Received 10 April 2022; | Revised 6 June 2016; | Accepted 6 June 2016

DOI: xxx/xxxx

RESEARCH ARTICLE

Some traveling wave solutions to the generalized (3+1)-dimensional Korteweg-de Vries-Zakharov-Kuznetsov equation in plasma physics†

Kalim U. Tariq* | Rizwan Javed

[1]Department of Mathematics, Mirpur University of Science and Technology, Mirpur-10250 (AJK), Pakistan

Correspondence
*Corresponding author name, Email: kalimulhaq@must.edu.pk

Present Address
MUST Mirpur-10250 (AJK), Pakistan

Summary

In this article, the generalized (3+1)-dimensional Korteweg-de Vries-Zakharov-Kuznetsov equation is investigated, which describes the influence of magnetic fields on weak ion-acoustic waves in plasma made up of cool and hot electrons. To find out some new traveling waves solutions and other exact solutions, the improved F-expansion approach and the $\exp(-\phi(\zeta))$-expansion approach is applied to above mentioned nonlinear higher dimensional model. Several solutions have been found, including dark soliton, periodic type solitons, bell shaped solitons, single bell shaped solitons. We also show a graphical representation of a number of exact solutions to the equation, together with a description of their behaviour. The proposed techniques can also be used to solve a range of nonlinear evolution problems in mathematical physics and plasma physics.

KEYWORDS:
Improved F-expansion method, the (3+1)-dimensional gKdV-ZK model, the $\exp(-\phi(\zeta))$-expansion technique, traveling waves solutions, exact wave solutions.

1 | INTRODUCTION

Nonlinear evolution equations (NLEEs) are of key importance due to their significant role in diverse disciplines of science and technology. The nonlinear wave structures have fascinated many researchers in recent decades due to their diverse properties observed in various disciplines of contemporary sciences. In the presence of solitary waves, the nonlinear evolution models are utilized to simulate the effect of surface for deep water and weakly nonlinear dispersive long waves. Therefore, the exact solutions of such models play a vital role of study of dynamical structures and further properties of physical phenomenon occurring several fields to name a few, electromagnetism, physical chemistry, geophysics, ionised physics, elastic medium, fluid motion, fluid mechanics, elastic medium, nuclear physics, electrochemistry, optical fibres, energy physics, chemical mechanics, gravity, biostatistics, statistical and natural physics [1,2,3,4,5,6].

With the recent developments in various contemporary analytical methodologies, solitons plays a key role to understand the nonlinear phenomenon of many crucial structures in an exceptional way. The major feature of solitons is that they have nearly the same forms and speeds after colliding; also, the production of optical solitons is linked to optical frequency. Kink solutions are asymptotic waves that ascend or descend from one asymptotic state to the next, and they also approach a constant at infinity. Kink solutions, like classical particles, have a constant shape; nevertheless, their widths shrink, which can

†The generalized (3+1)-dimensional Korteweg-de Vries-Zakharov-Kuznetsov equation in plasma physics.

change. Solitons are transmitted as dark ones in the normal dispersion domain, but as bright ones in the anomalous dispersion domain. With the rapid advancement of information technology and telecommunications, the optical solitons play an important rule in understanding the dynamics of nonlinear wave propagation through a variety of wave-guides. The polarization of pulse propagation over trans-oceanic and trans-continental distances is an inherent problem with the dynamics of pulse propagation. Recently, several analytical and numerical approaches have been established for creating exact traveling wave solutions, solitary waves solutions and the dynamics of these solutions to such models. The Jacobi elliptic function expansion technique[7,8], the Hirota bilinear technique[9], the generalized unified technique[10,11], the tanh-coth expansion technique[12,13], the sub ODE technique[14,15], the extended Sinh-Gordon equation technique[16,17,18], the first integral technique[19,20,21], the extended simplest equation technique[22,23,24],modified khater method[25,26], new Kudryashov technique[27,28,29], modified direct algebraic technique[30,31,32], generalized auxiliary equation technique[33], generalized Riccati equation technique[34,35], the modified simple equation technique[36,37,38], Lie symmetry technique[39,40,39], the extended Tanh technique[41,42], and the Homogeneous balance method[43], among others, have all been found in current decades using illustrative computation.

The purpose of this study is to use two efficient approaches to get traveling and solitary waves solutions to the (3+1)-dimensional Korteweg-de Vries-Zakharov Kuznetsov model (gKdV-ZK) the improved F-expansion technique[44,45] and the $\exp(-\phi(\zeta))$-expansion technique[46] which reads

$$u_t + \alpha \mathrm{u}^2 u_x + \beta \mathrm{u}_{\mathrm{xxx}} + \nu (u_{yy} + uzz)_x = 0, \tag{1}$$

here $u = u(x, y, z, t)$ is a wave profile and α, β, ν, are constants.
In literature, various researchers have studied this model that is used in plasma physics and apply different analytical techniques to obtain exact solitons solutions, such as the Lie symmetry technique and direct integration are used to obtain the solution of this complex model, which includes a non-topological soliton[47], the modified extended direct algebraic technique[48] for investigating the ion-acoustic wave structure from this model, to generate a closed-form solution the Kudryashove technique is used[49].

This paper is organized as follows: In section 2, a detail description of the proposed techniques is shown. While section 3 is devoted to the applications of the two proposed techniques to (3+1)-dimensional gKdV-ZKe. However section 4 contains discussion and results also section 5 contains conclusions.

2 | METHODOLOGY

Suppose the NPDE in the following form,

$$F\left(\phi, \phi_t, \phi_x, \phi_{tt}, \phi_{xx}, \phi_{xt}, \phi_{yy}, \phi_{zz}, \phi_{yt}, \cdots\right) = 0, \tag{2}$$

ϕ is a wave function of the variables x, y, z and t and F is a nonlinear polynomial. Suppose the wave transformation of form,

$$u(x, y, z, t) = U(\zeta), \quad \zeta = x + y + z - ct. \tag{3}$$

Putting Eq.(3) into Eq.(2), gives an ODE of the form,

$$Q\left(\phi, \phi', \phi'', \phi''', \cdots\right) = 0. \tag{4}$$

2.1 | The improved F-expansion technique

The major process of approach are outlined below,
Step 1. The solution of Eq. (4) is then assumed and may be written as,

$$U(\zeta) = \sum_{i=0}^{N} a_i (m + F(\zeta))^i + \sum_{j=1}^{N} b_j (m + F(\zeta))^{-j}, \tag{5}$$

where $a_0, a_n, b_n, (n = 1, 2, \cdots, N)$ are constants to be determine. Furthermore, N is balance term that can be calculated by using the balancing principle between the higher derivative and higher power in ODE Eq.(4) and $F(\zeta)$ is a function that satisfies the following Riccati equation,

$$F'(\zeta) - F^2(\zeta) - K = 0, \tag{6}$$

where K is a real parameter. Thus Eq.(6) has the following solutions that depend on the parameter K.
Case I. If $K < 0$, then the hyperbolic function solution are:

$$F_1(\zeta) = -\sqrt{-K}\tanh\left(\sqrt{-K}\zeta\right),$$
$$F_2(\zeta) = -\sqrt{-K}\coth\left(\sqrt{-K}\zeta\right).$$

Case II. If $K > 0$, then the trigonometric function solution are:

$$F_3(\zeta) = \sqrt{K}\tan\left(\sqrt{K}\zeta\right),$$
$$F_4(\zeta) = -\sqrt{K}\cot\left(\sqrt{K}\zeta\right).$$

Case III. If $K = 0$, then the rational function solution are:

$$F_5(\zeta) = -\frac{1}{\zeta}.$$

Step 2. Puting Eq. (5) with Eq. (6) into Eq. (4), a polynomial in powers of $F(\zeta)$ are obtained. After collecting the coefficients of the same power of $F(\zeta)$ and setting each term of them equals to zero. We get a set of the algebraic system equations.
Step 3. After solving the algebraic system of equations, we substitute the obtained constants together with the solution of Eq.(6) into Eq.(5) to obtain exact wave solutions of Eq.(2).

2.2 | The $\exp(-\phi(\zeta))$-expansion technique

The major process of $\exp(-\phi(\zeta))$-expansion technique are outlined below,
Step 1. The solution of Eq. (4) is then assumed and written in the following form

$$U(\zeta) = \sum_{i=0}^{N} A_i(\exp(-\phi(\zeta))^i, \tag{7}$$

where A_i $(i = 0, 1, \cdots, N)$ are arbitrary constants that can be calculated later and $\exp(-\phi(\zeta))$ satisfies the following Riccati equation,

$$\phi'(\zeta) = \exp(-\phi(\zeta)) + \mu\exp(\phi(\zeta)) + \theta, \tag{8}$$

where μ and θ is a arbitrary constants. Eq. (8) has following exact wave solutions.
Family 1. If $\theta^2 - 4\mu > 0$ and $\mu \neq 0$, then Eq. (8) has following solution:

$$\begin{aligned} \phi(\zeta) &= \ln\left(\frac{-\sqrt{(\theta^2-4\mu)}\tanh\left(\frac{\sqrt{(\theta^2-4\mu)}}{2}(\zeta+K)\right)-\theta}{2\mu}\right), \\ \phi(\zeta) &= \ln\left(\frac{-\sqrt{(\theta^2-4\mu)}\coth\left(\frac{(\sqrt{\theta^2-4\mu)}}{2}(\zeta+K)\right)-\theta}{2\mu}\right), \end{aligned} \tag{9}$$

where K is integration constant.
Family 2. If $\theta^2 - 4\mu < 0$ and $\mu \neq 0$, then Eq. (8) has following solution

$$\begin{aligned} \phi(\zeta) &= \ln\left(\frac{\sqrt{(4\mu-\theta^2)}\tan\left(\frac{\sqrt{(4\mu-\theta^2)}}{2}(\zeta+K)\right)-\theta}{2\mu}\right), \\ \phi(\zeta) &= \ln\left(\frac{\sqrt{(4\mu-\theta^2)}\cot\left(\frac{(\sqrt{4\mu-\theta^2)}}{2}(\zeta+K)\right)-\theta}{2\mu}\right). \end{aligned} \tag{10}$$

Family 3. If $\theta^2 - 4\mu > 0$, $\mu = 0$ and $\theta \neq 0$, then Eq. (8) has solution

$$\phi(\zeta) = -\ln\left(\frac{\theta}{\exp(\theta(\zeta + K)) - 1}\right). \tag{11}$$

Family 4. If $\theta^2 - 4\mu = 0$, $\mu \neq 0$ and $\theta \neq 0$ then Eq. (8) has solution

$$\phi(\zeta) = \ln\left(-\frac{2(\theta(\zeta + K) + 2)}{\theta^2(\zeta + K)}\right). \tag{12}$$

Family 5. If $\theta^2 - 4\mu = 0$, $\mu = 0$ and $\theta = 0$ then Eq. (8) has solution

$$\phi(\zeta) = \ln(\zeta + K) \tag{13}$$

Step 2. The balance term N in Eq. (7) can be calculated by homogeneous balancing principle between the higher derivatives and highest power term in ODE Eq.(4).
Step 3. Putting Eq.(7) combined with Eq.(8) in Eq.(4), we have a polynomial in $\exp(-\phi(\zeta))$. Collecting all coefficients of the same $(\exp(-\phi(\zeta)))^i$ $(i = 0, 1, 2, \cdots, M)$ and setting all of the terms equals to zero, we get group of the algebraic equations.
Step 4. After solving the group of algebraic equations, we substitute the obtained constants combined with the solution of Eq.(8) into Eq.(7) to obtain exact wave solutions of Eq.(2).

3 | MATHEMATICAL ANALYSIS

The main purpose of this part is to obtain the exact solutions of the 3-dimensional gKdV-ZK equation (1). We apply the transformation as follows

$$u(x, y, z, t) = U(\zeta), \quad \zeta = x + y + z - ct, \tag{14}$$

substituting values from Eq.(14) to Eq.(1) and we get the following form of ODE,

$$-cU'(\zeta) + \beta U^{(3)}(\zeta) + 2\nu U^{(3)}(\zeta) + \alpha U^2(\zeta)U'(\zeta) = 0. \tag{15}$$

Integrating Eq.(15) w.r.t (ζ), we get

$$-\mathrm{cU}(\zeta) + (\beta + 2\nu)U''(\zeta) + \frac{1}{3}\alpha U^3(\zeta) = 0. \tag{16}$$

3.1 | Application to the improved F-expansion method

By using the balance rule we obtain the balancing term $N = 1$. The solution of Eq.(16) is of the form

$$U(\zeta) = a_0 + a_1(m + F(\zeta)) + \frac{b_1}{(m + F(\zeta))}. \tag{17}$$

Substituting Eq.(17) and its necessary derivatives into Eq.(16) and gathering all same powers terms of $F(\zeta)$. Setting each $F(\zeta)$ polynomial equal to zero yields the following group of algebraic group of equations,

$$
\begin{aligned}
&-\alpha a_1^2 b_1 m^4 - 2\alpha a_0 a_1 b_1 m^3 + \alpha a_1 b_1^2 m^2 - \alpha a_0^2 b_1 m^2 + \alpha a_0 b_1^2 m - \\
&a_1 c m^4 - a_0 c m^3 + \frac{1}{3}\alpha a_1^3 m^6 + \alpha a_0 a_1^2 m^5 + \alpha a_0^2 a_1 m^4 + \\
&\frac{1}{3}\alpha a_0^3 m^3 + \frac{\alpha b_1^3}{3} - b_1 c m^2 + 2\beta b_1 K^2 + 4 b_1 K^2 \nu = 0, \\
&\alpha a_0 b_1^2 - 4\alpha a_1^2 b_1 m^3 - 6\alpha a_0 a_1 b_1 m^2 + 2\alpha a_1 b_1^2 m - 2\alpha a_0^2 b_1 m - \\
&4 a_1 c m^3 - 3 a_0 c m^2 + 2 a_1 \beta K m^3 + 4 a_1 K \nu m^3 + 2\alpha a_1^3 m^5 + 5\alpha a_0 a_1^2 m^4 + \\
&4\alpha a_0^2 a_1 m^3 + \alpha a_0^3 m^2 - 2 b_1 c m - 2\beta b_1 K m + 4 b_1 K \nu m = 0, \\
&\alpha a_1 b_1^2 - \alpha a_0^2 b_1 - 6\alpha a_1^2 b_1 m^2 - 6\alpha a_0 a_1 b_1 m - 6 a_1 c m^2 - \\
&3 a_0 c m + 6 a_1 \beta K m^2 + 12 a_1 K \nu m^2 + 5\alpha a_1^3 m^4 + 10\alpha a_0 a_1^2 m^3 + \\
&6\alpha a_0^2 a_1 m^2 + \alpha a_0^3 m - b_1 c + 2\beta b_1 K + 12 b_1 K \nu = 0, \\
&\frac{\alpha a_0^3}{3} - 2\alpha a_0 a_1 b_1 - 4\alpha a_1^2 b_1 m - 4 a_1 c m - a_0 c + \\
&6 a_1 \beta K m + 12 a_1 K \nu m + \frac{20}{3}\alpha a_1^3 m^3 + 2 a_1 \beta m^3 + \\
&4 a_1 \nu m^3 + 10\alpha a_0 a_1^2 m^2 + 4\alpha a_0^2 a_1 m + 2\beta b_1 m + 4 b_1 \nu m = 0, \\
&\alpha a_0^2 a_1 - \alpha a_1^2 b_1 - a_1 c + 2 a_1 \beta K + 4 a_1 K \nu + 5\alpha a_1^3 m^2 + 6 a_1 \beta m^2 + \\
&12 a_1 m^2 \nu + 5\alpha a_0 a_1^2 m + 4\beta b_1 + 8 b_1 \nu = 0, \\
&\alpha a_0 a_1^2 + 2\alpha a_1^3 m + 6 a_1 \beta m + 12 a_1 m \nu = 0, \\
&\frac{\alpha a_1^3}{3} + 2 a_1 \beta + 4 a_1 \nu = 0.
\end{aligned}
$$

Using computer software Mathematica to solve the above algebraic group of equations. The following are the outcomes,

$$a_0 = 0,\ m = 0,\ \nu = -\frac{1}{5}(2\beta),\ a_1 = -\frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}}{\sqrt{\alpha}},\ c = -\frac{1}{5}(8\beta K),\ b_1 = \frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}K}{\sqrt{\alpha}}.$$

Substituting above results into Eq. (17), produces following categories of traveling wave solutions.
Family I:
The following cases arise:
Case I. If $K < 0$, then the solution is

$$u_1(\zeta) = \frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}\sqrt{-K}\tanh\left(\sqrt{-K}\zeta\right)}{\sqrt{\alpha}} - \frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}K\coth\left(\sqrt{-K}\zeta\right)}{\sqrt{\alpha}\sqrt{-K}}, \tag{18}$$

$$u_2(\zeta) = \frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}\sqrt{-K}\coth\left(\sqrt{-K}\zeta\right)}{\sqrt{\alpha}} - \frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}K\tanh\left(\sqrt{-K}\zeta\right)}{\sqrt{\alpha}\sqrt{-K}}. \tag{19}$$

Case II. If $K > 0$, then the solution is

$$u_3(\zeta) = \frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}\sqrt{K}\cot\left(\sqrt{-K}\zeta\right)}{\sqrt{\alpha}} - \frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}\sqrt{K}\tan\left(\sqrt{-K}\zeta\right)}{\sqrt{\alpha}}, \tag{20}$$

$$u_4(\zeta) = \frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}\sqrt{K}\cot\left(\sqrt{-K}\zeta\right)}{\sqrt{\alpha}} - \frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}\sqrt{K}\tan\left(\sqrt{-K}\zeta\right)}{\sqrt{\alpha}}. \tag{21}$$

Case III. If $K = 0$, then the solution is

$$u_5(\zeta) = \frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}}{\sqrt{\alpha}\left(\sqrt{-K}\zeta\right)} + \frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}K\,(\zeta)}{\sqrt{\alpha}}. \tag{22}$$

We have following results in family II:

$$a_0 = 0,\ m = 0,\ \nu = -\frac{1}{3}(2\beta),\ a_1 = -\frac{\sqrt{2}\sqrt{\beta}}{\sqrt{\alpha}}\ c = -4\beta K,\ b_1 = -\frac{\sqrt{2}\sqrt{\beta}K}{\sqrt{\alpha}}.$$

Substituting above results into Eq. (17), produces following categories of traveling wave solutions.
Family II:
The following cases arise:
Case I. If $K < 0$, then the solution is

$$u_6(\zeta) = \frac{\sqrt{2}\sqrt{\beta}\sqrt{-K}\tanh\left(\sqrt{-K}\zeta\right)}{\sqrt{\alpha}} + \frac{\sqrt{2}\sqrt{\beta}K\coth\left(\sqrt{-K}\zeta\right)}{\sqrt{\alpha}\sqrt{-K}}, \tag{23}$$

$$u_7(\zeta) = \frac{\sqrt{2}\sqrt{\beta}K\tanh\left(\sqrt{-K}\zeta\right)}{\sqrt{\alpha}\sqrt{-K}} + \frac{\sqrt{2}\sqrt{\beta}\sqrt{-K}\coth\left(\sqrt{-K}\zeta\right)}{\sqrt{\alpha}}. \tag{24}$$

Case II. If $K > 0$, then the solution is

$$u_8(\zeta) = -\frac{\sqrt{2}\sqrt{\beta}\sqrt{K}\tan\left(\sqrt{-K}\zeta\right)}{\sqrt{\alpha}} - \frac{\sqrt{2}\sqrt{\beta}\sqrt{K}\cot\left(\sqrt{K}\zeta\right)}{\sqrt{\alpha}}, \tag{25}$$

$$u_9(\zeta) = \frac{\sqrt{2}\sqrt{\beta}\sqrt{K}\tan\left(\sqrt{-K}\zeta\right)}{\sqrt{\alpha}} + \frac{\sqrt{2}\sqrt{\beta}\sqrt{K}\cot\left(\sqrt{-K}\zeta\right)}{\sqrt{\alpha}}. \tag{26}$$

Case III. If $K = 0$, then the solution is

$$u_{10}(\zeta) = \frac{\sqrt{2}\sqrt{\beta}}{\sqrt{\alpha}\zeta} - \frac{\sqrt{2}\sqrt{\beta}K\zeta}{\sqrt{\alpha}}. \tag{27}$$

We have following results in family III:

$$a_0 = 0,\ m = 0,\ \nu = -\frac{1}{5}(2\beta),\ a_1 = \frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}}{\sqrt{\alpha}},\ c = -\frac{1}{5}(8\beta K),\ b_1 = -\frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}K}{\sqrt{\alpha}}.$$

Substituting above results into Eq. (17), produces following categories of traveling wave solutions.
Family III:
The following cases arise:
Case. I If $K < 0$, then the solution is

$$u_{11}(\zeta) = \frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}K\coth\left(\sqrt{-K}\zeta\right)}{\sqrt{\alpha}\sqrt{-K}} - \frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}\sqrt{-K}\tanh\left(\sqrt{-K}\zeta\right)}{\sqrt{\alpha}}, \tag{28}$$

$$u_{12}(\zeta) = \frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}K\tanh\left(\sqrt{-K}\zeta\right)}{\sqrt{\alpha}\sqrt{-K}} - \frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}\sqrt{-K}\coth\left(\sqrt{-K}\zeta\right)}{\sqrt{\alpha}}. \tag{29}$$

Case II. If $K > 0$, then the solution is

$$u_{13}(\zeta) = \frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}\sqrt{K}\tan\left(\sqrt{K}\zeta\right)}{\sqrt{\alpha}} - \frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}\sqrt{K}\cot\left(\sqrt{K}\zeta\right)}{\sqrt{\alpha}}, \tag{30}$$

$$u_{14}(\zeta) = \frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}\sqrt{K}\tan\left(\sqrt{K}\zeta\right)}{\sqrt{\alpha}} - \frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}\sqrt{K}\cot\left(\sqrt{K}\zeta\right)}{\sqrt{\alpha}}. \tag{31}$$

Case III. If $K = 0$, then the solution is

$$u_{15}(\zeta) = -\frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}}{\sqrt{\alpha}\zeta} - \frac{i\sqrt{\frac{6}{5}}\sqrt{\beta}K\zeta}{\sqrt{\alpha}}. \tag{32}$$

We have following results in family IV:

$$a_0 = 0,\ b_1 = 0,\ m = 0,\ \nu = -\frac{1}{3}(2\beta),\ a_1 = \frac{\sqrt{2}\sqrt{\beta}}{\sqrt{\alpha}},\ c = -\frac{1}{3}(2\beta K).$$

Substituting above results into Eq. (17), produces following categories of traveling wave solutions.
Family IV:
The following cases arise:
Case I. If $K < 0$, then the solution is

$$u_{16}(\zeta) = -\frac{\sqrt{2}\sqrt{\beta}\sqrt{-K}\tanh\left(\sqrt{-K}\zeta\right)}{\sqrt{\alpha}}, \tag{33}$$

$$u_{17}(\zeta) = -\frac{\sqrt{2}\sqrt{\beta}\sqrt{-K}\coth\left(\sqrt{-K}\zeta\right)}{\sqrt{\alpha}}. \tag{34}$$

Case II. If $K > 0$, then the solution is

$$u_{18}(\zeta) = \frac{\sqrt{2}\sqrt{\beta}\sqrt{K}\tan\left(\sqrt{K}\zeta\right)}{\sqrt{\alpha}}, \tag{35}$$

$$u_{19}(\zeta) = -\frac{\sqrt{2}\sqrt{\beta}\sqrt{K}\cot\left(\sqrt{K}\zeta\right)}{\sqrt{\alpha}}. \tag{36}$$

Case III. If $K = 0$, then the solution is

$$u_{20}(\zeta) = -\frac{\sqrt{2}\sqrt{\beta}}{\sqrt{\alpha}\zeta}. \tag{37}$$

3.2 | Application to the $\exp(-\phi(\zeta))$-expansion technique

By using the homogeneous balance rule we obtain the balancing term $N = 1$.
Thus solution of Eq.(16), of the form

$$U(\zeta) = A_0 + A_1 \exp(-\phi(\zeta)). \tag{38}$$

Substitute Eq.(38) with Eq.(8) into Eq.(16) and gathering all same powers terms of $(\exp(-\phi(\zeta)))^i$ $(i = 0, \pm 1, \pm 2, \cdots)$ together and setting all the obtained coefficients equals to be zero, we get the following group of algebraic equations,

$$\begin{aligned}
\frac{\alpha A_0^3}{3} + A_1\beta\theta\mu - A_0 c + 2A_1\theta\mu\nu &= 0,\\
\alpha A_0^2 A_1 + A_1\beta\theta^2 + 2A_1\beta\mu - A_1 c + 2A_1\theta^2\nu + 4A_1\mu\nu &= 0,\\
\alpha A_0 A_1^2 + 3A_1\beta\theta + 6A_1\theta\nu &= 0,\\
\frac{\alpha A_1^3}{3} + 2A_1\beta + 4A_1\nu &= 0.
\end{aligned}$$

Using Mathematica to solve the above group of equations. The following are the outcomes,

$$A_0 = -\frac{\sqrt{\frac{3}{2}}\sqrt{-\beta\theta^2 - 2\theta^2\nu}}{\sqrt{\alpha}},\ A_1 = -\frac{\sqrt{6}\sqrt{-\theta^2(\beta + 2\nu)}}{\sqrt{\alpha}\theta},\ c = -\frac{1}{2}(\beta + 2\nu)(\theta^2 - 4\mu).$$

Putting above results into Eq. (38), produces following categories of traveling wave solutions.
Family I:
The following Cases arise:
Case I. When $\theta^2 - 4\mu > 0$ and $\mu \neq 0$, we get the solution

$$u_{21}(\zeta) = -\frac{\sqrt{\frac{3}{2}}\sqrt{-\beta\theta^2 - 2\theta^2\nu}}{\sqrt{\alpha}} - \frac{2\sqrt{6}\mu\sqrt{-\theta^2(\beta + 2\nu)}}{\sqrt{\alpha}\theta\left(-\theta - \sqrt{\theta^2 - 4\mu}\tanh\left(\frac{1}{2}\sqrt{\theta^2 - 4\mu}(\zeta + K)\right)\right)}, \tag{39}$$

$$u_{22}(\zeta) = -\frac{\sqrt{\frac{3}{2}}\sqrt{-\beta\theta^2 - 2\theta^2\nu}}{\sqrt{\alpha}} - \frac{2\sqrt{6}\mu\sqrt{-\theta^2(\beta + 2\nu)}}{\sqrt{\alpha}\theta\left(-\theta - \sqrt{\theta^2 - 4\mu}\coth\left(\frac{1}{2}\sqrt{\theta^2 - 4\mu}(\zeta + K)\right)\right)}, \tag{40}$$

where $\zeta = \frac{1}{2}t(\beta + 2\nu)\left(\theta^2 - 4\mu\right) + x + y + z$ and μ, θ are arbitrary constant.
Case 2. When $\theta^2 - 4\mu < 0$, $\mu \neq 0$ we get the solution,

$$u_{23}(\zeta) = -\frac{\sqrt{\frac{3}{2}}\sqrt{-\beta\theta^2 - 2\theta^2\nu}}{\sqrt{\alpha}} - \frac{2\sqrt{6}\mu\sqrt{-\theta^2(\beta + 2\nu)}}{\sqrt{\alpha}\theta\left(\sqrt{4\mu - \theta^2}\tan\left(\frac{1}{2}\sqrt{4\mu - \theta^2}(\zeta + K)\right) - \theta\right)}, \tag{41}$$

$$u_{24}(\zeta) = -\frac{\sqrt{\frac{3}{2}}\sqrt{-\beta\theta^2 - 2\theta^2\nu}}{\sqrt{\alpha}} - \frac{2\sqrt{6}\mu\sqrt{-\theta^2(\beta + 2\nu)}}{\sqrt{\alpha}\theta\left(\sqrt{4\mu - \theta^2}\cot\left(\frac{1}{2}\sqrt{4\mu - \theta^2}(\zeta + K)\right) - \theta\right)}, \tag{42}$$

where $\zeta = \frac{1}{2}t(\beta + 2\nu)\left(\theta^2 - 4\mu\right) + x + y + z$ and μ, θ are arbitrary constant.
Case 3. When $\theta^2 - 4\mu > 0$, $\mu = 0$ and $\theta \neq 0$, we get the solution,

$$u_{25}(\zeta) = -\frac{\sqrt{\frac{3}{2}}\sqrt{-\beta\theta^2 - 2\theta^2\nu}}{\sqrt{\alpha}} - \frac{\sqrt{6}\sqrt{-\theta^2(\beta + 2\nu)}}{\sqrt{\alpha}\left(\exp\left(\theta\left(K + \zeta\right)\right) - 1\right)}. \tag{43}$$

Case 4. When $\theta^2 - 4\mu = 0$, $\mu \neq 0$ and $\theta \neq 0$, we get the solution,

$$u_{26}(\zeta) = -\frac{\sqrt{\frac{3}{2}}\sqrt{-\beta\theta^2 - 2\theta^2\nu}}{\sqrt{\alpha}} + \frac{\sqrt{\frac{3}{2}}\theta\sqrt{-\theta^2(\beta + 2\nu)}\left(K + \zeta\right)}{\sqrt{\alpha}\left(\theta\left(K + \zeta\right) + 2\right)}. \tag{44}$$

Case 5. When $\theta^2 - 4\mu = 0$, $\mu = 0$ and $\theta = 0$, we get the solution,

$$u_{27}(\zeta) = -\frac{\sqrt{\frac{3}{2}}\sqrt{-\beta\theta^2 - 2\theta^2\nu}}{\sqrt{\alpha}} - \frac{\sqrt{6}\sqrt{-\theta^2(\beta + 2\nu)}}{\sqrt{\alpha}\theta\left(K + \zeta\right)}. \tag{45}$$

We have following results in family II:

$$A_0 = \frac{\sqrt{\frac{3}{2}}\sqrt{-\beta\theta^2 - 2\theta^2\nu}}{\sqrt{\alpha}},\ A_1 = \frac{\sqrt{6}\sqrt{-\theta^2(\beta + 2\nu)}}{\sqrt{\alpha}\theta},\ c = -\frac{1}{2}(\beta + 2\nu)\theta^2 - 4\mu.$$

Putting above results into Eq. (38), produces following categories of traveling wave solutions.
Family II:

The following Cases arise:
Case I. When $\theta^2 - 4\mu > 0$ and $\mu \neq 0$, we get the solution

$$u_{28}(\zeta) = \frac{\sqrt{\frac{3}{2}}\sqrt{-\beta\theta^2 - 2\theta^2\nu}}{\sqrt{\alpha}} + \frac{2\sqrt{6}\mu\sqrt{-\theta^2(\beta+2\nu)}}{\sqrt{\alpha}\theta\left(-\theta - \sqrt{\theta^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\theta^2-4\mu}(\zeta+K)\right)\right)}, \tag{46}$$

$$u_{29}(\zeta) = \frac{\sqrt{\frac{3}{2}}\sqrt{-\beta\theta^2 - 2\theta^2\nu}}{\sqrt{\alpha}} + \frac{2\sqrt{6}\mu\sqrt{-\theta^2(\beta+2\nu)}}{\sqrt{\alpha}\theta\left(-\theta - \sqrt{\theta^2-4\mu}\coth\left(\frac{1}{2}\sqrt{\theta^2-4\mu}(\zeta+K)\right)\right)},$$

where $\zeta = \frac{1}{2}t(\beta+2\nu)\left(\theta^2-4\mu\right) + x + y + z$ and μ, θ are arbitrary constant.
Case 2. When $\theta^2 - 4\mu < 0$, $\mu \neq 0$ we get the solution:

$$u_{30}(\zeta) = \frac{\sqrt{\frac{3}{2}}\sqrt{-\beta\theta^2 - 2\theta^2\nu}}{\sqrt{\alpha}} + \frac{2\sqrt{6}\mu\sqrt{-\theta^2(\beta+2\nu)}}{\sqrt{\alpha}\theta\left(\sqrt{4\mu-\theta^2}\tan\left(\frac{1}{2}\sqrt{4\mu-\theta^2}(\zeta+K)\right) - \theta\right)}, \tag{47}$$

,

$$u_{31}(\zeta) = \frac{\sqrt{\frac{3}{2}}\sqrt{-\beta\theta^2 - 2\theta^2\nu}}{\sqrt{\alpha}} + \frac{2\sqrt{6}\mu\sqrt{-\theta^2(\beta+2\nu)}}{\sqrt{\alpha}\theta\left(\sqrt{4\mu-\theta^2}\cot\left(\frac{1}{2}\sqrt{4\mu-\theta^2}(\zeta+K)\right) - \theta\right)}, \tag{48}$$

where $\zeta = \frac{1}{2}t(\beta+2\nu)\left(\theta^2-4\mu\right) + x + y + z$ and μ, θ are arbitrary constant.
Case 3. When $\theta^2 - 4\mu > 0$, $\mu = 0$ and $\theta \neq 0$, we get the solution,

$$u_{32}(\zeta) = \frac{\sqrt{\frac{3}{2}}\sqrt{-\beta\theta^2 - 2\theta^2\nu}}{\sqrt{\alpha}} + \frac{\sqrt{6}\sqrt{-\theta^2(\beta+2\nu)}}{\sqrt{\alpha}\left(\exp\left(\theta\left(K+\zeta\right)\right) - 1\right)}. \tag{49}$$

Case 4. When $\theta^2 - 4\mu = 0$, $\mu \neq 0$ and $\theta \neq 0$, we get the solution.

$$u_{33}(\zeta) = \frac{\sqrt{\frac{3}{2}}\sqrt{-\beta\theta^2 - 2\theta^2\nu}}{\sqrt{\alpha}} - \frac{\sqrt{\frac{3}{2}}\theta\sqrt{-\theta^2(\beta+2\nu)}\,(K+\zeta)}{\sqrt{\alpha}\left(\theta\left(K+\zeta\right)+2\right)}. \tag{50}$$

Case 5. When $\theta^2 - 4\mu = 0$, $\mu = 0$ and $\theta = 0$, we get the solution,

$$u_{34}(\zeta) = \frac{\sqrt{\frac{3}{2}}\sqrt{-\beta\theta^2 - 2\theta^2\nu}}{\sqrt{\alpha}} + \frac{\sqrt{6}\sqrt{-\theta^2(\beta+2\nu)}}{\sqrt{\alpha}\theta\left(K+\zeta\right)}. \tag{51}$$

4 | DISCUSSION AND RESULTS

This section contain the graphical representation of some new exact solitary and traveling wave solutions of the generalized (3+1)-dimensional gKdV-ZK model have been illustrated. The software Mathematica 11.0 is used to describe the behaviour of waves as a function of numerous factors. The 3D, contour and 2D graphs visualizes the nature of nonlinear wave solutions constructed from Eq.(1). To demonstrate the solutions by the improved F-expansion approach and the $\exp(-\phi(\zeta))$-expansion approach is observed for the Family-I, Family-II, Family-III. The figure (1, 3) represents single bell shaped soliton for the set of appropriate values $\alpha = -1.3,\ \beta = 1.6,\ K = -0.1,\ y = 1,\ z = 0.3,\ t = 1$ and $\alpha = -3.3,\ \beta = 2.6,\ K = -0.15,\ y = 1,\ z = 0.7,\ t = 1$. The figure (2) represents double bell shaped soliton for $\alpha = -0.13,\ \beta = 1.16,\ K = 0.13,\ y = 1,\ z = 0.19,\ t = 1$. The figure (5) represents dark soliton for the set of appropriate values $\alpha = -1.3,\ \beta = 1.6,\ K = -0.5,\ y = 1,\ z = 0.3,\ t = 1$. The figure (4, 6, 7) represents periodic shaped soliton for the set of appropriate values $\alpha = -1.03,\ \beta = -1,\ K = 0.3,\ y = -1,\ z = 0.1,\ t = 1$ and $\alpha = 0.1,\ \beta = -2,\ K = 2,\ y = 1,\ z = -5,\ t = 1$ and $\alpha = -2.13,\ \beta = 1.006,\ K = 2.15,\ y = -1,\ z = -0.13,\ t = 1$. The figure (8) represents bright soliton for the set of appropriate values $\alpha = -1.05,\ \beta = 0.9,\ y = -1.11,\ \mu = 1.0015,\ \nu = 1.008,\ \theta = 2.11,\ z = -0.2,\ K = -6.03,\ t = 1$. The figure (9) represents dark soliton for the set of values

$\alpha = 1.03,\ \beta = -0.013,\ y = 1,\ \mu = -0.5,\ \nu = -0.8,\ \theta = 0.1,\ z = -0.12,\ K = 0.001,\ t = 1$. While figure (10) represents single bell shaped soliton for the set of parameter $\alpha = 1,\ \beta = -0.3,\ \mu = 0,\ y = -1, \nu = -0.8,\ z = -0.2,\ K = 1,\ \theta = -1.11,\ t = 1$.

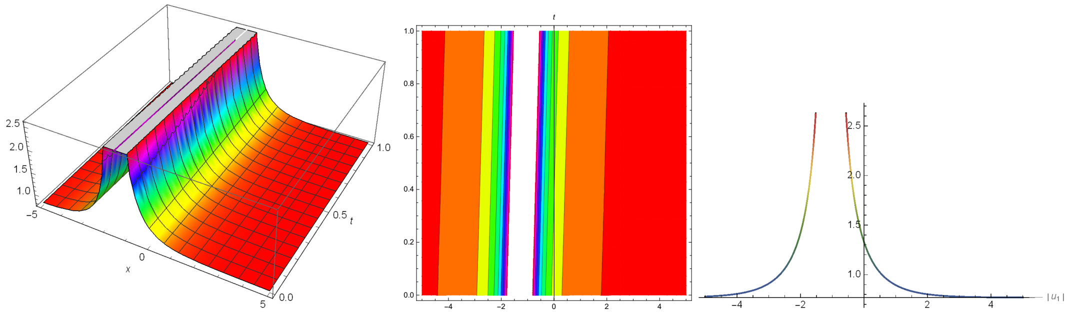

FIGURE 1 The 3D plot, contour plot and 2D plot for solution $u_1(x, y, z, t)$ of Eq. (18). Values chosen are $\alpha = -1.3,\ \beta = 1.6,\ y = 1,\ K = -0.1,\ z = 0.3,\ t = 1$.

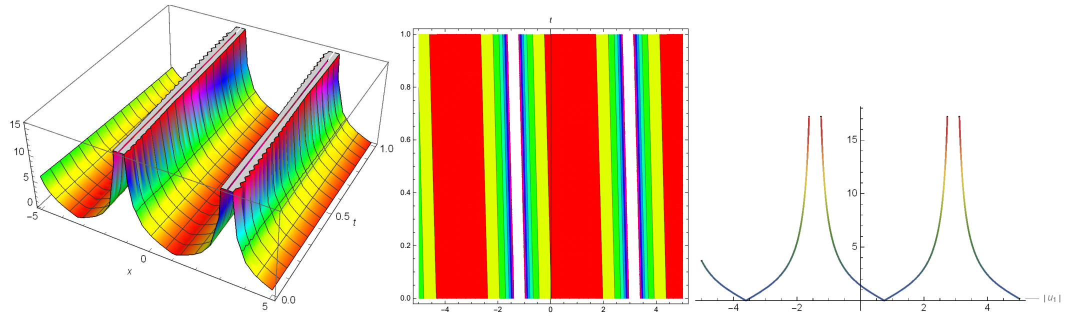

FIGURE 2 The 3D plot, contour plot and 2D plot for solution $u_3(x, y, z, t)$ of Eq. (20). Values chosen are $\alpha = -0.13,\ K = 0.13,\ y = 1,\ \beta = 1.16,\ z = 0.19,\ t = 1$.

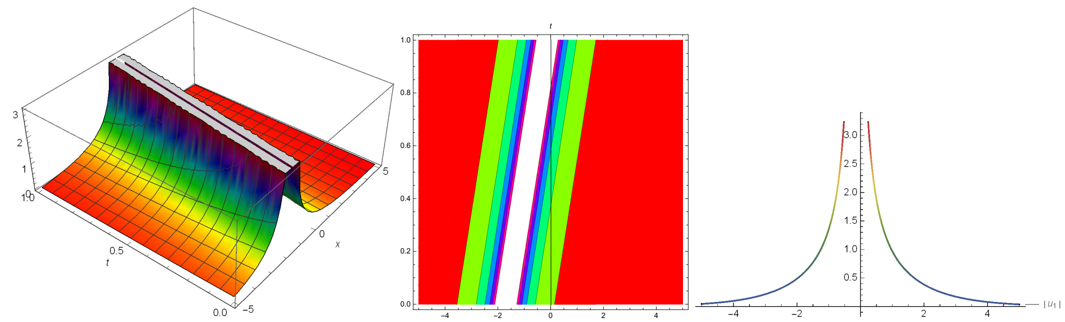

FIGURE 3 The 3D plot, contour plot and 2D plot for solution $u_6(x, y, z, t)$ of Eq. (23). Values chosen are $\alpha = -3.3,\ K = -0.15,\ y = 1,\ \beta = 2.6,\ z = 0.7,\ t = 1$.

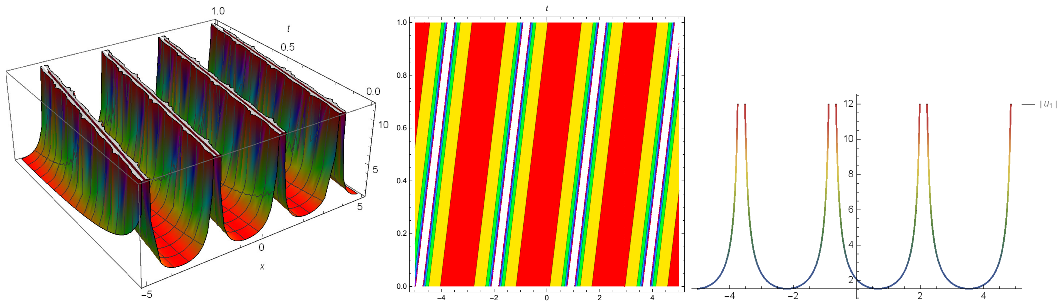

FIGURE 4 The 3D plot, contour plot and 2D plot for solution $u_8(x, y, z, t)$ of Eq. (25). Values chosen are $\alpha = -1.03$, $\beta = -1$, $K = 0.3$, $y = -1$, $z = 0.1$, $t = 1$.

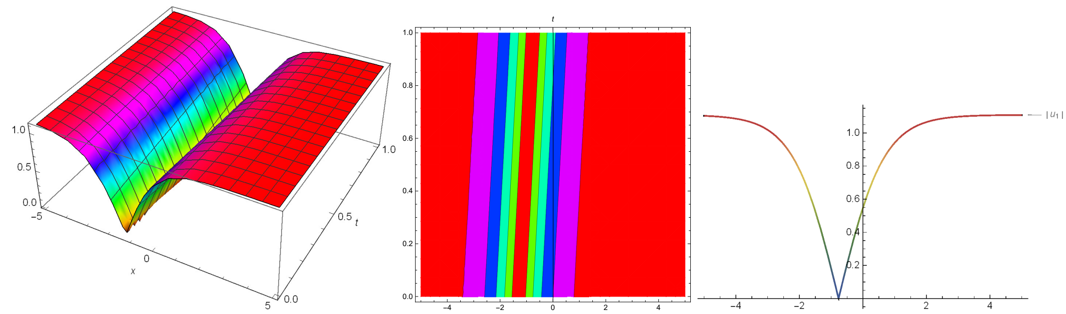

FIGURE 5 The 3D plot, contour plot and 2D plot for solution $u_{16}(x, y, z, t)$ of Eq. (33). Values chosen are $\alpha = -1.3$, $\beta = 1.6$, $K = -0.5$, $y = 1$, $z = 0.3$, $t = 1$.

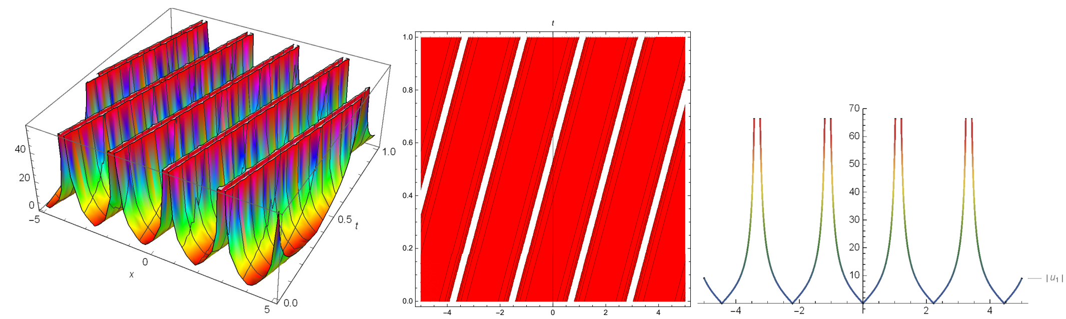

FIGURE 6 The 3D plot, contour plot and 2D plot for solution $u_{18}(x, y, z, t)$ of Eq. (35). Values chosen are $\alpha = 0.1$, $\beta = -2$, $K = 2$, $y = 1$, $z = -5$, $t = 1$.

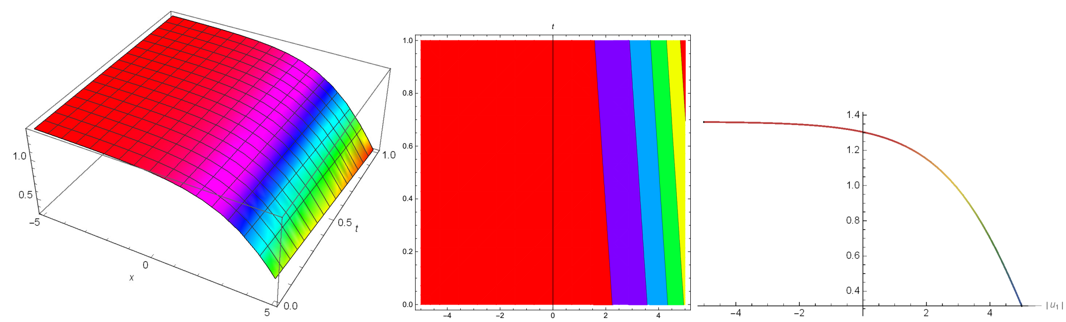

FIGURE 7 The 3D plot, contour plot and 2D plot for solution $u_{23}(x, y, z, t)$ of Eq. (41). Values chosen are $\alpha = -1.05$, $\beta = 0.9$, $y = -1.11$, $\mu = 1.0015$, $\nu = 1.008$, $\theta = 2.11$, $z = -0.2$, $K = -6.03$, $t = 1$.

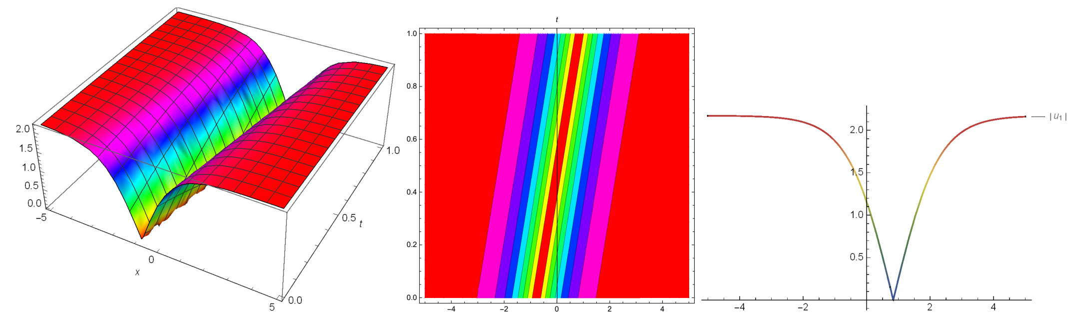

FIGURE 8 The 3D plot, contour plot and 2D plot for solution $u_{24}(x, y, z, t)$ of Eq. (42). Values chosen are $\alpha = 1.03,\ \beta = -0.013,\ y = 1,\ \mu = -0.5,\ \theta = 0.1,\ z = -0.12,\ \nu = -0.8,\ K = 0.001,\ t = 1$.

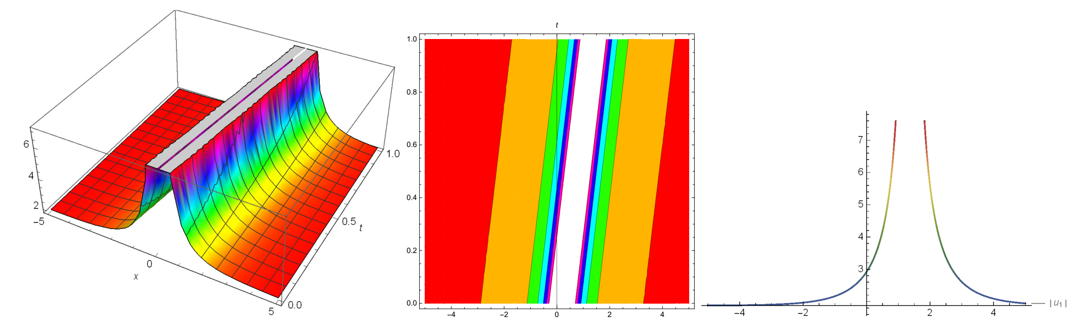

FIGURE 9 The 3D plot, contour plot and 2D plot for solution $u_{32}(x, y, z, t)$ of Eq. (49). Values chosen are $\alpha = 1,\ \mu = 0,\ \beta = -0.3,\ y = -1,\ \nu = -0.8,\ z = -0.2,\ K = 1,\ \theta = -1.11,\ t = 1$.

5 | CONCLUSIONS

In this study, a diverse number of exact wave solutions to the generalized (3+1)-dimensional Korteweg-de Vries-Zakharov Kuznetsov model have obtained, which is the most significant physical model in plasma physics, hydrodynamics. We used effective mathematical techniques called the improved F-expansion approach and the $\exp(-\phi(\zeta))$-expansion approach to develop the exact new solitary and traveling wave solutions for this nonlinear higher dimensional model. Soliton solutions, single bell shaped solitons, bright soliton, dark soliton, periodic type solitons, doubled bell shaped solitons, as well as various generalized three dimensional-Korteweg-de Vries-Zakharov Kuznetsov equation soliton solutions are obtained in different forms. Several of the results are written as hyperbolic, trigonometric functions with arbitrary constant parameters. Computerised symbolic calculation software Mathematica work supports the existing technique's effectiveness, reliability, and clarity. These two techniques can be used to solve a variety of nonlinear evolution equations in physics and mathematics.

Financial disclosure

The authors did not receive support from any organization for the submitted work.

Conflict of interest

The authors declare no potential conflict of interests.

References

1. Körpinar T, Cem Demirkol R, Körpinar Z, Asil V. Maxwellian evolution equations along the uniform optical fiber in Minkowski space. *Revista mexicana de física.* 2020;66(4):431–439.

2. Kumar Sachin, Almusawa Hassan, Hamid Ihsanullah, Abdou MA. Abundant closed-form solutions and solitonic structures to an integrable fifth-order generalized nonlinear evolution equation in plasma physics. *Results in Physics.* 2021;26:104453.

3. Abdelkawy MA, Bhrawy AH, Zerrad E, Biswas A. Application of tanh method to complex coupled nonlinear evolution equations. *Acta Phys. Pol. A.* 2016;129(3):278–283.

4. Hayward Rosie, Biancalana Fabio. Constructing new nonlinear evolution equations with supersymmetry. *Journal of Physics A: Mathematical and Theoretical.* 2018;51(27):275202.

5. Carillo Sandra, Ragnisco Orlando. *Nonlinear evolution equations and dynamical systems*. Springer Science & Business Media; 2012.

6. Sendi Cevat Teymuri, Manafian Jalil, Mobasseri Hasan, Mirzazadeh Mohammad, Zhou Qin, Bekir Ahmet. Application of the ITEM for solving three nonlinear evolution equations arising in fluid mechanics. *Nonlinear Dynamics.* 2019;95(1):669–684.

7. Shehata MS. Extended Jacobian elliptic function expansion method and its applications for solving some nonlinear evolution equations in mathematical physics. *International Journal of Computer Applications.* 2015;109(12):1–4.

8. Zahran Emad HM, Khater Mostafa MA, others . Exact traveling wave solutions for the system of shallow water wave equations and modified Liouville equation using extended Jacobian elliptic function expansion method. *American Journal of Computational Mathematics.* 2014;4(05):455.

9. Rizvi Syed Tahir Raza, Bibi Ishrat, Younis Muhammad, Bekir Ahmet. Interaction properties of solitons for a couple of nonlinear evolution equations. *Chinese Physics B.* 2021;30(1):010502.

10. Abdel-Gawad HI. Towards a unified method for exact solutions of evolution equations. An application to reaction diffusion equations with finite memory transport. *Journal of Statistical Physics.* 2012;147(3):506–518.

11. Majeed Abdul, Rafiq Muhammad Naveed, Kamran Mohsin, Abbas Muhammad, Inc Mustafa. Analytical solutions of the fifth-order time fractional nonlinear evolution equations by the unified method. *Modern Physics Letters B.* 2021;:2150546.

12. Rani Attia, Zulfiqar Aniqa, Ahmad Jamshad, Hassan Qazi Mahmod Ul. New soliton wave structures of fractional Gilson-Pickering equation using tanh-coth method and their applications. *Results in Physics.* 2021;29:104724.

13. Alquran Marwan. Physical properties for bidirectional wave solutions to a generalized fifth-order equation with third-order time-dispersion term. *Results in Physics.* 2021;28:104577.

14. Yang Xiao-Feng, Deng Zi-Chen, Wei Yi. A Riccati-Bernoulli sub-ODE method for nonlinear partial differential equations and its application. *Advances in Difference Equations.* 2015;2015(1):1–17.

15. Mirzazadeh Mohammad, Alqahtani Rubayyi T, Biswas Anjan. Optical soliton perturbation with quadratic-cubic nonlinearity by Riccati-Bernoulli sub-ODE method and Kudryashov's scheme. *Optik.* 2017;145:74–78.

16. Kumar Dipankar, Manafian Jalil, Hawlader Faisal, Ranjbaran Arash. New closed form soliton and other solutions of the Kundu–Eckhaus equation via the extended sinh-Gordon equation expansion method. *Optik.* 2018;160:159–167.

17. Kumar Dipankar, Joardar Atish Kumar, Hoque Ashabul, Paul Gour Chandra. Investigation of dynamics of nematicons in liquid crystals by extended sinh-Gordon equation expansion method. *Optical and Quantum Electronics.* 2019;51(7):1–36.

18. Akbar M Ali, Akinyemi Lanre, Yao Shao-Wen, et al. Soliton solutions to the Boussinesq equation through sine-Gordon method and Kudryashov method. *Results in Physics.* 2021;25:104228.

19. Javeed Shumaila, Riaz Sidra, Saleem Alimgeer Khurram, Atif Muhammad, Hanif Atif, Baleanu Dumitru. First integral technique for finding exact solutions of higher dimensional mathematical physics models. *Symmetry.* 2019;11(6):783.

20. Yépez-Martínez H, Gómez-Aguilar JF, Atangana Abdon. First integral method for non-linear differential equations with conformable derivative. *Mathematical Modelling of Natural Phenomena.* 2018;13(1):14.

21. Mirzazadeh M, Biswas Anjan. Optical solitons with spatio-temporal dispersion by first integral approach and functional variable method. *Optik.* 2014;125(19):5467–5475.

22. Bilige Sudao, Chaolu Temuer, Wang Xiaomin. Application of the extended simplest equation method to the coupled Schrödinger–Boussinesq equation. *Applied Mathematics and Computation.* 2013;224:517–523.

23. Zayed Elsayed ME, Shohib Reham MA. Optical solitons and other solutions to Biswas–Arshed equation using the extended simplest equation method. *Optik.* 2019;185:626–635.

24. Zayed Elsayed ME, Shohib Reham MA, Al-Nowehy Abdul-Ghani. Solitons and other solutions for higher-order NLS equation and quantum ZK equation using the extended simplest equation method. *Computers & Mathematics with Applications.* 2018;76(9):2286–2303.

25. Khater Mostafa MA, Anwar Sadia, Tariq Kalim U, Mohamed Mohamed S. Some optical soliton solutions to the perturbed nonlinear Schrödinger equation by modified Khater method. *AIP Advances.* 2021;11(2):025130.

26. Khater Mostafa, Inc Mustafa, Attia Raghda AM, Lu Dianchen. Computational Simulations; Abundant Optical Wave Solutions Atangana Conformable Fractional Nonlinear Schrödinger Equation. *Advances in Mathematical Physics.* 2022;2022.

27. Akinyemi Lanre, Hosseini Kamyar, Salahshour Soheil. The bright and singular solitons of (2+ 1)-dimensional nonlinear Schrödinger equation with spatio-temporal dispersions. *Optik.* 2021;242:167120.

28. Rezazadeh Hadi, Ullah Najib, Akinyemi Lanre, et al. Optical soliton solutions of the generalized non-autonomous nonlinear Schrödinger equations by the new Kudryashov's method. *Results in Physics.* 2021;24:104179.

29. Kudryashov Nikolay A. Implicit solitary waves for one of the generalized nonlinear Schrödinger equations. *Mathematics.* 2021;9(23):3024.

30. Seadawy Aly R, Arshad Muhammad, Lu Dianchen. The weakly nonlinear wave propagation of the generalized third-order nonlinear Schrödinger equation and its applications. *Waves in Random and Complex Media.* 2020;:1–13.

31. Arshad Muhammad, Seadawy Aly R, Lu Dianchen. Bright–dark solitary wave solutions of generalized higher-order nonlinear Schrödinger equation and its applications in optics. *Journal of Electromagnetic Waves and Applications.* 2017;31(16):1711–1721.

32. Seadawy Aly R, Iqbal Mujahid, Lu Dianchen. Construction of soliton solutions of the modify unstable nonlinear Schrödinger dynamical equation in fiber optics. *Indian Journal of Physics.* 2020;94(6):823–832.

33. Akinyemi Lanre, Inc Mustafa, Khater Mostafa, Rezazadeh Hadi. Dynamical behaviour of Chiral nonlinear Schrödinger equation. *Optical and Quantum Electronics.* 2022;54(3):1–15.

34. Lu Dianchen, Seadawy Aly R, Khater Mostafa MA. Structures of exact and solitary optical solutions for the higher-order nonlinear Schrödinger equation and its applications in mono-mode optical fibers. *Modern Physics Letters B.* 2019;33(23):1950279.

35. Nasreen Naila, Seadawy Aly R, Lu Dianchen, Albarakati Wafaa A. Dispersive solitary wave and soliton solutions of the gernalized third order nonlinear Schrödinger dynamical equation by modified analytical method. *Results in Physics.* 2019;15:102641.

36. Rasheed Noha M, Al-Amr Mohammed O, Az-Zo'bi Emad A, Tashtoush Mohammad A, Akinyemi Lanre. Stable optical solitons for the Higher-order Non-Kerr NLSE via the modified simple equation method. *Mathematics.* 2021;9(16):1986.

37. Younis Muhammad, Zafar Asim. The modified simple equation method for solving nonlinear Phi-Four equation. *International Journal of Innovation and Applied Studies.* 2013;2(4):661–664.

38. Liang Yuan. Exact solutions of the (3+ 1)-dimensional modified KdV-Zakharov-Kuznetsev equation and Fisher equations using the modified simple equation method. *Journal of Interdisciplinary Mathematics.* 2014;17(5-6):565–578.

39. Sahoo S, Garai G, Ray S Saha. Lie symmetry analysis for similarity reduction and exact solutions of modified KdV–Zakharov–Kuznetsov equation. *Nonlinear Dynamics.* 2017;87(3):1995–2000.

40. Zhang Tian-Tian. On Lie symmetry analysis, conservation laws and solitary waves to a longitudinal wave motion equation. *Applied Mathematics Letters.* 2019;98:199–205.

41. Zahran Emad HM, Khater Mostafa MA. Modified extended tanh-function method and its applications to the Bogoyavlenskii equation. *Applied Mathematical Modelling.* 2016;40(3):1769–1775.

42. Pandir Yusuf, Yildirim Ayse. Analytical approach for the fractional differential equations by using the extended tanh method. *Waves in Random and Complex Media.* 2018;28(3):399–410.

43. Song Chenwei, Liu Yinping. Extended homogeneous balance conditions in the sub-equation method. *Journal of Applied Analysis.* 2021;.

44. Islam Naima, Khan Kamruzzaman, Islam Md Hamidul. Travelling wave solution of Dodd-Bullough-Mikhailov equation: a comparative study between Generalized Kudryashov and improved F-expansion methods. *Journal of Physics Communications.* 2019;3(5):055004.

45. Karaman Bahar. The use of improved-F expansion method for the time-fractional Benjamin–Ono equation. *Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas.* 2021;115(3):1–7.

46. Khan Kamruzzaman, Akbar M Ali. The exp (- Φ (ξ))–expansion method for finding travelling wave solutions of Vakhnenko–Parkes equation. *International Journal of Dynamical Systems and Differential Equations.* 2014;5(1):72–83.

47. Khalique Chaudry Masood, Adeyemo Oke Davies. A study of (3+ 1)-dimensional generalized Korteweg-de Vries-Zakharov-Kuznetsov equation via Lie symmetry approach. *Results in Physics.* 2020;18:103197.

48. Rehman HU, Seadawy Aly R, Younis M, et al. Weakly nonlinear electron-acoustic waves in the fluid ions propagated via a (3+ 1)-dimensional generalized Korteweg–de-Vries–Zakharov–Kuznetsov equation in plasma physics. *Results in Physics.* 2022;33:105069.

49. Ray S Saha. New analytical exact solutions of time fractional KdV–KZK equation by Kudryashov methods. *Chinese Physics B.* 2016;25(4):040204.

50. Zayed Elsayed ME, SA Hoda Ibrahim. Modified simple equation method and its applications for some nonlinear evolution equations in mathematical physics. *International Journal of Computer Applications.* 2013;67(6).

51. Seadawy Aly R, Cheemaa Nadia. Applications of extended modified auxiliary equation mapping method for high-order dispersive extended nonlinear Schrödinger equation in nonlinear optics. *Modern Physics Letters B.* 2019;33(18):1950203.

52. Yildirim Yakup. Optical solitons of Biswas-Arshed equation by modified simple equation technique. *Optik.* 2019;182:986–994.

53. Osman MS, Almusawa Hassan, Tariq Kalim U, et al. On global behavior for complex soliton solutions of the perturbed nonlinear Schrödinger equation in nonlinear optical fibers. *Journal of Ocean Engineering and Science.* 2021;.

54. Savaissou Nestor, Gambo B, Rezazadeh Hadi, Bekir Ahmet, Doka Serge Y. Exact optical solitons to the perturbed nonlinear Schrödinger equation with dual-power law of nonlinearity. *Optical and Quantum Electronics.* 2020;52(6):1–16.

AUTHOR BIOGRAPHY

Kalim U. Tariq. The author is serving as Associate Professor at Mirpur University of Science & Technology (MUST), Mirpur-10250 (AJK), Pakistan. In addition to my extraordinary academic excellence, He have more than 15 years of university teaching exposure to graduate and post-graduate levels and published 40 articles in reputed international journals. In 2015, the Higher Education Commission of Pakistan has honored him with a full foreign scholarship for PhD studies. He did his PhD in Applied Mathematics from Huazhong University of Science & Technology, P. R. China in 2018 and privileged with two international awards.

Rizwan Javed. The author is M.Phil. research scholar at Mirpur University of Science & Technology (MUST), Mirpur-10250 (AJK), Pakistan.