Analytical study of the chiral nonlinear Schrödinger's equation for optical Soliton Solutions

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Abstract

The aim of the present paper is to extract the exact traveling wave solutions of the chiral nonlinear Schrödinger's equation (cNLSE). The $(\frac{G'}{G^2})-$expansion method and the first integral method along with symbolic computation package has been exerted to celebrate the exact solutions. As a consequence, the obtained solutions can be categorized into trigonometric, hyperbolic and rational with some free parameters of the problem studied. In addition, these types of the solutions lead to understand the physical phenomena of the problem such as solitary, periodic, complex function, singular optical solitons and dark-singular combo solitons.

Keywords: Extended $(\frac{G'}{G^2})-$expansion method; First integral method; The chiral nonlinear Schrödinger equation; Exact solutions.

1 Introduction

The model in the context of Quantum Hall effect where chiral excitations are known to appear is known as the Jackiw and Pi model and produces the chiral solitons which are of fundamental importance in nonlinear optics. The governing model is the chiral nonlinear Schrodinger's equation (cNLSE). More details are elucidated in [1]. There are several studies that are carried out for this equation such as in [2] perturbation of soliton due to the cNLSE by the aid of soliton perturbation theory is discussed. Topological and bright soliton solutions are obtained using the soliton ansatz method to carry out the derivation of the soliton in [3]. The author integrates the generalized cNLSE by the soliton ansatz method to find both topological as well as non topological 1-soliton solutions and the multiplier method approach is used to calculate a few conserved quantities of the cNLSE [4]. Chiral solitons with Bohm potential by lie group analysis and traveling wave hypothesis are observed in [5]. In presence of Bohm potential term the cNLSE is investigated by means of the exponential function method, $\frac{G'}{G}$ approach as well as the traveling wave hypothesis to obtain solution in terms of doubly periodic function where in the limiting case topological soliton solutions are retrieved[6]. Solitons and singular periodic solutions are obtained by

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the trial solution technique [7] and more recently rapidly convergent approximation method (RCAM) is applied for seek of exact solutions of cNLSE [8].
Nowadays, nonlinear dynamics is an engine of modern sciences which describes nonlinear phenomena. Many nonlinear phenomena in physics, chemistry, biology, engineering, economics and other fields are described by nonlinear partial differential equations (NPDEs). Nonlinear PDE is one of the main research area and interesting issue in mathematics and physics. Their exact and numerical solutions in the type of soliton solutions have vital importance among traveling wave solutions since they act as a bridge between mathematics and physics. Exact traveling wave solutions are considered best to understand the phenomena of natural sciences. For this reason a vast community of researchers have given their ability to issues of finding exact solutions for NPDEs and many new techniques for finding exact solution for nonlinear equations have been proposed. For example, Jacobi elliptic function method [9], tanh method [10], exp-function method [11], F-expansion method [12], Hirotas method [13], extended Fan sub-equation method [14], sine-cosine method [15], the first integral method [16, 17] and so on. In the present work, the main goal is to seek exact traveling wave solutions of nonlinear evolution equation using the extended $(\frac{G'}{G^2})-$expansion method and the first integral method. The (1+2)-dimensional chiral nonlinear schrödinger equation has been taken as an example to illustrate the effectiveness of these proposed methods. Based on $(\frac{G'}{G})-$expansion method [18], the extended $(\frac{G'}{G^2})-$expansion method is developed by Zhouzheng [19]. Based on the ring theory of commutative algebra, first integral method is introduced by Feng [20]. These are quite reliable direct methods to find exact solutions of nonlinear partial differential equations.
The rest of the paper is as follows: Algorithms of $(\frac{G'}{G^2})-$expansion method and first integral method have been discussed for finding the new exact traveling wave solutions of nonlinear PDEs, in Section 2. In Section 3, the solutions of cNSLE are obtained using these methods. The main findings are given in Section 4.

2 Analytical methods

In this section, a brief review has been given on two different analytical methods, individually. Step 1 is common step in both methods.
Step1: A nonlinear partial differential equation with a physical field Γ is considered, as

$$P(\Gamma, \Gamma_x, \Gamma_t, \Gamma_{xx}, \Gamma_{xt}, ...) = 0, \tag{1}$$

where $\Gamma = \Gamma(x, t)$ is a travelling wave solution.
Using the following transformation

$$\Gamma(x, y, t) = \Gamma(\psi), \qquad \psi = k_1 x + k_2 y \pm ct, \tag{2}$$

the nonlinear PDE (1) converted into an ordinary differential equation

$$Q(\Gamma, \Gamma^{'}, \Gamma^{'}, \Gamma^{''}, \Gamma^{''}, \Gamma^{''}, ...) = 0, \tag{3}$$

where k and c are constants to be determined and $'$ denotes $\frac{\partial}{\partial \psi}$.

2.1 Description of the extended $(\frac{\mathbf{G'}}{\mathbf{G^2}})-$expansion method

Step 2. Assume that Eq.(3) has the solution:

$$\Gamma(\psi) = a_0 + \sum_{i=1}^{i=m} \left[a_i \left(\frac{G'}{G^2} \right)^i + b_i \left(\frac{G'}{G^2} \right)^{-i} \right], \tag{4}$$

where $G = G(\psi)$ satisfies

$$\left(\frac{G'}{G^2} \right)' = \mu + \lambda \left(\frac{G'}{G^2} \right)^2, \tag{5}$$

in which $\mu \neq 1$ and $\lambda \neq 0$ are integers and a_0, a_i and b_i $(i = 1, 2, ..., m)$ are unknown constants to be determined. By homogeneous balance principle, the value of positive integer m can be found.

Step 3. To find the unknown constants by substituting Eq.(4) using Eq.(5) into Eq.(3), then collecting like powers of $(\frac{G'}{G^2})^j$ $(j = 0, \pm 1, \pm 2, ...)$. Setting all coefficients to zero yields a set of algebraic equations. The obtained algebraic system is solved to get the values of unknown constants.

Step 4. By the general solutions of Eq.(5), $\left(\frac{G'}{G^2}\right)$ has three possibilities:

If $\mu\lambda > 0$, then

$$\frac{G'}{G^2} = \sqrt{\frac{\mu}{\lambda}} \left[\frac{A_1 \cos(\sqrt{\lambda\mu}\psi) + A_2 \sin(\sqrt{\lambda\mu}\psi)}{A_2 \cos(\sqrt{\lambda\mu}\psi) - A_1 \sin(\sqrt{\lambda\mu}\psi)} \right]. \tag{6}$$

If $\mu\lambda < 0$, then

$$\frac{G'}{G^2} = -\frac{\sqrt{|\mu\lambda|}}{\lambda} \left[\frac{A_1 \sinh(2\sqrt{|\lambda\mu|}\psi) + A_1 \cosh(2\sqrt{|\lambda\mu|}\psi) + A_2}{A_1 \sinh(2\sqrt{|\lambda\mu|}\psi) + A_1 \cosh(2\sqrt{|\lambda\mu|}\psi) - A_2} \right]. \tag{7}$$

If $\mu = 0$ and $\lambda \neq 0$, then

$$\frac{G'}{G^2} = -\frac{A_1}{\lambda(A_1\psi + A_2)}, \tag{8}$$

in which A_1 and A_2 are arbitrary constants.

Step 5. The solution of the proposed Eq.(3) can be obtained by substituting the values of a_0, a_i, b_i and $\frac{G'}{G^2}$ into Eq.(4).

2.2 Description of first integral method

Step 2. Assume that the solution of Eq.(3)

$$\Gamma(x, t) = \zeta(\psi). \tag{9}$$

By introducing a new independent variable:

$$X(\psi) = \zeta(\psi), \quad Y(\psi) = \frac{\partial \zeta(\psi)}{\partial \psi}.$$

Step 3. Eq.(3) can be reduced to a system of nonlinear ODEs under the conditions of step 1:

$$X^{'}(\psi) = Y(\psi), \quad Y^{'} = S(X(\psi), Y(\psi)). \tag{10}$$

One can construct general solutions to Eq.(10) explicitly or directly, if two first independent integrals to Eq.(10) can be found [21]. Applying Division theorem to get one first integral to Equation (10) which converts Eq.(3) to a first order integrable ODE.
Step 4. An exact solution to Eq.(1) is then obtained by solving the first order integrable ordinary differential equation directly.
Division Theorem: Consider complex domain $\mathbb{C}[p,q]$ of two variables and $Q(p,q)$ and $M(p,q)$ are polynomials in $\mathbb{C}[p,q]$, where $Q(p,q)$ is not reducible. If $M(p,q)$ vanishes at all zero points of $Q(p,q)$, then there exists a polynomial T(p,q) in $\mathbb{C}[p,q]$ such that
$M(p,q) = Q(p,q)T(p,q)$.

3 The chiral (1+2)-dimensional nonlinear Schrödingers equation

Consider the chiral (1+2)-dimensional nonlinear Schrödingers equation:

$$\iota q_t + a(q_{xx} + q_{yy}) + \iota\left[d_1(qq_x^* - q^*q_x) + d_2(qq_y^* - q^*q_y)\right]q = 0, \tag{11}$$

where the first term is the evolution term, the parameter a is dispersion term, while d_1 and d_2 are the coefficients of nonlinear coupling terms. This nonlinearity represents the current density. It is to be noted that Eq.(11) is not Galilean invariant and furthermore is not integrable by the method of inverse scattering transform as it fails the Painleve test of integrability.
The following transformation is introduced

$$q(x,y,t) = u(\psi)e^{\iota\phi(x,y,t)}, \quad \psi = M_1x + M_2y - vt, \quad \phi(x,y,t) = k_1x + k_2y + \omega t + \theta, \tag{12}$$

where $u(\psi)$ and $\phi(x,y,t)$ are the amplitude portion and phase portion of the solution, respectively. The parameters k_1 and k_2 denote the frequencies in x- and y- directions, ω is the soliton frequency, while θ is the phase constant. Moreover, M_1, M_2 and v are unknown constants.
Using Eq.(12) into Eq.(11), the real part and the imaginary part can be separated, as

$$a(M_1 + M_2)u^{''} - (a(k_1 + k_2) + \omega)u + 2(d_1k_1 + d_2k_2)u^3 = 0, \tag{13}$$

$$(-v + 2a(k_1M_1 + k_2M_2))u^{'} = 0. \tag{14}$$

From Eq.(14), we get

$$v = 2a(k_1M_1 + k_2M_2) \tag{15}$$

3.1 Solitary wave solutions by the extended $(\frac{G'}{G^2})-$expansion method

By extended $(\frac{G'}{G^2})-$expansion method, applying homogeneous balance principle to Eq.(13), gives $m = 1$, then the solution of Eq.(13):

$$u(\psi) = a_0 + a_1 \left(\frac{G'}{G^2}\right) + b_1 \left(\frac{G'}{G^2}\right)^{-1}. \tag{16}$$

Substituting Eq.(16) along with Eq.(5) into Eq.(13), A set of algebraic equations are obtained:

$$\begin{aligned}
\left(\frac{G'}{G^2}\right)^{-3} &: 2\,aM_1{}^2b_1\mu^2 + 2\,aM_2{}^2b_1\mu^2 + 2\,k_1d_1b_1{}^3 + 2\,k_2d_2b_1{}^3 = 0,\\
\left(\frac{G'}{G^2}\right)^{-2} &: 6\,k_1d_1a_0b_1{}^2 + 6\,k_2d_2a_0b_1{}^2 = 0,\\
\left(\frac{G'}{G^2}\right)^{-1} &: 6\,k_1d_1a_0{}^2b_1 + 6\,k_1d_1a_1b_1{}^2 + 6\,k_2d_2a_0{}^2b_1 + 6\,k_2d_2a_1b_1{}^2 - ak_1{}^2b_1 - ak_2{}^2b_1\\
&\quad -wb_1 + 2\,aM_1{}^2b_1\lambda\,\mu + 2\,aM_2{}^2b_1\lambda\,\mu = 0,\\
\left(\frac{G'}{G^2}\right)^{0} &: 12\,k_1d_1a_0a_1b_1 + 12\,k_2d_2a_0a_1b_1 - ak_1{}^2a_0 - ak_2{}^2a_0 + 2\,k_1d_1a_0{}^3 + 2\,k_2d_2a_0{}^3\\
&\quad -wa_0 = 0,\\
\left(\frac{G'}{G^2}\right)^{1} &: 6\,k_1d_1a_0{}^2a_1 + 6\,k_1d_1a_1{}^2b_1 + 6\,k_2d_2a_0{}^2a_1 + 6\,k_2d_2a_1{}^2b_1 - ak_1{}^2a_1 - ak_2{}^2a_1\\
&\quad -wa_1 + 2\,aM_1{}^2a_1\lambda\,\mu + 2\,aM_2{}^2a_1\lambda\,\mu = 0,\\
\left(\frac{G'}{G^2}\right)^{2} &: 6\,k_1d_1a_0a_1{}^2 + 6\,k_2d_2a_0a_1{}^2 = 0,\\
\left(\frac{G'}{G^2}\right)^{3} &: 2\,aM_1{}^2a_1\lambda^2 + 2\,aM_2{}^2a_1\lambda^2 + 2\,k_1d_1a_1{}^3 + 2\,k_2d_2a_1{}^3 = 0.
\end{aligned}$$

Solving these algebraic equations leads to the results

$$\textbf{Case 1}: \mu = \frac{ak_1{}^2 + ak_2{}^2 + w}{2a\lambda\left(M_1{}^2 + M_2{}^2\right)},\ a_0 = 0,\ a_1 = \pm\sqrt{-\frac{aM_1{}^2 + aM_2{}^2}{k_1d_1 + k_2d_2}}\lambda, b_1 = 0.$$

Three types of traveling wave solutions of Eq.(11) can be calculated.
If $\lambda\mu > 0$, the trigonometric function solutions can be expressed:

$$u_1(x, y, t) = \pm\sqrt{-\frac{a(M_1{}^2 + M_2{}^2)\lambda\mu}{k_1d_1 + k_2d_2}}\left[\frac{A_1\cos(\sqrt{\lambda\mu}\psi) + A_2\sin(\sqrt{\lambda\mu}\psi)}{A_2\cos(\sqrt{\lambda\mu}\psi) - A_1\sin(\sqrt{\lambda\mu}\psi)}\right]e^{\iota\phi(x,y,t)}. \tag{17}$$

If $\lambda\mu < 0$, the hyperbolic function solutions can be expressed:

$$u_2(x, y, t) = \pm\sqrt{-\frac{a(M_1{}^2 + M_2{}^2)|\lambda\mu|}{k_1d_1 + k_2d_2}}\left[\frac{A_1\sinh(2\sqrt{|\lambda\mu|}\psi) + A_1\cosh(2\sqrt{|\lambda\mu|}\psi) + A_2}{A_1\sinh(2\sqrt{|\lambda\mu|}\psi) + A_1\cosh(2\sqrt{|\lambda\mu|}\psi) - A_2}\right] \times e^{\iota\phi(x,y,t)}. \tag{18}$$

To obtain soliton solution, choose $A_1 = A_2$, we get

$$u_2(x,y,t) = \pm\sqrt{-\frac{a({M_1}^2+{M_2}^2)|\lambda\mu|}{k_1d_1+k_2d_2}}.\coth\left(\sqrt{|\lambda\mu|}\psi\right)e^{\iota\phi(x,y,t)}, \tag{19}$$

these are singular optical solitons.
If $\lambda \neq 0$, $\mu = 0$, the rational function solutions can be expressed:

$$u_3(x,y,t) \;=\; \mp\sqrt{-\frac{a({M_1}^2+{M_2}^2)}{k_1d_1+k_2d_2}}.\frac{A_1e^{\iota\phi(x,y,t)}}{A_1\psi+A_2}, \tag{20}$$

where A_1 and A_2 are arbitrary constants.

$$\textbf{Case 2}:\quad \mu = -\frac{a{k_1}^2+a{k_2}^2+w}{4a\lambda\left({M_1}^2+{M_2}^2\right)},\; a_0=0,\; a_1=\pm\sqrt{-\frac{a{M_1}^2+a{M_2}^2}{k_1d_1+k_2d_2}}\lambda,$$
$$b_1 = \pm\frac{a{k_1}^2+a{k_2}^2+w}{4\sqrt{-a\left({M_1}^2+{M_2}^2\right)(k_1d_1+k_2d_2)\lambda}}.$$

If $\lambda\mu > 0$, the trigonometric function solutions can be expressed:

$$\begin{aligned}u_4(x,y,t) = &\pm\sqrt{-\frac{a({M_1}^2+{M_2}^2)\lambda\mu}{k_1d_1+k_2d_2}}\left[\frac{A_3\cos(\sqrt{\lambda\mu}\psi)+A_4\sin(\sqrt{\lambda\mu}\psi)}{A_4\cos(\sqrt{\lambda\mu}\psi)-A_3\sin(\sqrt{\lambda\mu}\psi)}\right]e^{\iota\phi(x,y,t)}\\ &\pm\frac{a{k_1}^2+a{k_2}^2+w}{4\sqrt{-a\left({M_1}^2+{M_2}^2\right)(k_1d_1+k_2d_2)\lambda}}\\ &\times\left[\sqrt{\frac{\mu}{\lambda}}\left(\frac{A_3\cos(\sqrt{\lambda\mu}\psi)+A_4\sin(\sqrt{\lambda\mu}\psi)}{A_4\cos(\sqrt{\lambda\mu}\psi)-A_3\sin(\sqrt{\lambda\mu}\psi)}\right)\right]^{-1}e^{\iota\phi(x,y,t)}.\end{aligned} \tag{21}$$

If $\lambda\mu < 0$, the hyperbolic function solutions can be expressed:

$$\begin{aligned}u_5(x,y,t) = &\mp\sqrt{-\frac{a({M_1}^2+{M_2}^2)|\lambda\mu|}{k_1d_1+k_2d_2}}\left[\frac{A_3\sinh(2\sqrt{|\lambda\mu|}\psi)+A_3\cosh(2\sqrt{|\lambda\mu|}\psi)+A_4}{A_3\sinh(2\sqrt{|\lambda\mu|}\psi)+A_3\cosh(2\sqrt{|\lambda\mu|}\psi)-A_4}\right]\\ &\times e^{\iota\phi(x,y,t)}\pm\frac{a{k_1}^2+a{k_2}^2+w}{4\sqrt{-a\left({M_1}^2+{M_2}^2\right)(k_1d_1+k_2d_2)\lambda}}\\ &\times\left[-\frac{\sqrt{|\lambda\mu|}}{\lambda}\left(\frac{A_3\sinh(\sqrt{|\lambda\mu|}\psi)+A_3\cosh(\sqrt{|\lambda\mu|}\psi+A_4)}{A_3\sinh(\sqrt{|\lambda\mu|}\psi)+A_3\cosh(\sqrt{\lambda\mu}\psi-A_4)}\right)\right]^{-1}e^{\iota\phi(x,y,t)}.\end{aligned} \tag{22}$$

Choose $A_3 = A_4$ to obtain soliton solution as

$$\begin{aligned}u_5(x,y,t) = &\pm\sqrt{-\frac{a({M_1}^2+{M_2}^2)|\lambda\mu|}{k_1d_1+k_2d_2}}\coth(\sqrt{|\lambda\mu|}\psi)e^{\iota\phi(x,y,t)}\\ &\pm\frac{a{k_1}^2+a{k_2}^2+w}{4\sqrt{-a\left({M_1}^2+{M_2}^2\right)(k_1d_1+k_2d_2)}\sqrt{|\lambda\mu|}}\tanh(\sqrt{|\lambda\mu|}\psi)e^{\iota\phi(x,y,t)}.\end{aligned} \tag{23}$$

These are dark-singular combo soliton pairs.
If $\lambda \neq 0$, $\mu = 0$, the rational function solutions can be expressed as

$$\begin{aligned} u_6(x,y,t) = \mp\sqrt{-\frac{a(M_1{}^2+M_2{}^2)}{k_1d_1+k_2d_2}}.\frac{A_3e^{\iota\phi(x,y,t)}}{A_3\psi+A_4} \pm \frac{ak_1{}^2+ak_2{}^2+w}{4\sqrt{-a\left(M_1{}^2+M_2{}^2\right)(k_1d_1+k_2d_2)}} \\ \times\left[\frac{-A_3}{A_4+A_3\psi}\right]^{-1} e^{\iota\phi(x,y,t)}, \end{aligned} \tag{24}$$

where A_3 and A_4 are arbitrary constants.

$$\begin{aligned} \textbf{Case 3}: \quad \mu = \frac{ak_1{}^2+ak_2{}^2+w}{8a\lambda\left(M_1{}^2+M_2{}^2\right)},\ a_0 = 0,\ a_1 = \pm\sqrt{-\frac{aM_1{}^2+aM_2{}^2}{k_1d_1+k_2d_2}}\lambda, \\ b_1 = \pm\frac{ak_1{}^2+ak_2{}^2+w}{8\sqrt{-a\left(M_1{}^2+M_2{}^2\right)(k_1d_1+k_2d_2)\lambda}}. \end{aligned}$$

If $\lambda\mu > 0$, the trigonometric function solutions can be expressed as

$$\begin{aligned} u_7(x,y,t) = \pm\sqrt{-\frac{a(M_1{}^2+M_2{}^2)\lambda\mu}{k_1d_1+k_2d_2}}\left[\frac{A_5\cos(\sqrt{\lambda\mu}\psi)+A_6\sin(\sqrt{\lambda\mu}\psi)}{A_6\cos(\sqrt{\lambda\mu}\psi)-A_5\sin(\sqrt{\lambda\mu}\psi)}\right]e^{\iota\phi(x,y,t)} \\ \pm\frac{ak_1{}^2+ak_2{}^2+w}{8\sqrt{-a\left(M_1{}^2+M_2{}^2\right)(k_1d_1+k_2d_2)\lambda}} \\ \times\left[\sqrt{\frac{\mu}{\lambda}}\left(\frac{A_5\cos(\sqrt{\lambda\mu}\psi)+A_6\sin(\sqrt{\lambda\mu}\psi)}{A_6\cos(\sqrt{\lambda\mu}\psi)-A_5\sin(\sqrt{\lambda\mu}\psi)}\right)\right]^{-1}e^{\iota\phi(x,y,t)}. \end{aligned} \tag{25}$$

If $\lambda\mu < 0$, the hyperbolic function solutions can be expressed as

$$\begin{aligned} u_8(x,y,t) = \mp\sqrt{-\frac{a(M_1{}^2+M_2{}^2)|\lambda\mu|}{k_1d_1+k_2d_2}}\left[\frac{A_5\sinh(2\sqrt{|\lambda\mu|}\psi)+A_5\cosh(2\sqrt{|\lambda\mu|}\psi)+A_6}{A_5\sinh(2\sqrt{|\lambda\mu|}\psi)+A_5\cosh(2\sqrt{|\lambda\mu|}\psi)-A_6}\right] \\ \times e^{\iota\phi(x,y,t)} \pm \frac{ak_1{}^2+ak_2{}^2+w}{8\sqrt{-a\left(M_1{}^2+M_2{}^2\right)(k_1d_1+k_2d_2)\lambda}} \\ \times\left[-\frac{\sqrt{|\lambda\mu|}}{\lambda}\left(\frac{A_5\sinh(\sqrt{|\lambda\mu|}\psi)+A_5\cosh(\sqrt{|\lambda\mu|}\psi+A_6)}{A_5\sinh(\sqrt{|\lambda\mu|}\psi)+A_5\cosh(\sqrt{\lambda\mu}\psi-A_6)}\right)\right]^{-1}e^{\iota\phi(x,y,t)}. \end{aligned} \tag{26}$$

Taking $A_5 = A_6$ to get soliton solutions:

$$\begin{aligned} u_8(x,y,t) = \mp\sqrt{-\frac{a(M_1{}^2+M_2{}^2)|\lambda\mu|}{k_1d_1+k_2d_2}}\coth(\sqrt{|\lambda\mu|}\psi)e^{\iota\phi(x,y,t)} \\ \pm\frac{ak_1{}^2+ak_2{}^2+w}{8\sqrt{-a\left(M_1{}^2+M_2{}^2\right)(k_1d_1+k_2d_2)\sqrt{|\lambda\mu|}}}\tanh(\sqrt{|\lambda\mu|}\psi)e^{\iota\phi(x,y,t)}. \end{aligned} \tag{27}$$

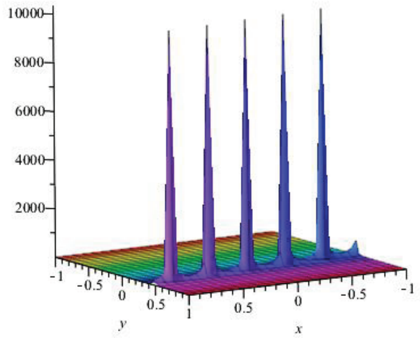


Figure 1: 3D graphics of $|u_1(x,y,t)|$ with $t=1$ and taking $a=0.05$, $d_1=0.05$, $d_2=0.054$, $M_1=-0.33$, $M_2=0.27$, $k_1=0.09$, $k_2=0.03$, $v=1$, $\omega=0.05$ and $\lambda=0.3$.

These are dark-singular combo soliton pairs.
If $\lambda \neq 0$, $\mu = 0$, the rational function solutions can be expressed as

$$\begin{aligned} u_9(x,y,t) = &\pm\sqrt{-\frac{a({M_1}^2+{M_2}^2)}{k_1d_1+k_2d_2}}.\frac{A_5e^{\iota\phi(x,y,t)}}{A_5\psi+A_6} \pm \frac{a{k_1}^2+a{k_2}^2+w}{8\sqrt{-a\left({M_1}^2+{M_2}^2\right)(k_1d_1+k_2d_2)}} \\ &\times\left[\frac{-A_5}{A_6+A_5\psi}\right]^{-1}e^{\iota\phi(x,y,t)}, \end{aligned} \tag{28}$$

where A_5 and A_6 are arbitrary constants. It is to be noted that absolute behavior of the solutions $u_1(x,y,t)$ and $u_5(x,y,t)$ are given in Figure 1 and Figure 2, respectively.

3.2 Solitary wave solutions by the first integral method

Assume that Eq.(13) can be written in the form, as

$$u^{''} - P_0u - R_0u^3 = 0, \tag{29}$$

where

$$\begin{aligned} P_0 &= \frac{a(k_1+k_2)+\omega}{a(M_1+M_2)}, \\ R_0 &= -\frac{2(d_1k_1+d_2k_2)}{a(M_1+M_2)} \end{aligned}$$

Let $X(\psi)=v(\psi),\ Y(\psi)=v^{'}(\psi)$, then Eq.(29) leads as

$$\begin{cases} \frac{dX}{d\psi} = Y(\psi), \\ \frac{dY}{d\psi} = P_0X(\psi)+R_0X^3(\psi). \end{cases} \tag{30}$$

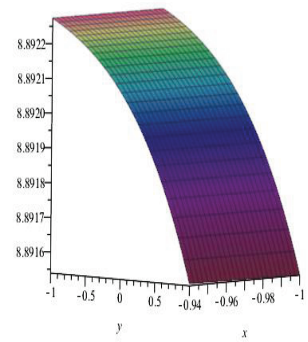


Figure 2: 3D graphics of $|u_5(x,y,t)|$ with $t=1$ and taking $a=-0.05$, $d_1=0.05$, $d_2=0.054$, $M_1=-0.33$, $M_2=0.27$, $k_1=0.09$, $k_2=0.03$, $v=1$, $\omega=0.05$ and $\lambda=0.3$.

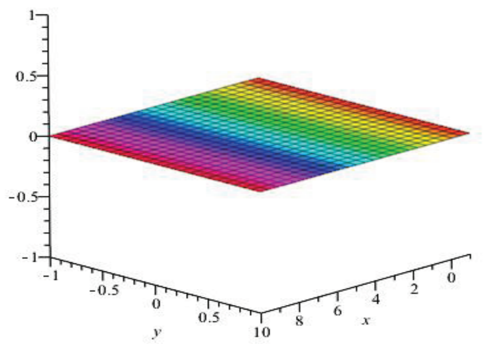


Figure 3: 3D graphics of $|u_{9(x,y,t)}|$ with $t=3$ and taking $a=0.05$, $d_1=0.05$, $d_2=0.054$, $M_1=-0.33$, $M_2=0.27$, $k_1=0.09$, $k_2=0.03$,$v=1$, $\omega=-ak_1{}^2-ak_2{}^2=-0.00045$ and $\lambda=0.3$.

Suppose that $X(\psi)$ and $Y(\psi)$ are nontrivial solutions of Eq.(30), and polynomial

$$Q(Y,X)=\sum_{i=0}^{m}a_i(X)Y^i,$$

is not reducible in $\mathbb{C}[X,Y]$, such that

$$Q[Y(\psi),X(\psi)]=\sum_{i=0}^{m}a_i(X(\psi))Y^i(\psi)=0, \tag{31}$$

in which $a_i(X)(i=0,1,2,...,m)$, are polynomials of X and $a_m(X)\neq 0$. Eq.(31) is said to be first integral to Eq.(30). Applying division theorem, there is a polynomial $g(X)+h(X)Y$ in $\mathbb{C}[X,Y]$, such that

$$\frac{dQ}{d\psi}=\frac{\partial Q}{\partial X}\frac{dX}{d\psi}+\frac{\partial Q}{\partial Y}\frac{dY}{d\psi}=(g(X)+h(X)Y)\sum_{i=0}^{m}a_i(X)Y^i. \tag{32}$$

Taking $m=1$ in Eq.(31), by the homogeneous balance principle and equating the like powers Y^i, $(i=0,1,2)$ of Eq. (32), leads as

$$\textit{Coefficient of } Y^0 \;:\; a_1(X)(P_0X+R_0X^3)=a_0(X)g(X), \tag{33}$$

$$\textit{Coefficient of } Y^1 \;:\; \dot{a}_0(X)=a_0(X)h(X)+a_1(X)g(X), \tag{34}$$

$$\textit{Coefficient of } Y^2 \;:\; \dot{a}_1(X)=a_1(X)h(X). \tag{35}$$

Since $a_i(X)(i=0,1)$ are polynomials in X, so from Eq.(35) it is deducted that $a_1(X)$ is a constant and $h(X)=0$. Take $a_1(X)=1$, for simplicity. At that point, adjusting of $g(X)$ and $a_0(X)$ in Eq.(34), it is derived that deg(g(X))=1 only. Assume that

$$\begin{aligned} g(X) &= P_1X+R_1,\\ P_0(X) &= P_1\frac{X^2}{2}+R_1X+D_0, \quad (P_1\neq 0)\end{aligned}$$

where P_1, R_1 and D_0 are unknown constants.
After substituting values of $a_0(X)$ and $g(X)$ in Eq.(33) and setting same powers of each coefficients of $X^i,(i=0,1,2,3)$ to zero, the resulting algebraic equations:

$$\textit{Coefficient of } X^3 \;:\; R_0-\frac{P_1^2}{2}=0. \tag{36}$$

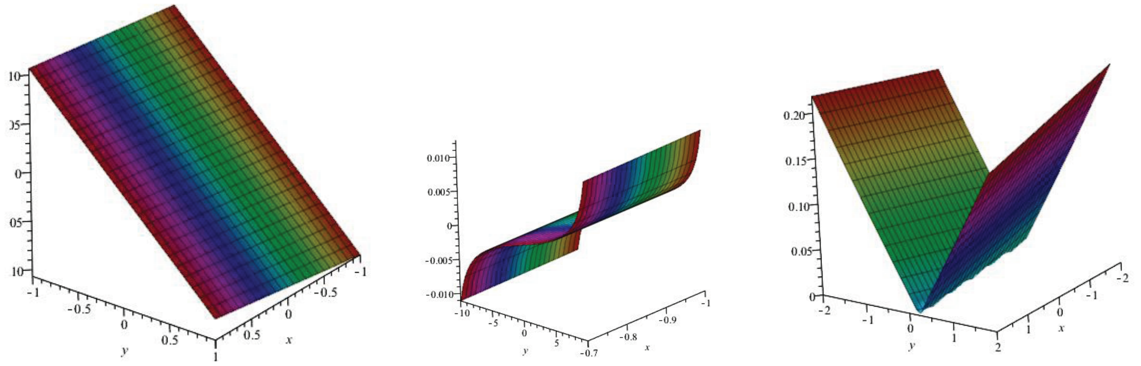
$$\textit{Coefficient of } X^2 \;:\; \frac{3}{2}P_1R_1=0. \tag{37}$$

$$\textit{Coefficient of } X^1 \;:\; P_0-D_0P_1-R_1^2=0. \tag{38}$$

$$\textit{Coefficient of } X^0 \;:\; -D_0R_1=0. \tag{39}$$

Solving above system of Eqns.(36-39) by using Eq.(30), simultaneously the following nontrivial solutions are obtained as

$$\begin{cases} P_1=\sqrt{2R_0}, \quad R_1=0, \quad D_0=\frac{P_0}{\sqrt{2R_0}},\\ P_1=-\sqrt{2R_0}, \quad R_1=0, \quad D_0=-\frac{P_0}{\sqrt{2R_0}}.\end{cases} \tag{40}$$



(a) (b) (c)

Figure 4: Taking $a = 0.05$, $d_1 = -0.57e^{-1}$, $d_2 = 0.054$, $M_1 = -0.33$, $M_2 = 0.27$, $k_1 = 0.09$, $k_2 = 0.03$,$v = 1$, $\omega = 0.05$ and $\lambda = 0.3$, (a) 3D graphics of real part of $u_{10}(x, y, t)$. (b) 3D graphics of imaginary part of $u_{10}(x, y, t)$. (c) 3D graphics of $|u_{10}(x, y, t)|$.

By the solution (40), Eq.(31) takes the form as

$$\begin{cases} Y(\psi) = -\sqrt{\frac{R_0}{2}}X^2 - \frac{P_0}{\sqrt{2R_0}}, \\ Y(\psi) = \sqrt{\frac{R_0}{2}}X^2 + \frac{P_0}{\sqrt{2R_0}}. \end{cases} \tag{41}$$

Combining Eq.(41) with the system given by Eq.(31), leads to

$$\begin{cases} X(\psi) = -\sqrt{\frac{P_0}{R_0}} \tan\left[\sqrt{\frac{P_0}{2R_0}}(\psi + P_0)\right], \\ X(\psi) = \sqrt{\frac{P_0}{R_0}} \tan\left[\sqrt{\frac{P_0}{2R_0}}(\psi + P_1)\right], \end{cases} \tag{42}$$

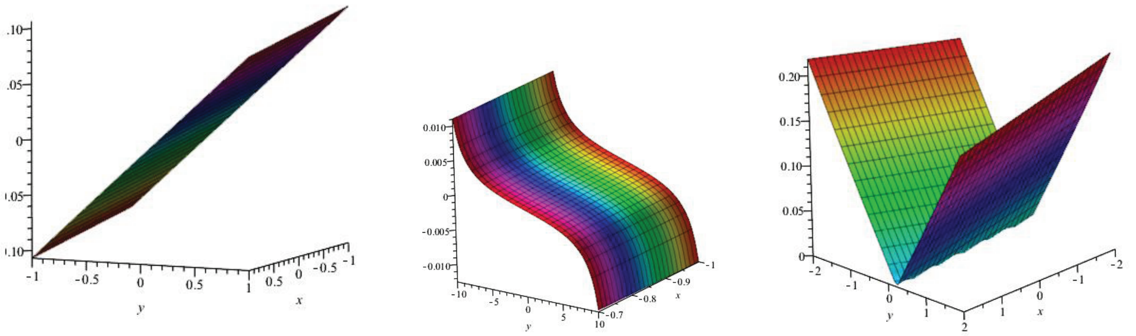
where P_0 and P_1 are arbitrary constants. To obtain exact solution of Eq.(11) by combining Eq.(42) with Eq.(30), as

$$u_{10}(x, y, t) = -\sqrt{-\frac{a(k_1 + k_2) + \omega}{2(d_1k_1 + d_2k_2)}} \tan\left[\sqrt{-\frac{a(k_1 + k_2) + \omega}{4(d_1k_1 + d_2k_2)}}(M_1x + M_2y - vt + C_0)\right] \times e^{\iota(k_1x+k_2y+\omega t+\theta)} \tag{43}$$

and

$$u_{11}(x, y, t) = \sqrt{-\frac{a(k_1 + k_2) + \omega}{2(d_1k_1 + d_2k_2)}} \tan\left[\sqrt{-\frac{a(k_1 + k_2) + \omega}{4(d_1k_1 + d_2k_2)}}(M_1x + M_2y - vt + C_1)\right] \times e^{\iota(k_1x+k_2y+\omega t+\theta)}, \tag{44}$$

where C_0 and C_1 are arbitrary constants. Figures 4 and 5 provide the graphical illustrations for $u_{10}(x, y, t)$ and $u_{11}(x, y, t)$ respectively.



(a) (b) (c)

Figure 5: Taking $a = 0.05$, $d_1 = -0.57e^{-1}$, $d_2 = 0.054$, $M_1 = -0.33$, $M_2 = 0.27$, $k_1 = 0.09$, $k_2 = 0.03$,$v = 1$, $\omega = 0.05$ and $\lambda = 0.3$, (a) 3D graphics of real part of $u_{11}(x, y, t)$. (b) 3D graphics of imaginary part of $u_{11}(x, y, t)$. (c) 3D graphics of $|u_{11}(x, y, t)|$.

4 Conclusion

Two novel analytical techniques: Extended $(\frac{G'}{G^2})$-expansion method and first integral method are used for extracting more general exact traveling wave solutions of the chiral nonlinear Schrödinger equation. These lead to several types of travelling wave solutions including solitary, periodic, singular optical solitons, complex function and dark-singular combo solitons. The proposed methods are almost well suited for constructing exact traveling wave solutions of other nonlinear evolution equations and give more solutions. In addition, the results of this study are generalized and extended version of previously reported solutions.

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