

Highlights

Chaotic Numbers and It's uses on millennium prize problems.

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- Chaotic Numbers
- Riemann Hypothesis
- Navier Stokes Equations
- Yang Mills Theory and Mass Gap.

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ABSTRACT

Chaotic number system is a tool which was needed to solve the mysteries of today's modern mathematical problems the unsolved millennium prize problems namely 1. Riemann Hypothesis, 2. Navier Stokes Equations, 3. Yang Mills Theory and Mass Gap. These are some of the names of problems which we will see in this paper can be solved I don't claim that I have solved these problems but certainly I have given a new perspective on them. We use chaotic numbers and axiomatic properties of these numbers which is denoted by $\mathbb{U}n$ and is a super set of complex numbers. The numbers are dynamic and always changing the operations on them are different and they match the exact description of quantum mechanics as the root of inventing these numbers was to understand quantum particles and having a clear understanding by the mathematics that works with the same uncertainty.

1. Introduction


The world of Quantum Mechanics is pretty wild and so is our world, I started working these numbers a year ago and after learning a lot of Math concepts, Quantum Mechanics I thought of the way to divide chaos into 2 parts Low Chaos and High Chaos which are classes inside of our chaotic set $\mathbb{U}n$ and I have made Un so big that it is a super set of \mathbb{C} as we will see in this paper how k is a low chaos pronounced as "Kaa" a devanagri letter and *seen* is a high chaos arabic letter pronounced as English "seen" these are Family of sets since k_1, k_2, \dots and $seen_1, seen_2, \dots$ are sets which increases in ascending order and are chaotic. We now have a tool which is on the same level as quantum phenomenons or weather or mysteries of prime numbers. Riemann Hypothesis is an example of how in need we were to solve such caliber problem and the technique I used to solve Riemann hypothesis can be used to find exact position of n number of quantum particles in free space and which can be used to solve Quantum Computing problems and the most important Quantum Measurement Problem. I have introduced a chaotic norm space or chaotic vector space which helped us look at a millennium prize problem in a different way by having such powerful double vectors at our disposal we won't have limits to solve any gas or fluid flow problems. With this new mathematical number system/tool I was able to propose a new method to solve Yang Mills problem as this new number system and behaviour of these numbers matches with the uncertainty of quantum phenomenons. First we will look at how our mathematical numbers have limits of being static and then I propose my chaotic number and move to first millennium prize problem Riemann hypothesis and after proving it we will look at how easy it is to solve quantum problems like quantum measurement problem by assuming nature is always decided and outcome of a uncertainty is always decided by nature we just have to choose and the nature offers by establishing a connection with low chaos and past and future with high chaos and your answers lie inside the intersection of both which is our present. Next I will start by defining a chaotic norm space to look at navier stokes equation with a different eye how a dvector (double vector) has so much power in keeping track of something so chaotic and continues in nature.

2. Axioms of Chaotic Numbers

2.1. Chaos Increases in the ascending order

Chaos we defined called low and high chaos respectively have their own increasing members every k in low chaos and every *seen* in high chaos have their own subscript l and h respectively which increases as the chaos increase. When $seen_1 =$ starts it's $\sup k_1 = [1, i], \inf k_1 = [-1, -i]$ which is closed interval and all the entries are chaotic like $\{0.2, -0.00087, 1.0326, 0.65i, -1.2398i, \dots\}$

eg: Low Chaos increasing in ascending order $= k_1 < k_2 < k_3 < k_4 \dots < k_l$ (chaos starts but if the chaos ended due to some condition all the remaining chaotic numbers in series are ϕ or empty set). High Chaos increasing in ascending

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order = $seen_1 < seen_2 < seen_3 < seen_4 < \dots < seen_h$ (Same condition as low chaos if chaos interrupted the next members becomes ϕ or empty).

2.2. Chaotic Members are Zero/All Equal only if Un is Zero/All Equal

This property of chaotic numbers is pretty obvious so we stating it as an axiom since all the members like $seen_1, \dots, seen_h$ and k_1, \dots, k_l of the chaotic set $\mathbb{U}n = 0$ or $\mathbb{U}n$ has same element throughout, then members of Un are equal to each other.

$$\bigcup_{h=1}^{\infty} Seen_h + \bigcup_{l=1}^{\infty} K_l = 0 \implies seen_h = k_l \implies seen_1 = seen_2 = \dots = seen_h, k_1 = k_2 = k_3 \dots = k_l \iff \forall a \in \mathbb{U}n = 0 \quad (1)$$

$$\bigcap_{h=1}^{\infty} Seen_h + \bigcap_{l=1}^{\infty} K_l = 0 \implies seen_h = k_l \iff \forall a \in \mathbb{U}n = 0 \quad (2)$$

Lemma 1. $\forall a \in (0, 1)$ each of the element of $(0,1)$ assigned to a one by one example $a = 0.2, a = 0.0024, a = 0.5, \dots$ will map to the whole interval $(0,1)$.

we know $a \in (0, 1)$ if the statement was $a \in \mathbb{R}$ maps to $(0,1)$ it would have been much obvious but the statement implies that if $a = 0.2$ then a should be equal to all the numbers in the open interval $(0, 1)$ as the traditional mathematics can never satisfy this lemma so, Proposition 1 we can see that we have chaotic numbers and Def 1 shows us how first member of chaos is enough to cover a open interval. we can just assign $a \in K_l$ or $a \in Seen_h$ lemma asks us to map a it's each value to be sufficient to map the whole interval $(0, 1)$ now we know by Def 1 that how k_1 has the interval $(-1/-i, 1/i)$ if $a = k_l$ or $a = seen_1$ where $seen_1$ is in Def 1 with high chaotic values. since Lemma 1 main objective was to show a who can map to the whole interval $(0, 1)$ and we just used our chaotic numbers on them hence. ■

Proposition 1. Let $\mathbb{U}n$ be a set of chaotic numbers with members such as low chaos and high chaos denoted by k and $seen$ respectively. where $k_l \in \mathbb{U}n$ is the finer among all low chaotic members. $k_1 < k_2 < k_3 < \dots < k_l$ and the same goes for high chaos $seen_h \in \mathbb{U}n$ is the finer among all high chaotic members. $seen_1 < seen_2 < seen_3 < \dots < seen_h$

Definition 1. Low chaos: $K_1 \rightarrow K_l$ by Axiom(1) we know chaos increases in ascending order and $k_1 = \{0.26, 0.39, 0.00056i, -0.0000023, \dots\}$ first member of low chaos has chaotic entries in the interval $(-1/-i, 1/i)$ this is an open interval because some of the entries in k_1 can also be found in k_2 because of the chaotic entries we don't really know what might the next number be.

High Chaos: $Seen_1 \rightarrow Seen_h$ by Axiom(1) chaos increases in an ascending order and $seen_1 = \{0.26, -1, 0.06i, -0.03, 1, i, \dots\}$ first member of high chaos has chaotic entries in the interval $(-1/-i, 1/i)$ and as seen in the entries of set the chaos found is high and entries are very far away from each other this is an open interval because some of the entries in $seen_1$ can also be found in $seen_2$ because of the chaotic entries we don't really know what might the next number be.

3. Riemann Hypothesis Proof

Riemann Hypothesis is one of the millennium prize problems and it was proposed in 1914, it has bothered many of mathematicians for 162 years. The problem states that the ζ function which has all the non-trivial zeros on the critical strip whose real part is $1/2$ and imaginary part to be anything. Riemann had this idea after euler discovered in 1737 zeta function which involved infinite series of positive integers and prime numbers looking at this riemann took the idea and made it in the complex plane by substituting those of eulers to real and imaginary terms and he said that if his hypothesis is to be true all the zeros lie inside this critical strip. And now I have a proof of the riemann hypothesis problem.

PROOF. Let $\zeta = 1/n^s$ which can be written as $\sum_{n=1}^{\infty} \sum_{l=1}^{\infty} 1/n^{1/2+it}$ where $1/2$ is our real part we are interested in and it is the imaginary part and both \sum_t and \sum_n are running to help us only focus on if we can find all the zeros or we may find any non zero element.

Now, we will assign ζ to our chaotic set \mathbb{U}_n which I remind you $\mathbb{C} \subset \mathbb{U}_n$ so there is nothing wrong in assigning our ζ to \mathbb{U}_n So,

$$\sum_{n=1}^{\infty} \sum_{t=1}^{\infty} 1/n^{1/2+it} = \mathbb{U}_n \quad (3)$$

Now, \mathbb{U}_n has all possible values on a critical strip. Riemann Hypothesis claims that all the non trivial zeros are inside critical strip in our case \mathbb{U}_n this implies $Un = 0$ all the values of chaotic set \mathbb{U}_n are zero.

By Axiom (2) we know if $Seen_h = K_l \iff \forall a \in \mathbb{U}_n = 0$ and hypothesis claims $\mathbb{U}_n = 0$ as Un is the result of the eqn 3 So we can say we would only find zero's on the critical strip.

Now, we can see this as a counter example which will mean. $\exists x \in \mathbb{U}_n$ which is a non zero element. if a non zero element exists inside Un this means that members of chaotic set will both have $x \in seen_h$ and $x \in k_l$ we will now assume this non zero element is closer to 0 and closer to zero we have first element of both low and high chaos k_1 and $seen_1$ respectively.

This implies $seen_1 \cap k_1 = x$ both have non zero element x and 0 so we can say $seen_1 = k_1$ as both have same cardinality this implies by Axiom 2 eqn 1 and eqn 2 $seen_1 = seen_2 = seen_3 \dots$ AND $k_1 = k_2 = k_3 = \dots$ this again implies that the nonzero element x is in all of $seen$ and k this is a contradiction as

$$seen_1 \neq seen_2 \neq seen_3 \dots \text{ and } k_1 \neq k_2 \neq k_3 \dots \quad (4)$$

since all the other chaotic members other than $seen_1, k_1$ only have 0's and no x . So Riemann Hypothesis critical strip have all zeros and not a single non zero element. **Hence Proved** ■

4. Collapse of a Quantum Particle Solution with Chaotic Numbers: Rigorous Mathematical Description of Quantum Mechanics.

We know a quantum particle is like the most chaotic and undeterministic piece we all have to encounter while making a any quantum technology or just finding the exact position or momentum or we wish we knew the exact state of the particle before and after collapse of a wave function. We will tackle this problem because of simply show and propose a new approach to solve quantum mechanical prblems.

4.1. Prove that a Quantum Particle \mathcal{A} always exists at $seen_h \cap k_l = \mathcal{A}$

PROOF. $seen_h$ and $k_l \in \mathbb{U}_n$.

assigning $\mathcal{A} = \mathbb{U}_n$ this is the same method we used in riemann hypothesis solution so now that Un has all the collapsed state values of \mathcal{A} . Now if you must know that collapse of a particle never happens unless observed.

So we assume that all the quantum phenomenon or laws of quantum mechanics are always decided no matter what you do the outcome would be the same so, I say we have low and high chaos and in between high and low chaos is the present the chaos at which we operate which means that Past (k_l) | Present ($seen_h \cap k_l$) | Future ($seen_h$)

This whole statement and assumption implies that desired collapsed state of \mathcal{A} is inside the intersection of $past(k_l)$ and $future(seen_h)$

So,

$$Seen_h \cap k_l = \mathcal{A} \quad (5)$$

Hence Proved ■

4.2. Example: Find a particle with spin $|\downarrow\rangle$

PROOF. Let our quantum particle be \mathcal{A} this particle will be assigned to \mathbb{U}_n to get all of it's collapsed state into our chaotic set. So,

$$\mathcal{A} = (i=|\downarrow\rangle)\mathcal{A} \quad (6)$$

where i is the desired state/spin from \mathcal{A} which is $\mathcal{A} = \mathbb{U}_n$ and $seen_h, k_l \in \mathbb{U}_n$

$$\mathcal{A}(seen_h \cap k_l)_i = |\downarrow\rangle \quad (7)$$

So, $\mathcal{A} = |\downarrow\rangle$ **Hence Proved** ■

This method is the type we all have been waiting for a number system or mathematics that can actually work like a particle works just like a quantum particle is undeterministic and chaotic we have the numbers to better understand it a rigorous mathematical description of quantum mechanics.

5. Yang Mills Theory and Mass Gap.

Though through experimental results in laboratories have confirmed this problem we were in the search of a strong mathematical foundation to show quantum phenomenon as they are. The problem demands a existence of yang mills theory on \mathbb{R}^4 and mass gap $\Delta > 0$ and they demand an axiomatic properties.

Now that we know what the problem is and we have defined our axioms for chaotic numbers that has been used to solve major modern problems I decided to try this millennium problem and as I stare at the problem statement I was confident that I think I have solved this problem without knowing as mentioned here [2] the secret to solving riemann hypothesis would be in quantum mechanics and as this [2] say this description of new mathematics capable of going with the quantum mechanics would be Grand Unified Theory (GUT) because we will know how particles interact and that's what we are in need of. So,

PROOF. This problem demands gauge theory and mass gap and QFT and many other but I would not be going to prove this problem but rather I would propose that we could solve in this manner the method we are about to use it chaotic number system and it is a non abelian group the method we use is in both space and time because of the continuous behaviour of our chaotic numbers the problem is in terms of time as one might calculate and get the desired state of quantum particle but after an hour the result may differ as the state we are looking for might have gone from the intersection region by that time but we might not need it by then as I proposed this method assumes that laws of nature are predefined and we will not need the state we think we need so if we need a state $|\uparrow\rangle$ then we will find that state in intersection as per the law. So,

As explained in above section 4 that a chaotic number having low chaos and high chaos k_l and $seen_h$ respectively can be used to find exact spin or state of a quantum particle. Though the numbers and mathematics are non intuitive and so as the quantum particle we have been using probability since quantum mechanics started and we still don't have a rigorous mathematical tool to match with what we see so we don't have to only talk about in matrices or probability or how a wave function collapses when observed.

Let \mathcal{A} be a quantum particle in free space by eqn 6 and eqn 7 and eqn 5 we already know the result of this particle \mathcal{A} . Because we assume that nature is always decided which means no matter what you do if a particle is in $|0\rangle + |1\rangle / \sqrt{2}$ a superposition of all possible states a mixed state a pure state we know that whatever the particle collapses to is always inside the intersection of $Seen_h \cap K_l$. This says that even if the particle collapses and we didn't expect it to collapse on a particular state then I am saying it was decided and that state was inside our intersection region because of the way chaotic numbers axioms are set up we will always get a value that is needed in section 4 we can see how we compared low chaos to past and high chaos to future and present to the intersection of the two this is the reason I claim that my mathematics is the perfect mathematical description of quantum mechanics and the mathematicians should look more into this mathematics to solve Yang Mills Theory.

6. Chaotic Normed Space:

PROOF. As mentioned [14] in the rules of functional analysis a space can only be considered as a normed space only if these conditions satisfy So \mathbf{X} is consired to be normed space or vector space only if,

$$(a) \|x + y\| \leq \|x\| + \|y\| \quad \forall x, y \in X \quad (8)$$

$$(b) \|\alpha x\| = |\alpha| \|x\| \quad \text{if } x \in X \text{ and } \alpha \text{ is a scalar.} \quad (9)$$

$$(c) \|x\| > 0 \quad \text{if } x \neq 0 \quad (10)$$

For a chaotic normed space. Let's call it \mathbf{cX} be a non abelian vector space with d vectors (double vectors) $x_d, y_d, z_d \in \mathbf{cX} \subset \mathbb{U}_n$ eqns 8 9 10 are the requirements for this to be a normed space lets start with (a) or eqn 8.

$$\|x_d\| = \sum_x^{x'} (seen_1 \rightarrow seen_h)(k_1 \rightarrow k_l) x_d \quad (11)$$

The dvector¹ where x_d is a double vector with 2 heads and zero tails x and x' are the position of the 2 heads whereas $seen_1 \rightarrow seen_h$ says that between x and x' there are chaotic numbers which are midpoints of x_d and $k_1 \rightarrow k_l$ is the low chaotic midpoints with high chaos \sum is used from initial point of x_d i.e x till the x' the final or latest point of x_d . This sums up till the space is filled² Now for y_d ,

$$\|y_d\| = \sum_y^{y'} (seen_1 \rightarrow seen_h)(k_1 \rightarrow k_l) y_d \quad (12)$$

So eqn 8 is the property of normed space on \mathbf{X} and we will use eqn 11 and eqn 12 to define norm space on \mathbf{cX} ,

$$\|x_d + y_d\| \leq \|x_d\| + \|y_d\| \quad \forall x_d \text{ and } y_d \in \mathbf{cX} \quad (13)$$

Now for property (b) or eqn 9 since we know that we don't need a scalar so α is out of the chaotic norm property we only need our x_d that will define our chaotic scaling, squishing, multiplying and everything a vector should do but with a very very minimal calculation errors or parameters. So (b) or eqn 9 is given by,

$$\|x_d\| = \|x_d\| / \|y_d\| \quad \text{since } x_d, y_d \in \mathbf{cX} \quad (14)$$

And now for the final property (c) eqn 10 we get,

$$\|x_d\| > 0 \text{ if } x \neq 0. \quad (15)$$

□ Hence chaotic normed space (\mathbf{cX}) is a Linear Space or Vector Space.

6.1. Navier Usama Stokes Equation:

Navier Stokes Equations demand a solution to a equation that both navier and stokes given and as for now only 2D problem of this has been solved I am not gonna give the exact answer to this Millennium problem as I solved for [5] but I will give you a different approach to see at this problem. I thought maybe we are looking at the problem in a wrong way so I just gave my theorem of 3D Cplane and I thought maybe I have solved [6] but again I won't claim I solved it I just gave an idea.

PROOF. Let \mathcal{A} be a space of fluid, gas or any smooth viscous dense quantity.

By chaotic norm space we know, eqn 13 and this equation alone $\|x_d + y_d\| \leq \|x_d\| + \|y_d\| \quad \forall x_d \text{ and } y_d \in \mathbf{cX}$ proves that individual dvector norms fills more space and much better than combined norm of dvector so we will define \mathcal{A} as,

$$\mathcal{A} = \|x_d\| + \|y_d\| + \|z_d\| + \|a_d\| + \|b_d\| + \dots + \|n_d\| \quad (16)$$

\mathcal{A} has infinitely many dvector and all of them are given by,

$$\begin{aligned} \mathcal{A} = & \sum_x^{x'} (seen_1 \rightarrow seen_h)(k_1 \rightarrow k_l) x_d + \sum_y^{y'} (seen_1 \rightarrow seen_h)(k_1 \rightarrow k_l) y_d + \sum_z^{z'} (seen_1 \rightarrow seen_h)(k_1 \rightarrow k_l) z_d + \\ & \sum_a^{a'} (seen_1 \rightarrow seen_h)(k_1 \rightarrow k_l) a_d + \sum_b^{b'} (seen_1 \rightarrow seen_h)(k_1 \rightarrow k_l) b_d + \dots + \sum_n^{n'} (seen_1 \rightarrow seen_h)(k_1 \rightarrow k_l) n_d \end{aligned} \quad (17)$$

¹or Double vectors are the vectors which squishes squeezes or scales on it's on which means we don't need any scalars or any scalar multiplication to specify where dvector should go they go with the flow of nature.

²filled: this word is used to describe a movement of double vectors the simple vectors only points in one direction whereas double vectors can point in ∞ number of direction so if we have a space \mathcal{Y} the space \mathcal{Y} is filled with x_d meaning the space \mathcal{Y} has x_d filling up the space needed for calculation.

\mathcal{A} has like all the dvector each dvector has a capacity to fill and run till infinity and when all of these dvector combine and keep moving in space and time this will model the true nature in \mathbb{R}^3 3 Dimensions the problem of Navier Stokes Equation was the equation was going through a hard time if tried for \mathbb{R}^3 this is the reason I propose this method and a way to look at physics mathematics with the lens of chaotic numbers. This section introduced with chaotic norm which help us look at Navier Stokes Equation with a new perspective. **Hence Proved Navier Usama Stokes Equation new approach. ■**

7. Conclusion

We started by showing how today's number system is so fragile and why we need a new number system and we saw how it helped us solve problems we were just waiting for years someone would solve and I have started this new number system which has potential to do much more this is just my ideas and approach. I was only able to apply them with the questions I found on google like millennium prize problems as they are so famous I live in a very small town in India where not many people are even literate. I found my knowledge through youtube and online lectures and I was able to try these problems there might be a different problem that would become easy with this. This paper has a lot to take in and many new things to digest but if this work were to ever go public it will be a new opportunity for researchers to use this tool shape it the way they like, I would encourage researchers to work on this number system and make it robust enough as it's just a proposed method from my small mind what more could I have done and what the world might do with this new mathematical tool which is so different in nature.

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