, more accurate unsaturated soil moisture profile functions are expected to be adopted in the GAMS to further improve the infiltration simulation and soil hydraulic parameters estimation. Moreover, the general solution of the GAMS should be extended from uniform soils to stratified soils or soils with non-uniform initial soil moisture profile, as well as the soils with more hydraulic property models such as the VG model (van Genuchten, 1980).

6 Conclusions

Ponding at the soil surface changes surface pressure head and affects soil moisture profile shape in infiltration. A novel analytical solution, the GAMS, is derived to one-dimensional vertical infiltration under ponding conditions for any forms of soil hydraulic properties models, which can describe the length of saturated zone versus infiltration time with a simple expression. The GAMS was evaluated with a special solution for Brooks-Corey soil hydraulic property model. Compared with the TGAM model (Green and Ampt, 1911) and GAME model (Ma et al., 2015), the GAMS showed a better performance in infiltration simulation indicated by higher agreement with the numerical solution by HYDRUS-1D along the infiltration period. Furthermore, the model inversion of the GAMS yields more accurate estimates of soil hydraulic property model parameters from a one-dimensional vertical infiltration experiment. Besides, the time-dependency of model parameter estimation by the GAMS is weaker in long infiltration time than the TGAM and GAME models. The novel GAMS is supposed to be used in irrigation design and rainfall-runoff simulation by providing more accurate data of cumulative infiltration and soil moisture distribution.

Acknowledgments

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Data Availability Statement

The data set used to validate the new solution was produced by the HYDRUS-1D software (version 3.0) (Šimůnek et al., 2005). The program to calculate and draw figures is edited in R language (version 4.1.2) (R Core Team, 2021). All the data set and program are available after requested.

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736