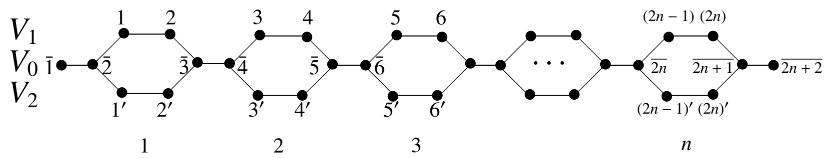

Fig. 1: The linear hexagonal chain H_n and its vertex labeling.

Let H_n be a linear hexagonal chain with n hexagons (see Fig.1) . In this paper, we give a decomposition theorem of Laplacian polynomial of weighted graphs and obtain that the Laplacian spectrum of H_n consists of the eigenvalues of a symmetric tridiagonal matrices of order $4n+2$ and the Laplacian eigenvalues of $2n$ K_2s. Together with the relationship between the roots and coefficients of the characteristic polynomials of the above matrices, explicit formula of the Kirchhoff index of H_n is derived. We also give the number of spanning trees of H_n, and show that the Kirchhoff index of H_n is approximately one half of its Wiener index.

The Kirchhoff index of linear hexagonal chains H_n as follows.

Theorem 0.1. *Let H_n be the linear hexagonal chain with n hexagons. Then the Kirchhoff index of H_n is*

$$Kf(H_n) = \frac{40}{3}n^3 + 23n^2 + \frac{22}{3}n + 11.$$

Meanwhile, we obtain the number of spanning trees of H_n, and also give the Winner index of H_n.

Theorem 0.2. *The number of spanning trees of H_n is $\tau(H_n) = 6^n$.*

Theorem 0.3. *The Winner index of H_n is*

$$W(H_n) = 24n^3 + 24n^2 + 13n + 1.$$

From Theorem 0.1 and Theorem 0.3 we can see that the ratio of the Kirchhoff index and Winner index of H_n is $5/9$ when $n \to \infty$.