Event-triggered synchronization and $\mathcal{H}_\infty$ synchronization of coupled delayed reaction-diffusion memristive neural networks

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Abstract

This paper settles event-triggered synchronization and $\mathcal{H}_\infty$ synchronization matters for two types of coupled delayed reaction-diffusion memristive neural networks (CDRDMNNs). First of all, several synchronization and $\mathcal{H}_\infty$ synchronization conditions are acquired for CDRDMNNs with state coupling in virtue of exploiting Lyapunov stability theory in combination with proper controllers of the triggering event. Then, for CDRDMNNs with spatial diffusion coupling, event-triggered synchronization and $\mathcal{H}_\infty$ synchronization are investigated as well. Finally, the correctness of the deduced synchronization and $\mathcal{H}_\infty$ synchronization results is verified by two given numerical examples.

Keywords: Synchronization, Event-triggered control, Reaction-diffusion terms, $\mathcal{H}_\infty$ synchronization, Spatial diffusion coupling.

1. Introduction

Recently, complex networks (CNs) have absorbed extensive attention from investigators because they are ubiquitous under the circumstance of our daily, for instance, public traffic networks, communication networks, social networks and multi-agent systems [1, 2]. As a special type of CNs, coupled neural networks (CNNs) have been applied triumphantly in various domains, e.g., image encryption, pattern recognition and secure communication [3]-[5]. Actually, these extensive applications in a certain extent depend on CNNs' behaviors of dynamics. It is understood that synchronization is an interesting and significant dynamical behavior, and thus has acquired more and more attention. At present, many crucial synchronization results on this topic have been derived recently [6]-[11]. [6] solved the synchronization problem of delayed CNNs and established some conditions to make sure of the synchronization of the considered network. [9] firstly introduced normalized left eigenvector approach, thereby synchronization stability criteria were established for coupled ODEs.

It is well known that disturbance or noise are unavoidable and maybe lead the networks to unanticipated states or destroy the synchronization in the real systems. Consequently, how to design disturbance attenuating synchronization controllers and reduce the influence of external disturbance has increasingly become a popular academic topic. In the past few years, $\mathcal{H}_\infty$ synchronization has proven to be an efficient strategy in order to reduce the impact of noise or disturbance, and relevant $\mathcal{H}_\infty$ synchronization results have been established [12]-[15]. [12] dealt with the $\mathcal{H}_\infty$ synchronization problem of chaotic neural networks, and $\mathcal{H}_\infty$ synchronization criteria were presented. In [13], some conditions for multi-weighted CNs with fixed and switching topologies were acquired to ensure $\mathcal{H}_\infty$ synchronization.

Note that the vast majority of synchronization result in above-mentioned works [6]-[15] have overlooked this diffusive effect in the spatial domain. To the best of our knowledge, the diffusive effect should not be ignored in cellular neural networks [16, 17], especially when electrons in a nonuniform electromagnetic domain are highly running. Consequently, it is very important and worthwhile to study reaction-diffusion neural networks (RDNNs).

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So far, numerous significant synchronization results of coupled RDNNs (CRDNNs) have been reported [18]-[21]. Wang et al. [18] not only concerned the synchronization issue of CRDNNs but also proposed some synchronization criteria for the raised network. By means of designing applicable pinning controllers, several adequate conditions were acquired for ensuring CRDNNs realize synchronization in [21]. Furthermore, a number of scholars have discussed the $\mathcal{H}_\infty$ synchronization problem for RDNNs [22]-[25]. Liu et al. [22, 23] addressed $\mathcal{H}_\infty$ synchronization of RDNNs, and $\mathcal{H}_\infty$ synchronization criteria were presented. However, the study of $\mathcal{H}_\infty$ synchronization for CRDNNs was conducted by only few authors. In [24], adequate conditions were acquired for making sure that CRDNNs achieve lag $\mathcal{H}_\infty$ synchronization with the aid of Lyapunov functional strategy.

As we all know, Chua first put forward the notion of memristor in 1971. Compared with the traditional resistor, the past dynamic history of memristor can be remembered since the number of this memristor relies on the quantity of charge running through here. Thus, there are wide applications referred to the memristor in many domains involving image processing, signal processing and optimization [26]-[28]. Moreover, by means of replacing resistor by the memristor, memristive neural networks (MNNs) can show the neural transmission in the brain primely [29]. So far, numerous significant and valuable research results for MNNs have been reported [30]-[33]. By exploiting control theory and nonsmooth analysis, several Lagrange stability criteria reliant on the parameters of MNNs were derived in [31]. In [33], some adequate conditions were acquired for ensuring MNNs exponential stabilization. However, only a small number of works discussed the dynamical behaviors of coupled memristive neural networks (CMNNs) [34]-[37]. In [34], some new sufficient conditions were established to guarantee the periodicity and synchronization of CMNNs with supremums. Wang and Shen [35] addressed exponential synchronization of CMNNs, and several exponential synchronization criteria were derived. There is no doubt that the synchronization and $\mathcal{H}_\infty$ synchronization results for CRDMNNs have been reported so far.

To our the best knowledge, numerous CNs under the natural state cannot be synchronized ($\mathcal{H}_\infty$ synchronized) through themselves in actual circumstances. Hence, the strategy of designing appropriate controllers is very necessary and meaningful to ensure the CNs achieve synchronization and $\mathcal{H}_\infty$ synchronization. Up to now, the control method based event-triggered mechanism has become increasingly prevalent. More specifically, event-triggered control is the distributed triggering mechanism which has received extensive attention since it can overcome a number of consecutive control's defects in system theory, and avoid some needless communication when data information is exchanging. Therefore, many significant and interesting synchronization results for CNs with event-triggered control have been acquired recently [38]-[42]. In [39], several conditions were established for ensuring the CNs reach synchronization based on event-triggered mechanism. By means of exploiting event-triggered control strategy, [40] addressed the problem of synchronization for CNs, and presented some event-triggered synchronization criteria. By now, only a few event-triggered synchronization results for CNNs have been derived [43, 44]. Through employing event-triggered control and Lyapunov functional method, several passivity and synchronization criteria were acquired for partially CNNs in [43]. Nevertheless, the issues of event-triggered $\mathcal{H}_\infty$ synchronization and synchronization for CDRDMNNs have not been yet addressed.

Motivated by the aforementioned statements, we respectively study synchronization and $\mathcal{H}_\infty$ synchronization of CDRDMNNs with state coupling and spatial diffusion coupling via event-triggered control in this paper. The four main contributions of the presented work are as follows:

1. First, two types of CDRDMNNs models are proposed that the first type is CDRDMNNs with state coupling and the second one is CDRDMNNs with spatial diffusion coupling in virtue of an event-triggered approach. Compared with [34]-[37], the considered model includes reaction-diffusion terms in this paper, named as CDRDMNNs, which is more general and complicated in many real networks.
2. By making use of Lyapunov functional and the appropriate event-driven strategy, some conditions are derived for guaranteeing CDRDMNNs with state coupling realize synchronization. To our best knowledge, the majority of works in existence investigated the synchronization issue of CNNs and CRDNNs by some conventional methods [6]-[11], [18]-[21]. Nevertheless, the acquired synchronization results in our presented works are focused on a discrete control strategy, i.e., event-triggered control is capable of taking great advantages on discontinuous control approach to the full and decrease some superfluous communication when information is transmitted.
3. synchronization criteria for the discussed CDRDMNNs are proposed. As we know, the external disturbance and environmental noise are unavoidable and they are likely to lead the networks to unexpected states, and even break the synchronization. In comparison to existing $\mathcal{H}_\infty$ synchronization results [12]-[15], [22]-[25], the event-based

mechanism is adopted to study $\mathcal{H}_\infty$ synchronization of CDRDMNNs, and the event-triggered $\mathcal{H}_\infty$ synchronization problem for the proposed network is firstly solved in our paper.

4. The event-triggered synchronization and $\mathcal{H}_\infty$ synchronization for CDRDMNNs with spatial diffusion coupling are discussed. Compared with synchronization results on CNs with triggering event [38]-[44], we study the event-triggered synchronization and $\mathcal{H}_\infty$ synchronization for CDRDMNNs with spatial diffusion coupling in this paper, and these derived $\mathcal{H}_\infty$ synchronization and synchronization results are less conservative and more general in our presented work.

The framework of this remaining paper is displayed as follows. In Section 2, several necessary notations and lemmas are introduced. In Section 3, we firstly propose the model of CDRDMNNs with state coupling based on event-triggered mechanism. Then, for CDRDMNNs with state coupling, event-triggered synchronization and $\mathcal{H}_\infty$ synchronization criteria are derived respectively. Furthermore, we develop event-triggered synchronization and $\mathcal{H}_\infty$ synchronization of the network with spatial diffusion coupling in Section 4. Section 5 provides two examples to show the correctness of these acquired results. Current works are summarized in Section 6.

2. Preliminaries

Define a weighted connected digraph $\mathcal{F} = (\varsigma, \nu, H)$ with a set of nodes $\varsigma = \{1, 2, \cdots, N\}$ and a set of edges $\nu \subseteq \varsigma \times \varsigma$. $(q, p) \in \nu$ represents a directed edge in digraph $\mathcal{F}$. The set of neighbors of node q is denoted by $\mathcal{N}_q = \{p \in \varsigma | (q, p) \in \nu\}$, where $p \neq q$. The weighted adjacency matrix is defined as $H = (H_{qp})_{N\times N}$ with nonnegative elements $H_{qq} = 0$ and $H_{qp} > 0$ when $p \in \mathcal{N}_q$. In addition, $\mathcal{W} > 0$ ($\mathcal{W} \geqslant 0, \mathcal{W} < 0, \mathcal{W} \leqslant 0$) denotes the symmetric matrix $\mathcal{W}$ is positive (semi-positive, negative, semi-negative) definite. $\otimes$ is the Kronecker product. The smallest (largest) eigenvalue of the corresponding matrix is represented by $\lambda_m(\cdot)(\lambda_M(\cdot))$. $\Omega = \{m = (m_1, m_2, \cdots, m_\eta)^T \| |m_\beta| < \varphi_\beta, \beta = 1, 2, \cdots, \eta\}$ is an open bounded domain in $\mathbb{R}^\eta$ with smooth boundary $\partial\Omega$. For $e(m, t) = (e_1(m, t), e_2(m, t), \cdots, e_n(m, t))^T \in \mathbb{R}^n$,

$$\|e(\cdot, t)\| = \Big(\int_\Omega \sum_{i=1}^{n} e_i^2(m, t)dm\Big)^{\frac{1}{2}}.$$

Lemma 2.1. (See [45]) Define $\Omega = \{m = (m_1, m_2, \cdots, m_\eta)^T \| |m_\beta| < \varphi_\beta, \beta = 1, 2, \cdots, \eta\}$. Then,

$$\int_\Omega w^2(m)dm \leqslant \varphi_\beta^2 \int_\Omega \Big(\frac{\partial w(m)}{\partial m_\beta}\Big)^2 dm,$$

where $w(m) \in C^1(\Omega)$ is a real-valued function and $w(m)|_{\partial\Omega} = 0$.

Lemma 2.2. (See [46]) For $\forall \alpha_1, \alpha_2 \in \mathbb{R}^n$ and $U \in \mathbb{R}^{n\times n} > 0$, we can get the inequality:

$$2\alpha_1^T \alpha_2 \leqslant \alpha_1^T U \alpha_1 + \alpha_2^T U^{-1} \alpha_2.$$

3. Event-triggered synchronization and $\mathcal{H}_\infty$ synchronization of CDRDMNNs with state coupling

3.1. Event-triggered synchronization

In this section, we consider the following CDRDMNNs with state coupling which is given by:

$$\begin{cases} \frac{\partial z_q(m,t)}{\partial t} &= \sum_{\beta=1}^{\eta} Y_\beta \frac{\partial^2 z_q(m,t)}{\partial m_\beta^2} - K z_q(m, t) + I + A(z_q(m, t)) f(z_q(m, t)) + \gamma_q(m, t) + D(z_q(m, t)) g(\overline{z_q(m, t)}) \\ & \quad + \xi \sum_{p=1}^{N} J_{qp} \Gamma z_p(m, t) + v_q(m, t), \\ z_q(m, t) &= 0, \qquad (m, t) \in \partial\Omega \times [-\tau, +\infty), \\ z_q(m, t) &= \vartheta_q(m, t), \qquad (m, t) \in \Omega \times [-\tau, 0], \end{cases} \tag{1}$$

where $q = 1, 2, \cdots, N$ and $z_q(m, t) = (z_{q1}(m, t), z_{q2}(m, t), \cdots, z_{ql}(m, t))^T \in \mathbb{R}^l$ denotes the state vector of qth neuron; $Y_\beta = \mathrm{diag}(y_{1\beta}, y_{2\beta}, \cdots, y_{l\beta}) \in \mathbb{R}^{l\times l} > 0$; $K = \mathrm{diag}(k_1, k_2, \cdots, k_l) \in \mathbb{R}^{l\times l} > 0$; the activation functions in neural network q are denoted by $f(z_q(m, t)) = (f_1(z_{q1}(m, t)), f_2(z_{q2}(m, t)), \cdots, f_l(z_{ql}(m, t)))^T \in \mathbb{R}^l$ and $g(\overline{z_q(m, t)}) = (g_1(z_{q1}(m, t -$

$\tau_1(t))), g_2(z_{q2}(m, t - \tau_2(t))), \cdots, g_l(z_{ql}(m, t - \tau_l(t))))^T \in \mathbb{R}^l$; $I = (I_1, I_2, \cdots, I_l)^T \in \mathbb{R}^l$ stands for the constant external input; the global coupling strength is $0 < \xi \in \mathbb{R}$; the inner coupling matrix is given by $\Gamma \in \mathbb{R}^{l\times l} > 0$; $v_q(m,t) \in \mathbb{R}^l$ and $\gamma_q(m,t) \in \mathbb{R}^l$ signify the controllers; $J = (J_{qp})_{N\times N}$ is the coupling weight satisfying $J_{qp} = J_{pq} > 0 (p \neq q)$ if there is a link from p-th node to q-th node, or else $J_{qp} = 0$, and $J_{qq} = -\sum_{\substack{p=1 \\ p\neq q}}^{N} J_{qp}$; In addition, the time varying delay $\tau_j(t)(j = 1, 2, \cdots, l)$ satisfies $0 \leqslant \tau_j(t) \leqslant \tau_j, \tau = \max_{j=1,2,\cdots,l}\{\tau_j\}$ and $\dot{\tau}_j(t) \leqslant \varpi_j < 1$; $\vartheta_q(m,t) \in C(\Omega \times [-\tau, 0])$ is continuous and bounded; $A(z_q(m,t)) = (a_{gh}(z_{qg}(m,t)))_{l\times l}$ and $D(z_q(m,t)) = (d_{gh}(z_{qg}(m,t)))_{l\times l}$ are the feedback connection weights, they are derived by memristors and can be defined as

$$a_{gh}(z_{qg}(m,t)) = \frac{\mathcal{X}_{gh}}{\mathcal{D}_g} \times \text{sign}_{gh}, \quad d_{gh}(z_{qg}(m,t)) = \frac{\mathcal{Y}_{gh}}{\mathcal{D}_g} \times \text{sign}_{gh},$$

where $g, h \in \{1, 2, \cdots, l\}$, $\text{sign}_{gh} = -1$ when $g = h$, otherwise $\text{sign}_{gh} = 1$. $\mathcal{X}_{gh}, \mathcal{Y}_{gh}$ represent the memductances of memristors $\mathcal{K}_{gh}, \mathcal{U}_{gh}$. $\mathcal{K}_{gh}$ and $\mathcal{U}_{gh}$ denote the memristors between $f_h(z_{qh}(m,t))$ and $z_{qg}(m,t)$ as well as $g_h(z_{qh}(m, t - \tau_h(t)))$ and $z_{qg}(m,t)$, respectively. By means of the voltage current characteristic of memristor, we can obtain

$$a_{gh}(z_{qg}(m,t)) = \begin{cases} \hat{a}_{gh}, & |z_{qg}(m,t)| \leqslant \chi_{qg}, \\ \check{a}_{gh}, & |z_{qg}(m,t)| > \chi_{qg}, \end{cases} \quad d_{gh}(z_{qg}(m,t)) = \begin{cases} \hat{d}_{gh}, & |z_{qg}(m,t)| \leqslant \chi_{qg}, \\ \check{d}_{gh}, & |z_{qg}(m,t)| > \chi_{qg}, \end{cases}$$

where the switching jumps $\chi_{qg} > 0$, and $g, h \in \{1, 2, \cdots, l\}$ and $\hat{a}_{gh}, \check{a}_{gh}, \hat{d}_{gh}, \check{d}_{gh}$ are constants.

For convenience, we define

$$\begin{aligned} \bar{A} &= \text{diag}(\sum_{h=1}^{l} \bar{a}_{1h}^2, \sum_{h=1}^{l} \bar{a}_{2h}^2, \cdots, \sum_{h=1}^{l} \bar{a}_{lh}^2), \bar{a}_{gh} = \max\{|\check{a}_{gh}|, |\hat{a}_{gh}|\}, \\ \bar{D} &= \text{diag}(\sum_{h=1}^{l} \bar{d}_{1h}^2, \sum_{h=1}^{l} \bar{d}_{2h}^2, \cdots, \sum_{h=1}^{l} \bar{d}_{lh}^2), \bar{d}_{gh} = \max\{|\check{d}_{gh}|, |\hat{d}_{gh}|\}, \\ \tilde{A} &= (\tilde{a}_{gh})_{l\times l}, \tilde{a}_{gh} = |\hat{a}_{gh} - \check{a}_{gh}|, \tilde{D} = (\tilde{d}_{gh})_{l\times l}, \tilde{d}_{gh} = |\hat{d}_{gh} - \check{d}_{gh}|. \end{aligned}$$

Suppose the functions $f_j(\cdot)$ and $g_j(\cdot)(j = 1, 2, \cdots, l)$ are globally Lipschitz continuous, i.e., there exist $0 < \psi_j \in \mathbb{R}, 0 < \phi_j \in \mathbb{R}, 0 < \breve{\psi}_j \in \mathbb{R}, 0 < \breve{\phi}_j \in \mathbb{R}$ such that

$$\begin{aligned} |f_j(\mu_1) - f_j(\mu_2)| &\leqslant \psi_j |\mu_1 - \mu_2|, \quad |f_j(\mu)| \leqslant \breve{\psi}_j, \\ |g_j(\mu_1) - g_j(\mu_2)| &\leqslant \phi_j |\mu_1 - \mu_2|, \quad |g_j(\mu)| \leqslant \breve{\phi}_j \end{aligned}$$

for all $\mu_1, \mu_2, \mu \in \mathbb{R}$.

Assume the equilibrium solution of (1) is $z^*(m) = (z_1^*(m), z_2^*(m), \cdots, z_l^*(m))^T \in \mathbb{R}^l$, and it dynamics

$$\sum_{\beta=1}^{\eta} Y_\beta \frac{\partial^2 z^*(m)}{\partial m_\beta^2} - K z^*(m) + A(z^*(m)) f(z^*(m)) + D(z^*(m)) g(z^*(m)) + I = 0.$$

In addition, the control inputs $v_q(m,t)$ and $\gamma_q(m,t)$ are designed for network (1) as follows:

$$\begin{cases} v_q(m,t) = \varepsilon \sum_{p\in\mathcal{N}_q} H_{qp}(z_q(m,t) - z_p(m,t)), \\ \gamma_q(m,t) = -\text{sign}(z_q(m,t) - z^*(m))(\tilde{A}\breve{\psi} + \tilde{D}\breve{\phi}), \end{cases} \tag{2}$$

where $\varepsilon > 0, \breve{\psi} = (\breve{\psi}_1, \breve{\psi}_2, \cdots, \breve{\psi}_l)^T \in \mathbb{R}^l, \breve{\phi} = (\breve{\phi}_1, \breve{\phi}_2, \cdots, \breve{\phi}_l)^T \in \mathbb{R}^l$ and $\text{sign}(z_q(m,t) - z^*(m)) = \text{diag}(\text{sign}(z_{q1}(m,t) - z_1^*(m)), \text{sign}(z_{q2}(m,t) - z_2^*(m)), \cdots, \text{sign}(z_{ql}(m,t) - z_l^*(m))) \in \mathbb{R}^{l\times l}$.

Let $\{t_k^q\}_{k=1}^{\infty}$ be a sequence of incremental event-triggered time, where $t_{k+1}^q > t_k^q$. According to the sample data and event-triggered strategy, $v_q(m,t)$ can be rewritten as follows:

$$v_q(m,t) = \varepsilon \sum_{p\in\mathcal{N}_q} H_{qp}(z_q(m, t_k^q) - z_p(m, t_k^p)), \tag{3}$$

in which $H=(H_{qp})_{N\times N}$ denotes the coupling matrix and meets $H_{qq}=0, H_{qp}>0$ when $p\in\mathcal{N}_q$, $t\in[t_k^q,t_{k+1}^q)$, the state of node q at t_k^q is represented by $z_q(m,t_k^q)$, where t_k^q corresponds to the event-triggered instant of node q. Similar as the result [15], for any $\varrho'\in\mathbb{R}>0$, Zeno behavior does not display as $t_{k+1}^q-t_k^q\geqslant\varrho'$.

Let the measure error of triggering event be $\pi_q(m,t)=z_q(m,t_k^q)-z_q(m,t)$. For $t\in[t_k^q,t_{k+1}^q)$ and $\theta\in\mathbb{R}>0$, the event-triggered condition is designed as follows:

$$t_{k+1}^q=\inf\Big\{t:t>t_k^q,\|\pi_q(m,t)\|>\theta\Big\|\sum_{p\in\mathcal{N}_q}H_{qp}(z_q(m,t_k^q)-z_p(m,t_k^p))\Big\|\Big\}. \tag{4}$$

For the error vector $e_q(m,t)=z_q(m,t)-z^*(m)=(e_{q1}(m,t),e_{q2}(m,t),\cdots,e_{ql}(m,t))^T$, we have

$$\begin{aligned}\frac{\partial e_q(m,t)}{\partial t}=&\sum_{\beta=1}^{\eta}Y_\beta\frac{\partial^2e_q(m,t)}{\partial m_\beta^2}+A(z_q(m,t))\hat{f}(e_q(m,t))+D(z_q(m,t))\hat{g}(\overline{e_q(m,t)})-Ke_q(m,t)+[A(z_q(m,t))\\&-A(z^*(m))]f(z^*(m))+[D(z_q(m,t))-D(z^*(m))]g(z^*(m))+\xi\sum_{p=1}^{N}J_{qp}\Gamma e_p(m,t)\\&-\mathrm{sign}(e_q(m,t))(\tilde{A}\breve{\psi}+\tilde{D}\breve{\phi})+\varepsilon\sum_{p\in\mathcal{N}_q}H_{qp}(e_q(m,t_{k_q(t)}^q)-e_p(m,t_{k_p(t)}^p)),\end{aligned}\tag{5}$$

where $q=1,2,\cdots,N$; $\hat{f}(e_q(m,t))=f(z_q(m,t))-f(z^*(m))$, $\hat{g}(\overline{e_q(m,t)})=g(\overline{z_q(m,t)})-g(z^*(m))$ and $\overline{e_q(m,t)}=(e_{q1}(m,t-\tau_1(t)),e_{q2}(m,t-\tau_2(t)),\cdots,e_{ql}(m,t-\tau_l(t)))^T$, $t\in[t_k^q,t_{k+1}^q)$. Each node q collects its adjacent state at time t with regard to the point of similar time $t_{k_q(t)}^q$, where $k_q(t)=\arg\max_{k^*}\{t_{k^*}^q\leqslant t\}$.

As $\pi_q(m,t)=z_q(m,t_{k_q(t)}^q)-z_q(m,t)$, we can derive from (5) that

$$\begin{aligned}\frac{\partial e_q(m,t)}{\partial t}=&\sum_{\beta=1}^{\eta}Y_\beta\frac{\partial^2e_q(m,t)}{\partial m_\beta^2}+A(z_q(m,t))\hat{f}(e_q(m,t))+D(z_q(m,t))\hat{g}(\overline{e_q(m,t)})-Ke_q(m,t)+[A(z_q(m,t))\\&-A(z^*(m))]f(z^*(m))+[D(z_q(m,t))-D(z^*(m))]g(z^*(m))+\xi\sum_{p=1}^{N}J_{qp}\Gamma e_p(m,t)\\&+\varepsilon\sum_{p\in\mathcal{N}_q}H_{qp}\Big(e_q(m,t)-e_p(m,t)+\pi_q(m,t)-\pi_p(m,t)\Big)-\mathrm{sign}(e_q(m,t))(\tilde{A}\breve{\psi}+\tilde{D}\breve{\phi}).\end{aligned}\tag{6}$$

By means of the event-triggered condition (4), we have

$$\begin{aligned}\|\pi_q(m,t)\|\leqslant&\theta\Big\|\sum_{p\in\mathcal{N}_q}H_{qp}(e_q(m,t_{k_q(t)}^q)-e_p(m,t_{k_p(t)}^p))\Big\|\\\leqslant&\theta\Big[\Big\|\sum_{p\in\mathcal{N}_q}H_{qp}(e_q(m,t)-e_p(m,t))\Big\|+\Big\|\sum_{p\in\mathcal{N}_q}H_{qp}(\pi_q(m,t)-\pi_p(m,t))\Big\|\Big]\\\leqslant&\theta\Big[\sum_{p\in\mathcal{N}_q}H_{qp}\Big\|e_q(m,t)\Big\|+\sum_{p\in\mathcal{N}_q}H_{qp}\Big\|e_p(m,t)\Big\|+\sum_{p\in\mathcal{N}_q}H_{qp}\Big\|\pi_q(m,t)\Big\|+\sum_{p\in\mathcal{N}_q}H_{qp}\Big\|\pi_p(m,t)\Big\|\Big]\\\leqslant&2\theta\mathbf{h}(\|e(m,t)\|+\|\pi(m,t)\|),\end{aligned}\tag{7}$$

where $\mathbf{h}=\max\{\sum_{p\in\mathcal{N}_q}H_{qp}\}$, $e(m,t)=(e_1^T(m,t),e_2^T(m,t),\cdots,e_N^T(m,t))^T$ and $\pi(m,t)=(\pi_1^T(m,t),\pi_2^T(m,t),\cdots,\pi_N^T(m,t))^T$. Then,

$$\|\pi(m,t)\|\leqslant2\theta N\mathbf{h}(\|e(m,t)\|+\|\pi(m,t)\|).$$

Finally,

$$\|\pi(m,t)\|\leqslant\frac{2\theta N\mathbf{h}}{1-2\theta N\mathbf{h}}\|e(m,t)\|, \tag{8}$$

where $0 < \theta < \frac{1}{2N\mathbf{h}}$.

Furthermore, we define

$$\Psi = \mathrm{diag}(\psi_1^2, \psi_2^2, \cdots, \psi_l^2), \Phi = \mathrm{diag}(\phi_1^2, \phi_2^2, \cdots, \phi_l^2), L = \mathrm{diag}(\frac{1}{1-\varpi_1}, \frac{1}{1-\varpi_2}, \cdots, \frac{1}{1-\varpi_l}).$$

Remark 1. As we know, Chua first put forward the concept of memristor. In fact, the memristor takes place the of resistor that can be used to simulate the human brain in neural networks since it has great performance [29]. Up to date, lots of significant and worthwhile synchronization results on MNNs have been obtained [30]-[33]. But just a few results on the dynamical behaviors of CMNNs have been reported [34]-[37]. Unfortunately, the synchronization and $\mathcal{H}_\infty$ synchronization of CDRDMNNs have not been studied. As far as we know, our paper is the first to address the $\mathcal{H}_\infty$ synchronization and synchronization of CDRDMNNs in this present work.

Theorem 3.1. *The network* (1) *reaches event-triggered synchronization under the controller* (2) *if there exist a matrix* $P = \mathrm{diag}(p_1, p_2, \cdots, p_l) \in \mathbb{R}^{l\times l} > 0$ *and a constant* $0 < \theta < \frac{1}{2N\mathbf{h}}$ *such that*

$$I_N \otimes \Theta_1 + \xi J \otimes (P\Gamma + \Gamma P) < 0, \tag{9}$$

where $\Theta_1 = -\sum_{\beta=1}^{\eta} \frac{2}{\varphi_\beta^2} PY_\beta - 2PK + P^2(\bar{A} + \bar{D}) + \Psi + \Phi L + 2\varepsilon P^2 + 4\varepsilon N\mathbf{h}^2(1 + (\frac{2\theta N\mathbf{h}}{1-2\theta N\mathbf{h}})^2)I_l$ *and* $\mathbf{h} = \max\{\sum_{p\in\mathcal{N}_q} H_{qp}\}$.

Proof. Select the following candidate functional for network (6):

$$V(t) = \sum_{q=1}^{N}\sum_{j=1}^{l} \frac{\phi_j^2}{1-\varpi_j}\int_{t-\tau_j(t)}^{t}\int_\Omega e_{qj}^2(m,s)dmds + \sum_{q=1}^{N}\int_\Omega e_q^T(m,t)Pe_q(m,t)dm. \tag{10}$$

Take the Dini derivative of $V(t)$, one can get

$$\begin{aligned}
D^+V(t) =& 2\sum_{q=1}^{N}\int_\Omega e_q^T(m,t)P\frac{\partial e_q(m,t)}{\partial t}dm + \sum_{q=1}^{N}\sum_{j=1}^{l}\frac{\phi_j^2}{1-\varpi_j}\int_\Omega e_{qj}^2(m,t)dm \\
&- \sum_{q=1}^{N}\sum_{j=1}^{l}\frac{\phi_j^2(1-\dot{\tau}_j(t))}{1-\varpi_j}\int_\Omega e_{qj}^2(m,t-\tau_j(t))dm \\
\leqslant& 2\sum_{q=1}^{N}\int_\Omega e_q^T(m,t)P\Big(\sum_{\beta=1}^{\eta} Y_\beta\frac{\partial^2 e_q(m,t)}{\partial m_\beta^2} + A(z_q(m,t))\hat{f}(e_q(m,t)) + [A(z_q(m,t)) - A(z^*(m))]f(z^*(m)) \\
&+ [D(z_q(m,t)) - D(z^*(m))]g(z^*(m)) + \xi\sum_{p=1}^{N} J_{qp}\Gamma e_p(m,t) - Ke_q(m,t) + D(z_q(m,t))\hat{g}(\overline{e_q(m,t)}) \\
&+ \varepsilon\sum_{p\in\mathcal{N}_q} H_{qp}(e_q(m,t) - e_p(m,t) + \pi_q(m,t) - \pi_p(m,t)) - \mathrm{sign}(e_q(m,t))(\tilde{A}\breve{\psi} + \tilde{D}\breve{\phi})\Big)dm \\
&+ \sum_{q=1}^{N}\int_\Omega e_q^T(m,t)\Phi Le_q(m,t)dm - \sum_{q=1}^{N}\int_\Omega \overline{e_q(m,t)}^T\Phi\overline{e_q(m,t)}dm.
\end{aligned}$$

In light of the boundary condition and Green's formula,

$$\sum_{\beta=1}^{\eta}\int_\Omega e_{q\epsilon}(m,t)\frac{\partial^2 e_{q\epsilon}(m,t)}{\partial m_\beta^2}dm = -\sum_{\beta=1}^{\eta}\int_\Omega\Big(\frac{\partial e_{q\epsilon}(m,t)}{\partial m_\beta}\Big)^2dm, \tag{11}$$

where $q = 1, 2, \cdots, N, \epsilon = 1, 2, \cdots, l$. Then,

$$\sum_{q=1}^{N}\int_\Omega e_q^T(m,t)P\sum_{\beta=1}^{\eta} Y_\beta\frac{\partial^2 e_q(m,t)}{\partial m_\beta^2}dm = \sum_{\beta=1}^{\eta}\sum_{q=1}^{N}\sum_{\epsilon=1}^{l} p_\epsilon y_{\epsilon\beta}\int_\Omega e_{q\epsilon}(m,t)\frac{\partial^2 e_{q\epsilon}(m,t)}{\partial m_\beta^2}dm$$

$$
\begin{aligned}
&= -\sum_{\beta=1}^{\eta}\sum_{q=1}^{N}\sum_{\epsilon=1}^{l} p_\epsilon y_{\epsilon\beta}\int_\Omega \Big(\frac{\partial e_{q\epsilon}(m,t)}{\partial m_\beta}\Big)^2 dm\\
&= -\sum_{\beta=1}^{\eta}\sum_{q=1}^{N}\int_\Omega \Big(\frac{\partial e_q(m,t)}{\partial m_\beta}\Big)^T PY_\beta \frac{\partial e_q(m,t)}{\partial m_\beta} dm.
\end{aligned}
$$

Then, we obtain

$$
2\sum_{q=1}^{N}\int_\Omega e_q^T(m,t)P\sum_{\beta=1}^{\eta} Y_\beta \frac{\partial^2 e_q(m,t)}{\partial m_\beta^2} dm = -\sum_{\beta=1}^{\eta}\int_\Omega \Big(\frac{\partial e(m,t)}{\partial m_\beta}\Big)^T (I_N\otimes 2PY_\beta)\frac{\partial e(m,t)}{\partial m_\beta} dm.
$$

It is not difficult to find a real matrix $\Pi\in\mathbb{R}^{lN\times lN}$,

$$
I_N\otimes 2PY_\beta = \Pi^T\Pi.
$$

Thus,

$$
\Big(\frac{\partial e(m,t)}{\partial m_\beta}\Big)^T (I_N\otimes 2PY_\beta)\frac{\partial e(m,t)}{\partial m_\beta} = \Big(\frac{\partial(\Pi e(m,t))}{\partial m_\beta}\Big)^T \frac{\partial(\Pi e(m,t))}{\partial m_\beta}.
$$

Let $\varrho(m,t) = \Pi e(m,t)$, for $(m,t)\in\partial\Omega\times[-\tau,+\infty)$. In terms of the boundary condition in (1), we deduce $\varrho(m,t) = \Pi e(m,t) = 0$. Based on Lemma 2.1, one has

$$
\sum_{\beta=1}^{\eta}\int_\Omega \Big(\frac{\partial \varrho(m,t)}{\partial m_\beta}\Big)^T \frac{\partial \varrho(m,t)}{\partial m_\beta} dm \geqslant \sum_{\beta=1}^{\eta}\frac{1}{\varphi_\beta^2}\int_\Omega \varrho^T(m,t)\varrho(m,t)dm = \sum_{\beta=1}^{\eta}\frac{2}{\varphi_\beta^2}\int_\Omega e^T(m,t)(I_N\otimes PY_\beta)e(m,t)dm.
$$

Therefore,

$$
2\sum_{q=1}^{N}\int_\Omega e_q^T(m,t)P\sum_{\beta=1}^{\eta} Y_\beta \frac{\partial^2 e_q(m,t)}{\partial m_\beta^2} dm \leqslant -\sum_{\beta=1}^{\eta}\frac{2}{\varphi_\beta^2}\int_\Omega e^T(m,t)(I_N\otimes PY_\beta)e(m,t)dm. \tag{12}
$$

Moreover, it follows from Lemma 2.2 that

$$
\begin{aligned}
2\sum_{q=1}^{N}\int_\Omega e_q^T(m,t)PA(z_q(m,t))\hat{f}(e_q(m,t))dm =& 2\sum_{q=1}^{N}\sum_{g=1}^{l}\sum_{h=1}^{l} p_g \int_\Omega e_{qg}(m,t)a_{gh}(z_{qg}(m,t))f_h(e_{qh}(m,t))dm\\
\leqslant& 2\sum_{q=1}^{N}\sum_{h=1}^{l}\sum_{g=1}^{l} p_g\bar{a}_{gh}\int_\Omega |e_{qg}(m,t)||f_h(e_{qh}(m,t))|dm\\
\leqslant& \sum_{q=1}^{N}\sum_{h=1}^{l}\sum_{g=1}^{l}\int_\Omega p_g^2\bar{a}_{gh}^2 e_{qg}^2(m,t)dm + \sum_{q=1}^{N}\sum_{h=1}^{l}\int_\Omega \psi_h^2 e_{qh}^2(m,t)dm\\
\leqslant& \sum_{q=1}^{N}\int_\Omega e_q^T(m,t)P^2\bar{A}e_q(m,t)dm + \sum_{q=1}^{N}\int_\Omega e_q^T(m,t)\Psi e_q(m,t)dm\\
=& \int_\Omega e^T(m,t)[I_N\otimes(P^2\bar{A}+\Psi)]e(m,t)dm.
\end{aligned} \tag{13}
$$

Similarly, we have

$$
2\sum_{q=1}^{N}\int_\Omega e_q^T(m,t)PD(z_q(m,t))\hat{g}(\overline{e_q(m,t)})dm = 2\sum_{q=1}^{N}\sum_{g=1}^{l}\sum_{h=1}^{l} p_g\int_\Omega e_{qg}(m,t)d_{gh}(z_{qg}(m,t))g_h(e_{qh}(m,t-\tau_h(t)))dm
$$

$$
\begin{aligned}
&\leqslant 2\sum_{q=1}^{N}\sum_{g=1}^{l}\sum_{h=1}^{l} p_g \bar{d}_{gh}\int_{\Omega}|e_{qg}(m,t)||g_h(e_{qh}(m,t-\tau_h(t)))|dm\\
&\leqslant \sum_{q=1}^{N}\sum_{g=1}^{l}\sum_{h=1}^{l}\int_{\Omega} p_g^2\bar{d}_{gh}^2 e_{qg}^2(m,t)dm + \sum_{q=1}^{N}\sum_{h=1}^{l}\int_{\Omega}\phi_h^2 e_{qh}^2(m,t-\tau_h(t))dm\\
&\leqslant \sum_{q=1}^{N}\int_{\Omega} e_q^T(m,t)P^2\bar{D}e_q(m,t)dm + \sum_{q=1}^{N}\int_{\Omega}\overline{e_q(m,t)}^T\Phi\overline{e_q(m,t)}dm\\
&= \sum_{q=1}^{N}\int_{\Omega}\overline{e_q(m,t)}^T\Phi\overline{e_q(m,t)}dm + \int_{\Omega} e^T(m,t)[I_N\otimes(P^2\bar{D})]e(m,t)dm. \qquad (14)
\end{aligned}
$$

Furthermore,

$$
\begin{aligned}
&2\sum_{q=1}^{N}\int_{\Omega} e_q^T(m,t)P[A(z_q(m,t))-A(z^*(m))]f(z^*(m))dm\\
&=2\sum_{q=1}^{N}\sum_{g=1}^{l}\sum_{h=1}^{l} p_g\int_{\Omega} e_{qg}(m,t)(a_{gh}(z_{qg}(m,t))-a_{gh}(z_g^*(m)))f_h(z_h^*(m))dm\\
&\leqslant 2\sum_{q=1}^{N}\sum_{g=1}^{l}\sum_{h=1}^{l} p_g|\hat{a}_{gh}-\check{a}_{gh}|\breve{\psi}_h\int_{\Omega}|e_{qg}(m,t)|dm\\
&\leqslant 2\sum_{q=1}^{N}\int_{\Omega}|e_q(m,t)|P\tilde{A}\breve{\psi}dm. \qquad (15)
\end{aligned}
$$

Similarly, one has

$$
\begin{aligned}
&2\sum_{q=1}^{N}\int_{\Omega} e_q^T(m,t)P[D(z_q(m,t))-D(z^*(m))]g(z^*(m))dm\\
&=2\sum_{q=1}^{N}\sum_{g=1}^{l}\sum_{h=1}^{l} p_g\int_{\Omega} e_{qg}(m,t)(d_{gh}(z_{qg}(m,t))-d_{gh}(z_g^*(m)))g_h(z_h^*(m))dm\\
&\leqslant 2\sum_{q=1}^{N}\sum_{g=1}^{l}\sum_{h=1}^{l} p_g|\hat{d}_{gh}-\check{d}_{gh}|\breve{\phi}_h\int_{\Omega}|e_{qg}(m,t)|dm\\
&\leqslant 2\sum_{q=1}^{N}\int_{\Omega}|e_q(m,t)|P\tilde{D}\breve{\phi}dm. \qquad (16)
\end{aligned}
$$

In addition, we have

$$
\begin{aligned}
&2\sum_{q=1}^{N}\int_{\Omega} e_q^T(m,t)P\varepsilon\sum_{p\in\mathcal{N}_q} H_{qp}(e_q(m,t)-e_p(m,t))dm\\
&\leqslant \sum_{q=1}^{N}\int_{\Omega} e_q^T(m,t)\varepsilon P^2 e_q(m,t)dm + \sum_{q=1}^{N}\varepsilon\|\sum_{p\in\mathcal{N}_q} H_{qp}(e_q(m,t)-e_p(m,t))\|^2\\
&\leqslant \sum_{q=1}^{N}\varepsilon\Big(\sum_{p\in\mathcal{N}_q} H_{qp}\|e_q(m,t)\| + \sum_{p\in\mathcal{N}_q} H_{qp}\|e_p(m,t)\|\Big)^2 + \sum_{q=1}^{N}\int_{\Omega} e_q^T(m,t)\varepsilon P^2 e_q(m,t)dm\\
&\leqslant \varepsilon(2\mathbf{h})^2 N\|e(m,t)\|^2 + \int_{\Omega} e^T(m,t)(I_N\otimes\varepsilon P^2)e(m,t)dm
\end{aligned}
$$

$$= \int_\Omega e^T(m,t)(I_N \otimes \varepsilon P^2 + 4\varepsilon N\mathbf{h}^2 I_{lN})e(m,t)dm. \tag{17}$$

From (8), one has

$$\begin{aligned}
&2\sum_{q=1}^{N}\int_\Omega e_q^T(m,t)P\varepsilon\sum_{p\in N_q}H_{qp}\Big(\pi_q(m,t)-\pi_p(m,t)\Big)dm\\
&\leqslant \int_\Omega e^T(m,t)(I_N\otimes\varepsilon P^2)e(m,t)dm + \int_\Omega 4\varepsilon N\mathbf{h}^2\pi^T(m,t)\pi(m,t)dm\\
&\leqslant \int_\Omega e^T(m,t)(4\varepsilon N\mathbf{h}^2\Big(\frac{2\theta N\mathbf{h}}{1-2\theta N\mathbf{h}}\Big)^2 I_{lN} + I_N\otimes\varepsilon P^2)e(m,t)dm.
\end{aligned}\tag{18}$$

From (12)-(18), it is derived that

$$\begin{aligned}
D^+V(t) \leqslant& \int_\Omega e^T(m,t)\Big\{I_N\otimes\Big[-\sum_{\beta=1}^{\eta}\frac{2}{\varphi_\beta^2}PY_\beta - 2PK + P^2(\bar{A}+\bar{D}) + \Psi + \Phi L + 2\varepsilon P^2 + 4\varepsilon N\mathbf{h}^2\Big(1+\Big(\frac{2\theta N\mathbf{h}}{1-2\theta N\mathbf{h}}\Big)^2\Big)I_l\Big]\\
&+\xi J\otimes(P\Gamma+\Gamma P)\Big\}e(m,t)dm\\
\leqslant&\Xi\|e(\cdot,t)\|^2,
\end{aligned}\tag{19}$$

where $\Xi = \lambda_M(I_N\otimes[-\sum_{\beta=1}^{\eta}\frac{2}{\varphi_\beta^2}PY_\beta - 2PK + P^2(\bar{A}+\bar{D}) + \Psi + \Phi L + 2\varepsilon P^2 + 4\varepsilon N\mathbf{h}^2(1+(\frac{2\theta N\mathbf{h}}{1-2\theta N\mathbf{h}})^2)I_l] + \xi J\otimes(P\Gamma+\Gamma P)) < 0$.

By means of (10) and (19), we have $V(t)$ is bounded and monotonically decreasing. Hence, $V(t)$ asymptotically approaches to a finite positive value. Based on (19), one can deduce

$$\|e(\cdot,t)\|^2 \leqslant \frac{D^+V(t)}{\Xi}. \tag{20}$$

Then, we can obtain $\lim_{t\to+\infty}\int_0^t\|e(\cdot,t)\|^2dt \geqslant 0$. Because the time delays $\tau_j(t)$ are bounded, we can get

$$\lim_{t\to+\infty}\sum_{q=1}^{N}\sum_{j=1}^{l}\frac{\phi_j^2}{1-\varpi_j}\int_{t-\tau_j(t)}^{t}\int_\Omega e_{qj}^2(m,s)dmds = 0.$$

Thus, we can derive $\lim_{t\to+\infty}\sum_{q=1}^{N}\int_\Omega e_q^T(m,t)Pe_q(m,t)dm$ exists. In the next step, $\lim_{t\to+\infty}\sum_{q=1}^{N}\int_\Omega e_q^T(m,t)Pe_q(m,t)dm$ $= 0$ will be proved. If not,

$$\lim_{t\to+\infty}\sum_{q=1}^{N}\int_\Omega e_q^T(m,t)Pe_q(m,t)dm = \ell > 0.$$

Next, there exists a positive scalar T satisfying

$$\sum_{q=1}^{N}\int_\Omega e_q^T(m,t)Pe_q(m,t)dm > \frac{\ell}{2}$$

when $t \geqslant T$, where T is a time interval based on the event-triggered time sequence. Therefore,

$$\|e(\cdot,t)\|^2 > \frac{\ell}{2\kappa}, \qquad t \geqslant T, \tag{21}$$

where $\kappa = \lambda_M(P)$. On the basis of (19)-(21), one has

$$D^+V(t) < \frac{\Xi\ell}{2\kappa}, \qquad t \geqslant T. \tag{22}$$

By integrating (22) with respect to t over the time period T to $\Im$, it is easily derived that

$$\begin{aligned}\int_T^{\Im}\frac{\Xi\ell}{2\kappa}dt > \int_T^{\Im} D^+V(t)dt &= \int_T^{t_{k'}^q} D^+V(t)dt + \int_{t_{k'}^q}^{t_{k'+1}^q} D^+V(t)dt + \cdots + \int_{t_{k_q(\Im)}^q}^{\Im} D^+V(t)dt\\ &= V(t_{k'}^q) - V(T) + V(t_{k'+1}^q) - V(t_{k'}^q) + \cdots + V(\Im) - V(t_{k_q(\Im)}^q)\\ &= V(\Im) - V(T)\\ &> - V(T),\end{aligned} \tag{23}$$

in which $k' = \arg\min_k\{t_k^q \geqslant T\}$ and $k_q(\Im) = \arg\max_{k^*}\{t_{k^*}^q \leqslant \Im\}$. In accordance with (23), one gets $-V(T) < \lim_{\Im\to+\infty}\int_T^{\Im}\frac{\Xi\ell}{2\kappa}dt = -\infty$, this is unreasonable. Thus,

$$\lim_{t\to+\infty}\sum_{q=1}^{N}\int_\Omega e_q^T(m,t)Pe_q(m,t)dm = 0,$$

then $\lim_{t\to+\infty}\|e_q(\cdot,t)\| = 0$ can be obatined. Hence, the network (1) reaches event-triggered synchronization under the controller (2).

Remark 2. As one of unique classes of CNs, CNNs have been extensive attention owing to their potential and wide application prospects in image encryption, pattern recognition, and secure communication [3]-[5]. Strictly speaking, these successful applications mainly depend on CNNs' behaviours of dynamics, especially synchronization. Thus, synchronization has become an important research topic. In recent years, many crucial and meaningful synchronization results for CNNs and CRDNNs have been acquired [6]-[11], [18]-[21]. Nevertheless, the considered synchronization results in the above-mentioned works did not consider the memristor. As we know, the problem of event-triggered synchronization for CDRDMNNs has not been yet solved, thereby this promotes us to develop the research work in our paper.

Remark 3. The problem of event-triggered synchronization for CDRDMNNs becomes more complicated in this paper since the memristive term is introduced to the considered model, it is obviously hard to be addressed by exploiting the previous strategy of triggering event for CNNs without memristive term. Thus, a general event-triggered condition (4) is designed for conquering this difficulty through making use of the own characteristic of the considered model, which is improved significantly on existing event-triggered conditions. In addition, due to the original event-triggered condition and the memristive term, some novel inequality techniques ought to be utilized in (7) and the testify of the obtained synchronization conditions.

3.2. *Event-triggered $\mathcal{H}_\infty$ synchronization*

The following CDRDMNNs with state coupling is considered in this subsection:

$$\begin{aligned}\frac{\partial z_q(m,t)}{\partial t} =& \sum_{\beta=1}^{\eta} Y_\beta\frac{\partial^2 z_q(m,t)}{\partial m_\beta^2} - Kz_q(m,t) + A(z_q(m,t))f(z_q(m,t)) + I + \gamma_q(m,t) + D(z_q(m,t))g(\overline{z_q(m,t)})\\ &+ \xi\sum_{p=1}^{N} J_{qp}\Gamma z_p(m,t) + v_q(m,t) + \omega_q(m,t),\end{aligned} \tag{24}$$

where $z_q(m,t), Y_\beta, K, A(z_q(m,t)), f(z_q(m,t)), D(z_q(m,t)), g(\overline{z_q(m,t)}), I, \xi, J_{qp}, \Gamma, \gamma_q(m,t)$ and $v_q(m,t)$ are denoted in (1); $\omega_q(m,t) = (\omega_{q1}(m,t), \omega_{q2}(m,t), \cdots, \omega_{ql}(m,t))^T$ states the external disturbance and is square integrable, i.e., for any $\mathbb{R} \ni t_r \geqslant 0$,

$$\int_0^{t_r}\int_\Omega \omega_q^T(m,t)\omega_q(m,t)dmdt < +\infty.$$

Assume the equilibrium solution of (24) is $z^*(m) = (z_1^*(m), z_2^*(m), \cdots, z_l^*(m))^T \in \mathbb{R}^l$, then

$$\sum_{\beta=1}^{\eta} Y_\eta \frac{\partial^2 z^*(m)}{\partial m_\beta^2} - Kz^*(m) + A(z^*(m))f(z^*(m)) + D(z^*(m))g(z^*(m)) + I = 0.$$

Let $e_q(m,t) = z_q(m,t) - z^*(m)$. Based on the event-triggered condition (4), the error system $e_q(m,t)$ of network (24) can be described by:

$$\begin{aligned}\frac{\partial e_q(m,t)}{\partial t} =& \sum_{\beta=1}^{\eta} Y_\beta \frac{\partial^2 e_q(m,t)}{\partial m_\beta^2} + A(z_q(m,t))\hat{f}(e_q(m,t)) + D(z_q(m,t))\hat{g}(\overline{e_q(m,t)}) - Ke_q(m,t)\\ &+ [A(z_q(m,t)) - A(z^*(m))]f(z^*(m)) + [D(z_q(m,t)) - D(z^*(m))]g(z^*(m))\\ &+ \xi \sum_{p=1}^{N} J_{qp}\Gamma e_p(m,t) + \varepsilon \sum_{p\in N_q} H_{qp}\Big(e_q(m,t) - e_p(m,t) + \pi_q(m,t) - \pi_p(m,t)\Big)\\ &+ \omega_q(m,t) - \mathrm{sign}(e_q(m,t))(\tilde{A}\breve{\psi} + \tilde{D}\breve{\phi}), \end{aligned} \tag{25}$$

where $t \in [t_k^q, t_{k+1}^q)$, $\hat{f}(e_q(m,t))$ and $\hat{g}(\overline{e_q(m,t)})$ have the same meanings as those in system (5), $q = 1, 2, \cdots, N$.

Definition 3.1. *For the network* (24)*, we denote a nonnegative function $V(\cdot)$ satisfying*

$$\int_0^{t_r}\int_\Omega e^T(m,t)e(m,t)dmdt \leqslant V(0) + \delta^2 \int_0^{t_r}\int_\Omega \omega(m,t)\omega(m,t)dmdt$$

for any $t_r \in [0, +\infty)$ and the disturbance attenuation level $\delta \in \mathbb{R} > 0$, where $\omega(m,t) = (\omega_1^T(m,t), \omega_2^T(m,t), \cdots, \omega_N^T(m,t))^T$. Thus, network (24) *can realize $\mathcal{H}_\infty$ synchronization under the controller* (2).

Theorem 3.2. *For the network* (24)*, we suppose matrix $P = \mathrm{diag}(p_1, p_2, \cdots, p_l) \in \mathbb{R}^{l\times l} > 0$ and two constants $0 < \theta < \frac{1}{2N\mathbf{h}}, \delta > 0$ satisfying*

$$I_N \otimes \Theta_2 + \xi J \otimes (P\Gamma + \Gamma P) < 0, \tag{26}$$

where $\Theta_2 = -\sum_{\beta=1}^{\eta} \frac{2}{\varphi_\beta^2} PY_\beta - 2PK + P^2(\bar{A} + \bar{D}) + \Psi + \Phi L + (4\varepsilon N\mathbf{h}^2(1 + (\frac{2\theta N\mathbf{h}}{1-2\theta N\mathbf{h}})^2) + 1)I_l + (2\varepsilon + \frac{1}{\delta^2})P^2$ and $\mathbf{h} = \max\{\sum_{p\in N_q} H_{qp}\}$, then event-triggered $\mathcal{H}_\infty$ synchronization can be realized under the controller (2).

Proof. The Lyapunov functional is defined as (10) for network (25), one obtains

$$\begin{aligned} D^+V(t) \leqslant& 2\sum_{q=1}^{N}\int_\Omega e_q^T(m,t)P\Big(\sum_{\beta=1}^{\eta} Y_\beta \frac{\partial^2 e_q(m,t)}{\partial m_\beta^2} - Ke_q(m,t) + A(z_q(m,t))\hat{f}(e_q(m,t)) + D(z_q(m,t))\hat{g}(\overline{e_q(m,t)})\\ &+ [A(z_q(m,t)) - A(z^*(m))]f(z^*(m)) + [D(z_q(m,t)) - D(z^*(m))]g(z^*(m)) + \varepsilon \sum_{p\in N_q} H_{qp}(e_q(m,t)\\ &- e_p(m,t) + \pi_q(m,t) - \pi_p(m,t)) + \xi \sum_{p=1}^{N} J_{qp}\Gamma e_p(m,t) - \mathrm{sign}(e_q(m,t))(\tilde{A}\breve{\psi} + \tilde{D}\breve{\phi}) + \omega_q(m,t)\Big)dm\\ &+ \sum_{q=1}^{N}\int_\Omega e_q^T(m,t)\Phi Le_q(m,t)dm - \sum_{q=1}^{N}\int_\Omega \overline{e_q(m,t)}^T \Phi \overline{e_q(m,t)}dm. \end{aligned}$$

According to (12)-(18), we have

$$D^+V(t) \leqslant \int_\Omega e^T(m,t)\Big\{I_N \otimes \Big[-\sum_{\beta=1}^{\eta} \frac{2}{\varphi_\beta^2} PY_\beta - 2PK + P^2(\bar{A} + \bar{D}) + \Psi + \Phi L + 2\varepsilon P^2 + 4\varepsilon N\mathbf{h}^2\Big(1 + \Big(\frac{2\theta N\mathbf{h}}{1-2\theta N\mathbf{h}}\Big)^2\Big)I_l\Big]$$

$$+\xi J\otimes(P\Gamma+\Gamma P)\Big\}e(m,t)dm+2\int_{\Omega}e^{T}(m,t)(I_N\otimes P)\omega(m,t)dm,$$

where $\omega(m,t)=(\omega_1^T(m,t),\omega_2^T(m,t),\cdots,\omega_N^T(m,t))^T$. In addition,

$$\begin{aligned}
&\int_0^{t_r}\int_{\Omega}\Big[e^T(m,t)e(m,t)-\delta^2\omega^T(m,t)\omega(m,t)\Big]dmdt\\
=&\int_0^{t_r}\int_{\Omega}\Big[e^T(m,t)e(m,t)-\delta^2\omega^T(m,t)\omega(m,t)\Big]dmdt+\int_0^{t_r}D^+V(t)dt+V(0)-V(t_r)\\
=&\int_0^{t_r}\int_{\Omega}\Big[e^T(m,t)e(m,t)-\delta^2\omega^T(m,t)\omega(m,t)\Big]dmdt+\int_0^{t_1^q}D^+V(t)dt+\int_{t_1^q}^{t_2^q}D^+V(t)dt+\cdots\\
&+\int_{t^q_{k_q(t_r)}}^{t_r}D^+V(t)dt+V(0)-V(t_r)\\
\leqslant&\int_0^{t_r}\int_{\Omega}e^T(m,t)e(m,t)dmdt-\int_0^{t_r}\int_{\Omega}\delta^2\omega^T(m,t)\omega(m,t)dmdt+V(0)-V(t_r)+\int_0^{t_r}\int_{\Omega}e^T(m,t)\Big\{I_N\otimes\Big[-2PK\\
&-\sum_{\beta=1}^{\eta}\frac{2}{\varphi_\beta^2}PY_\beta+P^2(\bar{A}+\bar{D})+\Psi+4\varepsilon N\mathbf{h}^2\Big(1+\Big(\frac{2\theta N\mathbf{h}}{1-2\theta N\mathbf{h}}\Big)^2\Big)I_l+2\varepsilon P^2+\Phi L\Big]+\xi J\otimes(P\Gamma+\Gamma P)\Big\}e(m,t)dmdt\\
&+2\int_0^{t_r}\int_{\Omega}e^T(m,t)(I_N\otimes P)\omega(m,t)dmdt\\
=&-\int_0^{t_r}\int_{\Omega}\Big[\delta\omega(m,t)-\frac{I_N\otimes P}{\delta}e(m,t)\Big]^T\Big[\delta\omega(m,t)-\frac{I_N\otimes P}{\delta}e(m,t)\Big]dmdt+\int_0^{t_r}\int_{\Omega}e^T(m,t)\Big\{I_N\otimes[-\sum_{\beta=1}^{\eta}\frac{2}{\varphi_\beta^2}PY_\beta\\
&-2PK+P^2(\bar{A}+\bar{D})+\Psi+\Phi L+(2\varepsilon+\frac{1}{\delta^2})P^2+(4\varepsilon N\mathbf{h}^2\Big(1+\Big(\frac{2\theta N\mathbf{h}}{1-2\theta N\mathbf{h}}\Big)^2\Big)+1)I_l]\\
&+\xi J\otimes(P\Gamma+\Gamma P)\Big\}e(m,t)dmdt+V(0)-V(t_r)\\
\leqslant&V(0),
\end{aligned}\tag{27}$$

where $k_q(t_r)=\arg\max_{k^*}\{t^q_{k^*}\leqslant t_r\}$. In terms of (27), one can get

$$\int_0^{t_r}\int_{\Omega}e^T(m,t)e(m,t)dmdt\leqslant V(0)+\delta^2\int_0^{t_r}\int_{\Omega}\omega^T(m,t)\omega(m,t)dmdt.$$

Therefore, the network (24) realizes event-triggered $\mathcal{H}_\infty$ synchronization under (2) according to Definition 3.1.

Remark 4. As we al know, the disturbance or noise are unavoidable and maybe lead the networks to unanticipated states or destroy the synchronization in the real systems. Consequently, how to design disturbance attenuating synchronization controllers and reduce the influence of external disturbance has received considerable attention. In recent years, $\mathcal{H}_\infty$ synchronization has been verified to be a very effective strategy, which can reduce the effect of disturbances, and important results of CNs have been established [12]-[15], [22]-[25]. In addition, as a discontinuous control strategy, event-triggered control has been widely applied for ensuring CNs synchronization and $\mathcal{H}_\infty$ synchronization synchronization [38]-[44] since it can overcome a number of consecutive control's defects in system theory, and avoid some needless communication when data is sent and exchanged for transmission. In other words, the triggering event control is a very efficient tool to solve the unnecessary communication problem and overcome the disadvantages of continuous control in many real system. Unfortunately, synchronization and $\mathcal{H}_\infty$ synchronization problems of CDRDMNNs via event-triggered control have not been yet addressed. In this section, event-triggered synchronization and $\mathcal{H}_\infty$ synchronization criteria are established for CDRDMNNs with state coupling in Theorems 3.1 and 3.2, respectively.

4. Event-triggered synchronization and $\mathcal{H}_\infty$ synchronization of CDRDMNNs with spatial diffusion coupling

4.1. Event-triggered synchronization

The following CDRDMNNs with spatial diffusion coupling in this section is described by:

$$\begin{cases} \frac{\partial z_q(m,t)}{\partial t} &= \sum_{\beta=1}^{\eta} Y_\beta \frac{\partial^2 z_q(m,t)}{\partial m_\beta^2} - Kz_q(m,t) + I + A(z_q(m,t))f(z_q(m,t)) + \gamma_q(m,t) + D(z_q(m,t))g(\overline{z_q(m,t)}) \\ & \quad +\zeta \sum_{\beta=1}^{\eta} \sum_{p=1}^{N} G_{qp}\Upsilon \frac{\partial^2 z_p(m,t)}{\partial m_\beta^2} + v_q(m,t), \\ z_q(m,t) &= \tilde{\vartheta}_q(m,t), \qquad (m,t) \in \Omega \times [-\tau, 0], \\ z_q(m,t) &= 0, \qquad (m,t) \in \partial\Omega \times [-\tau, +\infty), \end{cases} \tag{28}$$

where $z_q(m,t), Y_\beta, K, A(z_q(m,t)), f(z_q(m,t)), D(z_q(m,t)), g(\overline{z_q(m,t)}), I, \tau_j(t), \gamma_q(m,t), v_q(m,t)$ denote the same as those in model (1); $\zeta > 0$ is the overall coupling strength; $0 < \Upsilon \in \mathbb{R}^{l\times l}$ denotes the inner coupling matrix; $G = (G_{qp})_{N\times N}$ is the coupling weight, which satisfies $G_{qp} = G_{pq} > 0 (p \neq q)$ if there exists a connection from p-th node to q-th node, or else $G_{qp} = 0$ and $G_{qq} = -\sum_{\substack{p=1\\p\neq q}}^{N} G_{qp}$; $\tilde{\vartheta}_q(m,t) \in C(\Omega \times [-\tau, 0])$ is continuous and bounded.

Assume the equilibrium solution of (28) is $z^*(m) = (z_1^*(m), z_2^*(m), \cdots, z_l^*(m))^T \in \mathbb{R}^l$. Then

$$\sum_{\beta=1}^{\eta} Y_\eta \frac{\partial^2 z^*(m)}{\partial m_\beta^2} - Kz^*(m) + A(z^*(m))f(z^*(m)) + D(z^*(m))g(z^*(m)) + I = 0.$$

Let $e_q(m,t) = z_q(m,t) - z^*(m)$. By means of the event-triggered condition (4), the error system $e_q(m,t)$ of network (28) is represented by:

$$\begin{aligned} \frac{\partial e_q(m,t)}{\partial t} =& \sum_{\beta=1}^{\eta} Y_\beta \frac{\partial^2 e_q(m,t)}{\partial m_\beta^2} + A(z_q(m,t))\hat{f}(e_q(m,t)) + D(z_q(m,t))\hat{g}(\overline{e_q(m,t)}) - Ke_q(m,t) + [A(z_q(m,t)) \\ & - A(z^*(m))]f(z^*(m)) + [D(z_q(m,t)) - D(z^*(m))]g(z^*(m)) + \zeta \sum_{\beta=1}^{\eta}\sum_{p=1}^{N} G_{qp}\Upsilon \frac{\partial^2 e_p(m,t)}{\partial m_\beta^2} \\ & + \varepsilon \sum_{p\in N_q} H_{qp}\Big(e_q(m,t) - e_p(m,t) + \pi_q(m,t) - \pi_p(m,t)\Big) - \mathrm{sign}(e_q(m,t))(\tilde{A}\breve{\psi} + \tilde{D}\breve{\phi}), \end{aligned} \tag{29}$$

in which $t \in [t_k^q, t_{k+1}^q)$, $\hat{f}(e_q(m,t))$ and $\hat{g}(\overline{e_q(m,t)})$ have the same meanings as that in system (5), $q = 1, 2, \cdots, N$.

Theorem 4.1. *The network* (28) *reaches event-triggered synchronization under the controller* (2) *if there exist a matrix* $P = \mathrm{diag}(p_1, p_2, \cdots, p_l) \in \mathbb{R}^{l\times l} > 0$ *and a constant* $0 < \theta < \frac{1}{2N\mathbf{h}}$ *such that*

$$I_N \otimes 2PY_\beta + \zeta G \otimes (P\Upsilon + \Upsilon P) \geqslant 0, \tag{30}$$

$$I_N \otimes W_1 - \sum_{\beta=1}^{\eta} \frac{1}{\varphi_\beta^2}[\zeta G \otimes (P\Upsilon + \Upsilon P)] < 0, \tag{31}$$

where $W_1 = -\sum_{\beta=1}^{\eta} \frac{2}{\varphi_\beta^2} PY_\beta - 2PK + P^2(\bar{A} + \bar{D}) + \Psi + \Phi L + 2\varepsilon P^2 + 4\varepsilon N\mathbf{h}^2(1 + (\frac{2\theta N\mathbf{h}}{1-2\theta N\mathbf{h}})^2)I_l$ *and* $\mathbf{h} = \max\{\sum_{p\in N_q} H_{qp}\}$.

Proof. The same Lyapunov functional is constructed as (10) for network (29). Then,

$$\begin{aligned} D^+V(t) \leqslant& 2\sum_{q=1}^{N}\int_\Omega e_q^T(m,t)P\Big(\sum_{\beta=1}^{\eta} Y_\beta \frac{\partial^2 e_q(m,t)}{\partial m_\beta^2} - Ke_q(m,t) + A(z_q(m,t))\hat{f}(e_q(m,t)) + D(z_q(m,t))\hat{g}(\overline{e_q(m,t)}) \\ & + [A(z_q(m,t)) - A(z^*(m))]f(z^*(m)) + [D(z_q(m,t)) - D(z^*(m))]g(z^*(m)) + \varepsilon \sum_{p\in N_q} H_{qp}(e_q(m,t) \\ & - e_p(m,t) + \pi_q(m,t) - \pi_p(m,t)) + \zeta \sum_{\beta=1}^{\eta}\sum_{p=1}^{N} G_{qp}\Upsilon \frac{\partial^2 e_p(m,t)}{\partial m_\beta^2} - \mathrm{sign}(e_q(m,t))(\tilde{A}\breve{\psi} + \tilde{D}\breve{\phi})\Big)dm \end{aligned}$$

$$+\sum_{q=1}^{N}\int_{\Omega}e_q^T(m,t)\Phi Le_q(m,t)dm-\sum_{q=1}^{N}\int_{\Omega}\overline{e_q(m,t)}^T\Phi\overline{e_q(m,t)}dm.$$

From (11), one gets

$$2\sum_{q=1}^{N}\int_{\Omega}e_q^T(m,t)P\zeta\sum_{\beta=1}^{\eta}\sum_{p=1}^{N}G_{qp}\Upsilon\frac{\partial^2 e_p(m,t)}{\partial m_\beta^2}=-\sum_{\beta=1}^{\eta}\int_{\Omega}\Big(\frac{\partial e(m,t)}{\partial m_\beta}\Big)^T[\zeta G\otimes(P\Upsilon+\Upsilon P)]\frac{\partial e(m,t)}{\partial m_\beta}dm.$$

Then, as the proof in Theorem 3.1, it is not difficult to derive that

$$\begin{aligned}&2\sum_{q=1}^{N}\int_{\Omega}e_q^T(m,t)P\Big(\sum_{\beta=1}^{\eta}Y_\beta\frac{\partial^2 e_q(m,t)}{\partial m_\beta^2}+\zeta\sum_{\beta=1}^{\eta}\sum_{p=1}^{N}G_{qp}\Upsilon\frac{\partial^2 e_p(m,t)}{\partial m_\beta^2}\Big)dm\\&\leqslant-\sum_{\beta=1}^{\eta}\frac{1}{\varphi_\beta^2}\int_{\Omega}e^T(m,t)[I_N\otimes 2PY_\beta+\zeta G\otimes(P\Upsilon+\Upsilon P)]e(m,t)dm.\end{aligned}\tag{32}$$

In light of (12)-(18) and (32), one can derive

$$\begin{aligned}D^+V(t)\leqslant&\int_{\Omega}e^T(m,t)\Big\{I_N\otimes\Big[-\sum_{\beta=1}^{\eta}\frac{2}{\varphi_\beta^2}PY_\beta-2PK+P^2(\bar{A}+\bar{D})+\Phi L+\Psi+2\varepsilon P^2+4\varepsilon N\mathbf{h}^2\Big(1+\Big(\frac{2\theta N\mathbf{h}}{1-2\theta N\mathbf{h}}\Big)^2\Big)I_l\Big]\\&-\sum_{\beta=1}^{\eta}\frac{1}{\varphi_\beta^2}[\zeta G\otimes(P\Upsilon+\Upsilon P)]\Big\}e(m,t)dm\\\leqslant&\Lambda\|e(\cdot,t)\|^2,\end{aligned}\tag{33}$$

in which $\Lambda=\lambda_M(I_N\otimes[-\sum_{\beta=1}^{\eta}\frac{2}{\varphi_\beta^2}PY_\beta-2PK+P^2(\bar{A}+\bar{D})+\Psi+\Phi L+2\varepsilon P^2+4\varepsilon N\mathbf{h}^2(1+(\frac{2\theta N\mathbf{h}}{1-2\theta N\mathbf{h}})^2)I_l]-\sum_{\beta=1}^{\eta}\frac{1}{\varphi_\beta^2}[\zeta G\otimes(P\Upsilon+\Upsilon P)])<0$.

Then, the rest of this proof for $\lim_{t\to+\infty}\|e(\cdot,t)\|=0$ runs as Theorem 3.1. Therefore, the network (28) reaches event-triggered synchronization under the controller (2).

4.2. Event-triggered $\mathcal{H}_\infty$ synchronization

The following CDRDMNNs with spatial diffusion coupling is stated in this subsection:

$$\begin{aligned}\frac{\partial z_q(m,t)}{\partial t}=&\sum_{\beta=1}^{\eta}Y_\beta\frac{\partial^2 z_q(m,t)}{\partial m_\beta^2}-Kz_q(m,t)+A(z_q(m,t))f(z_q(m,t))+I+\gamma_q(m,t)+D(z_q(m,t))g(\overline{z_q(m,t)})\\&+\zeta\sum_{\beta=1}^{\eta}\sum_{p=1}^{N}G_{qp}\Upsilon\frac{\partial^2 z_p(m,t)}{\partial m_\beta^2}+v_q(m,t)+\omega_q(m,t),\end{aligned}\tag{34}$$

where $z_q(m,t),Y_\beta,K,A(z_q(m,t)),f(z_q(m,t)),D(z_q(m,t)),g(\overline{z_q(m,t)}),I,\tau_j(t),\gamma_q(m,t),v_q(m,t),\zeta,G_{qp}$ and Υ are expressed well in (28); $\omega_q(m,t)$ is square integrable.

Assume the equilibrium solution of (34) is $z^*(m)=(z_1^*(m),z_2^*(m),\cdots,z_l^*(m))^T\in\mathbb{R}^l$, and it dynamics

$$\sum_{\beta=1}^{\eta}Y_\eta\frac{\partial^2 z^*(m)}{\partial m_\beta^2}-Kz^*(m)+A(z^*(m))f(z^*(m))+D(z^*(m))g(z^*(m))+I=0.$$

Let $e_q(m,t)=z_q(m,t)-z^*(m)$. Based on the event-triggered condition (4), the error system $e_q(m,t)$ for network (34) is given as follows:

$$\frac{\partial e_q(m,t)}{\partial t}=\sum_{\beta=1}^{\eta}Y_\beta\frac{\partial^2 e_q(m,t)}{\partial m_\beta^2}+A(z_q(m,t))\hat{f}(e_q(m,t))+D(z_q(m,t))\hat{g}(\overline{e_q(m,t)})-Ke_q(m,t)+[A(z_q(m,t))$$

$$- A(z^*(m))]f(z^*(m)) + [D(z_q(m,t)) - D(z^*(m))]g(z^*(m)) + \zeta \sum_{\beta=1}^{\eta}\sum_{p=1}^{N} G_{qp}\Upsilon \frac{\partial^2 e_p(m,t)}{\partial m_\beta^2}$$
$$+ \varepsilon \sum_{p\in\mathcal{N}_q} H_{qp}\Big(e_q(m,t) - e_p(m,t) + \pi_q(m,t) - \pi_p(m,t)\Big) + \omega_q(m,t) - \mathrm{sign}(e_q(m,t))(\tilde{A}\breve{\psi} + \tilde{D}\breve{\phi}), \qquad (35)$$

in which $t \in [t_k^q, t_{k+1}^q)$, $\hat{f}(e_q(m,t))$ and $\hat{g}(\overline{e_q(m,t)})$ have the same meanings as that in system (5).

Theorem 4.2. *For the network* (34), *we assume matrix* $P = \mathrm{diag}(p_1, p_2, \cdots, p_l) \in \mathbb{R}^{l\times l} > 0$ *and two constants* $0 < \theta < \frac{1}{2N\mathbf{h}}, \delta > 0$ *satisfying*

$$I_N \otimes 2PY_\beta + \zeta G \otimes (P\Upsilon + \Upsilon P) \geqslant 0, \qquad (36)$$

$$I_N \otimes W_2 - \sum_{\beta=1}^{\eta} \frac{1}{\varphi_\beta^2}[\zeta G \otimes (P\Upsilon + \Upsilon P)] < 0, \qquad (37)$$

where $W_2 = -\sum_{\beta=1}^{\eta}\frac{2}{\varphi_\beta^2}PY_\beta - 2PK + P^2(\bar{A} + \bar{D}) + \Psi + \Phi L + (4\varepsilon N\mathbf{h}^2(1 + (\frac{2\theta N\mathbf{h}}{1-2\theta N\mathbf{h}})^2) + 1)I_l + (2\varepsilon + \frac{1}{\delta^2})P^2$ *and* $\mathbf{h} = \max\{\sum_{p\in\mathcal{N}_q} H_{qp}\}$, *then event-triggered* $\mathcal{H}_\infty$ *synchronization can be reached under the controller* (2).

Proof. The Lyapunov functional is also chosen as (10) for network (35), we obtain

$$D^+V(t) \leqslant 2\sum_{q=1}^{N}\int_\Omega e_q^T(m,t)P\Big(\sum_{\beta=1}^{\eta} Y_\beta \frac{\partial^2 e_q(m,t)}{\partial m_\beta^2} - Ke_q(m,t) + A(z_q(m,t))\hat{f}(e_q(m,t)) + D(z_q(m,t))\hat{g}(\overline{e_q(m,t)})$$
$$+ [A(z_q(m,t)) - A(z^*(m))]f(z^*(m)) + [D(z_q(m,t)) - D(z^*(m))]g(z^*(m)) + \varepsilon\sum_{p\in\mathcal{N}_q} H_{qp}(e_q(m,t)$$
$$- e_p(m,t) + \pi_q(m,t) - \pi_p(m,t)) - \mathrm{sign}(e_q(m,t))(\tilde{A}\breve{\psi} + \tilde{D}\breve{\phi}) + \zeta\sum_{\beta=1}^{\eta}\sum_{p=1}^{N} G_{qp}\Upsilon\frac{\partial^2 e_p(m,t)}{\partial m_\beta^2}$$
$$+ \omega_q(m,t)\Big)dm + \sum_{q=1}^{N}\int_\Omega e_q^T(m,t)\Phi L e_q(m,t)dm - \sum_{q=1}^{N}\int_\Omega \overline{e_q(m,t)}^T\Phi\overline{e_q(m,t)}dm.$$

According to (12)-(18) and (32), one gets

$$D^+V(t) \leqslant \int_\Omega e^T(m,t)\Big\{I_N \otimes \Big[-\sum_{\beta=1}^{\eta}\frac{2}{\varphi_\beta^2}PY_\beta - 2PK + P^2(\bar{A} + \bar{D}) + \Psi + \Phi L + 2\varepsilon P^2 + 4\varepsilon N\mathbf{h}^2\Big(\Big(\frac{2\theta N\mathbf{h}}{1 - 2\theta N\mathbf{h}}\Big)^2 + 1\Big)I_l\Big]$$
$$- \sum_{\beta=1}^{\eta}\frac{1}{\varphi_\beta^2}[\zeta G \otimes (P\Upsilon + \Upsilon P)]\Big\}e(m,t)dm + 2\int_\Omega e^T(m,t)(I_N \otimes P)\omega(m,t)dm.$$

In addition,

$$\int_0^{t_r}\int_\Omega\Big[e^T(m,t)e(m,t) - \delta^2\omega^T(m,t)\omega(m,t)\Big]dmdt$$
$$\leqslant \int_0^{t_r}\int_\Omega e^T(m,t)e(m,t)dmdt - \int_0^{t_r}\int_\Omega \delta^2\omega^T(m,t)\omega(m,t)dmdt + V(0) - V(t_r) + \int_0^{t_r}\int_\Omega e^T(m,t)\Big\{I_N \otimes \Big[-2PK$$
$$- \sum_{\beta=1}^{\eta}\frac{2}{\varphi_\beta^2}PY_\beta + P^2(\bar{A} + \bar{D}) + \Psi + 4\varepsilon N\mathbf{h}^2\Big(\Big(\frac{2\theta N\mathbf{h}}{1 - 2\theta N\mathbf{h}}\Big)^2 + 1\Big)I_l + \Phi L + 2\varepsilon P^2\Big] - \sum_{\beta=1}^{\eta}\frac{1}{\varphi_\beta^2}[\zeta G \otimes (P\Upsilon$$
$$+ \Upsilon P)]\Big\}e(m,t)dmdt + 2\int_0^{t_r}\int_\Omega e^T(m,t)(I_N \otimes P)\omega(m,t)dmdt$$
$$= -\int_0^{t_r}\int_\Omega\Big[\delta\omega(m,t) - \frac{I_N \otimes P}{\delta}e(m,t)\Big]^T\Big[\delta\omega(m,t) - \frac{I_N \otimes P}{\delta}e(m,t)\Big]dmdt + \int_0^{t_r}\int_\Omega e^T(m,t)\Big\{I_N \otimes [-\sum_{\beta=1}^{\eta}\frac{2}{\varphi_\beta^2}PY_\beta$$

$$
\begin{aligned}
&-2PK+P^2(\bar{A}+\bar{D})+\Psi+\Phi L+(2\varepsilon+\frac{1}{\delta^2})P^2+(4\varepsilon N\mathbf{h}^2\Big(1+\big(\frac{2\theta N\mathbf{h}}{1-2\theta N\mathbf{h}}\big)^2\Big)+1)I_l]\\
&-\sum_{\beta=1}^{\eta}\frac{1}{\varphi_\beta^2}[\zeta G\otimes(P\Upsilon+\Upsilon P)]\Big\}e(m,t)dmdt+V(0)-V(t_r)\\
\leqslant& V(0).
\end{aligned}\tag{38}
$$

According to (38), one can acquire

$$
\int_0^{t_r}\int_\Omega e^T(m,t)e(m,t)dmdt\leqslant V(0)+\delta^2\int_0^{t_r}\int_\Omega\omega^T(m,t)\omega(m,t)dmdt.
$$

Therefore, the network (34) realizes event-triggered $\mathcal{H}_\infty$ synchronization with the controller (2).

Remark 5. Recently, most of synchronization results associated with the nodes of CRDNNs are coupled with their state [18]-[21]. It is well known that different diffusion of single RDNNs are likely to have an effect on other adjacent RDNNs in the coupling form. Therefore, it is indispensable to take dynamical behaviors of spatial diffusion coupled networks into account. As far as we know, this is the first paper to discuss synchronization and $\mathcal{H}_\infty$ synchronization of CDRDMNNs with spatial diffusion coupling via event-triggered protocol. Some event-triggered synchronization and $\mathcal{H}_\infty$ synchronization conditions are derived for CDRDMNNs with spatial diffusion coupling in Theorems 4.1 and 4.2 of this section, respectively.

5. Numerical Examples

Example 5.1. Consider the CDRDMNNs with state coupling via event-triggered communication as follows:

$$
\begin{aligned}
\frac{\partial z_q(m,t)}{\partial t}=&Y\frac{\partial^2 z_q(m,t)}{\partial m^2}-Kz_q(m,t)+I+A(z_q(m,t))f(z_q(m,t))+\gamma_q(m,t)+D(z_q(m,t))g(\overline{z_q(m,t)})\\
&+\xi\sum_{p=1}^{6}J_{qp}\Gamma z_p(m,t)+v_q(m,t),
\end{aligned}\tag{39}
$$

where $\Omega=\{m|-0.25<m<0.25\}, l=3, q=1,2,\cdots,6; f_\alpha(\mu)=\frac{|\mu+1|-|\mu-1|}{8}, g_\alpha(\mu)=\frac{|\mu+1|-|\mu-1|}{4}, \alpha=1,2,3; K=\mathrm{diag}(4,3,5), Y=\mathrm{diag}(0.6,0.7,0.9), I=(0,0,0)^T; \tau_j(t)=1-\frac{1}{j+4}e^{-t}, \varpi_j=\frac{1}{j+4}, j=1,2,3; \Gamma=\mathrm{diag}(0.6,0.5,0.8), \xi=2.4; v_q(m,t)=\varepsilon\sum_{p\in\mathcal{N}_q}H_{qp}(z_q(m,t_k^q)-z_p(m,t_k^p)); \gamma_q(m,t)=-\mathrm{sign}(z_q(m,t)-z^*(m))(\tilde{A}\breve{\psi}+\tilde{D}\breve{\phi}), \varepsilon=3.2$. The matrices $A(z_q(m,t)), D(z_q(m,t)), J=(J_{qp})_{6\times6}$ and $H=(H_{qp})_{6\times6}$ are chosen, respectively:

$$
a_{11}(z_{q1}(m,t))=\begin{cases}-0.18, & |z_{q1}(m,t)|\leqslant 1.2,\\ 0.35, & |z_{q1}(m,t)|>1.2,\end{cases}\quad a_{12}(z_{q1}(m,t))=\begin{cases}-0.28, & |z_{q1}(m,t)|\leqslant 1.2,\\ 0.45, & |z_{q1}(m,t)|>1.2,\end{cases}
$$

$$
a_{13}(z_{q1}(m,t))=\begin{cases}-0.12, & |z_{q1}(m,t)|\leqslant 1.2,\\ 0.33, & |z_{q1}(m,t)|>1.2,\end{cases}\quad a_{21}(z_{q2}(m,t))=\begin{cases}0.45, & |z_{q2}(m,t)|\leqslant 1.2,\\ -0.36, & |z_{q2}(m,t)|>1.2,\end{cases}
$$

$$
a_{22}(z_{q2}(m,t))=\begin{cases}-0.54, & |z_{q2}(m,t)|\leqslant 1.2,\\ 0.22, & |z_{q2}(m,t)|>1.2,\end{cases}\quad a_{23}(z_{q2}(m,t))=\begin{cases}-0.53, & |z_{q2}(m,t)|\leqslant 1.2,\\ -0.62, & |z_{q2}(m,t)|>1.2,\end{cases}
$$

$$
a_{31}(z_{q3}(m,t))=\begin{cases}0.25, & |z_{q3}(m,t)|\leqslant 1.2,\\ -0.55, & |z_{q3}(m,t)|>1.2,\end{cases}\quad a_{32}(z_{q3}(m,t))=\begin{cases}0.43, & |z_{q3}(m,t)|\leqslant 1.2,\\ -0.27, & |z_{q3}(m,t)|>1.2,\end{cases}
$$

$$
a_{33}(z_{q3}(m,t))=\begin{cases}-0.21, & |z_{q3}(m,t)|\leqslant 1.2,\\ -0.46, & |z_{q3}(m,t)|>1.2,\end{cases}\quad d_{11}(z_{q1}(m,t))=\begin{cases}0.57, & |z_{q1}(m,t)|\leqslant 1.2,\\ -0.45, & |z_{q1}(m,t)|>1.2,\end{cases}
$$

$$
d_{12}(z_{q1}(m,t))=\begin{cases}-0.64, & |z_{q1}(m,t)|\leqslant 1.2,\\ 0.33, & |z_{q1}(m,t)|>1.2,\end{cases}\quad d_{13}(z_{q1}(m,t))=\begin{cases}-0.36, & |z_{q1}(m,t)|\leqslant 1.2,\\ 0.55, & |z_{q1}(m,t)|>1.2,\end{cases}
$$

$$d_{21}(z_{q2}(m,t))=\begin{cases}-0.33, & |z_{q2}(m,t)|\leqslant 1.2,\\ -0.15, & |z_{q2}(m,t)|>1.2,\end{cases}\quad d_{22}(z_{q2}(m,t))=\begin{cases}0.42, & |z_{q2}(m,t)|\leqslant 1.2,\\ -0.27, & |z_{q2}(m,t)|>1.2,\end{cases}$$

$$d_{23}(z_{q2}(m,t))=\begin{cases}-0.54, & |z_{q2}(m,t)|\leqslant 1.2,\\ 0.45, & |z_{q2}(m,t)|>1.2,\end{cases}\quad d_{31}(z_{q3}(m,t))=\begin{cases}0.29, & |z_{q3}(m,t)|\leqslant 1.2,\\ -0.35, & |z_{q3}(m,t)|>1.2,\end{cases}$$

$$d_{32}(z_{q3}(m,t))=\begin{cases}0.32, & |z_{q3}(m,t)|\leqslant 1.2,\\ -0.15, & |z_{q3}(m,t)|>1.2,\end{cases}\quad d_{33}(z_{q3}(m,t))=\begin{cases}-0.38, & |z_{q3}(m,t)|\leqslant 1.2,\\ 0.51, & |z_{q3}(m,t)|>1.2,\end{cases}$$

$$J=\begin{pmatrix}-0.7 & 0.1 & 0 & 0.3 & 0.2 & 0.1\\ 0.1 & -0.6 & 0.2 & 0.2 & 0.1 & 0\\ 0 & 0.2 & -0.4 & 0.2 & 0 & 0\\ 0.3 & 0.2 & 0.2 & -0.9 & 0.2 & 0\\ 0.2 & 0.1 & 0 & 0.2 & -0.7 & 0.2\\ 0.1 & 0 & 0 & 0 & 0.2 & -0.3\end{pmatrix}, H=\begin{pmatrix}0 & 0.02 & 0.03 & 0.06 & 0.04 & 0.05\\ 0.03 & 0 & 0.04 & 0.02 & 0.03 & 0.04\\ 0.05 & 0.06 & 0 & 0.01 & 0.02 & 0.03\\ 0.08 & 0.07 & 0.03 & 0 & 0.03 & 0.04\\ 0.06 & 0.02 & 0.03 & 0.02 & 0 & 0.01\\ 0.03 & 0.21 & 0.02 & 0.01 & 0.04 & 0\end{pmatrix}.$$

Hence,

$$\tilde{A}=\begin{pmatrix}0.53 & 0.73 & 0.45\\ 0.81 & 0.76 & 0.09\\ 0.8 & 0.7 & 0.25\end{pmatrix}, \tilde{D}=\begin{pmatrix}1.02 & 0.97 & 0.91\\ 0.18 & 0.69 & 0.99\\ 0.64 & 0.47 & 0.89\end{pmatrix}.$$

Case 1: Evidently, $z^*(m)=(0,0,0)^T, \phi_j=\breve{\phi}_j=0.5$ and $\psi_j=\breve{\psi}_j=0.25$. Take $\theta=0.15$ in the condition (4), it is easy to get the matrix P satisfying (9) as follows:

$$P=\begin{pmatrix}0.8342 & 0 & 0\\ 0 & 0.8106 & 0\\ 0 & 0 & 0.5779\end{pmatrix}.$$

According to Theorem 3.1, the network (39) achieves synchronization under the controller (2). Fig. 1 displays the evolutions of the error vector when the network (39) is synchronized.
Case 2: $\psi_j, \breve{\psi}_j, \phi_j, \breve{\phi}_j, z^*(m), \theta$ are similar as those in *Case 1*. $\omega_q(m,t)=(0.6q\sqrt{t}\cos(\pi m), 0.8q\sqrt{t}\cos(\pi m), 1.2q\sqrt{t}\cos(\pi m))^T$. In terms of exploiting MATLAB, the matrices P satisfying (26) with $\delta=1.2$ can be computed as follows:

$$P=\begin{pmatrix}1.5059 & 0 & 0\\ 0 & 1.5168 & 0\\ 0 & 0 & 1.4837\end{pmatrix}.$$

From Theorem 3.2, the network (39) with the disturbance attenuation level $\delta=1.2$ reaches event-triggered $\mathcal{H}_\infty$ synchronization. Fig. 2 displays the evolutions of $e_q(m,t), \omega_q(m,t)$ when the network (39) is $\mathcal{H}_\infty$ synchronized, and the corresponding instants of triggering event are shown in Fig. 3.

Example 5.2. The following CDRDMNNs with spatial diffusion coupling via event-triggered control is considered:

$$\begin{aligned}\frac{\partial z_q(m,t)}{\partial t}=&Y\frac{\partial^2 z_q(m,t)}{\partial m^2}-Kz_q(m,t)+I+A(z_q(m,t))f(z_q(m,t))+\gamma_q(m,t)+D(z_q(m,t))g(\overline{z_q(m,t)})\\ &+\zeta\sum_{p=1}^{6}G_{qp}\Upsilon\frac{\partial^2 z_p(m,t)}{\partial m^2}+v_q(m,t),\end{aligned}\tag{40}$$

in which $l=3, q=1,2,\cdots,6, \alpha=1,2,3, f_\alpha(\mu)=\frac{|\mu+1|-|\mu-1|}{4}, g_\alpha(\mu)=\frac{|\mu+1|-|\mu-1|}{8}, \Omega=\{m|-0.5<m<0.5\}; K=\mathrm{diag}(2,6,8), Y=\mathrm{diag}(0.8,0.5,0.7), I=(0,0,0)^T; \tau_j(t)=1-\frac{1}{j+3}e^{-t}, \varpi_j=\frac{1}{j+3}, j=1,2,3; \zeta=0.09, \Upsilon=\mathrm{diag}(0.4,0.8,$

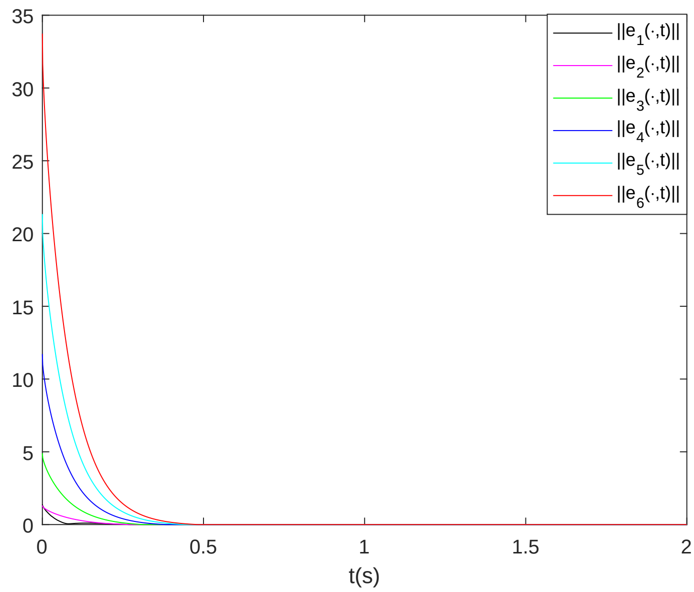

Figure 1: The evolutions of $e_q(m,t)$ in (39).

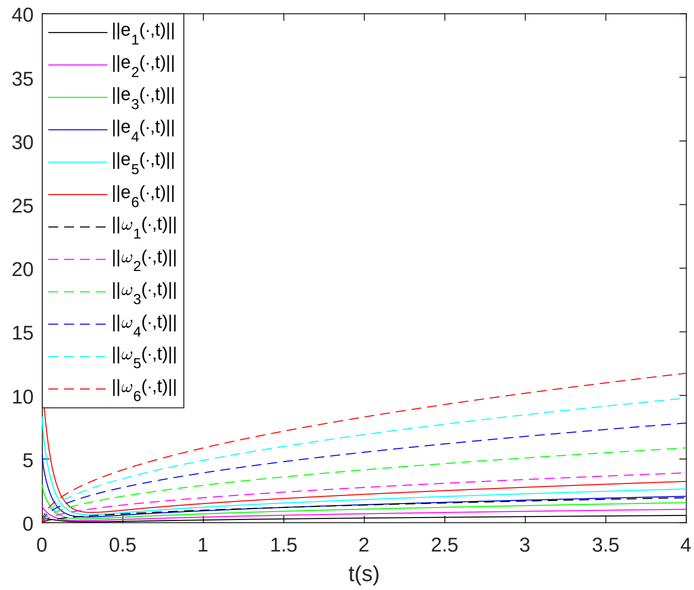

Figure 2: The evolutions of $e_q(m,t)$, $\omega_q(m,t)$ in (39).

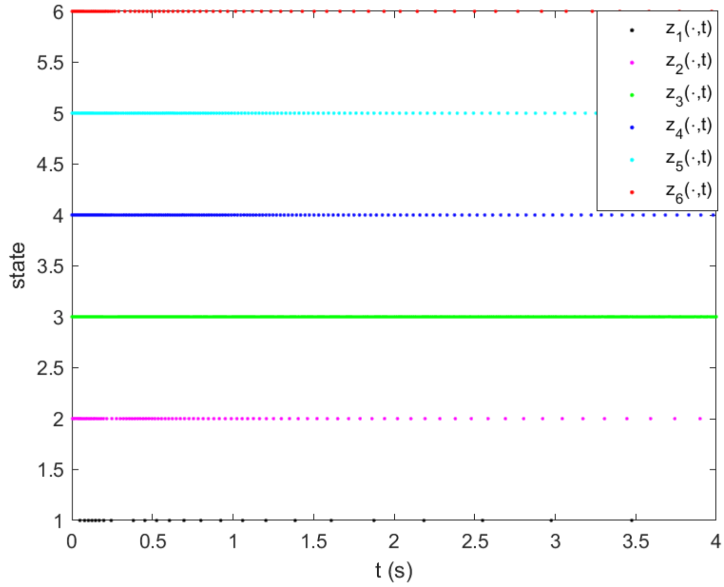

Figure 3: The change instants of triggering event of $z_q(m,t)$.

0.6); $v_q(m,t) = \varepsilon \sum_{p\in N_q} H_{qp}(z_q(m,t_k^q) - z_p(m,t_k^p))$; $\gamma_q(m,t) = -\mathrm{sign}(z_q(m,t) - z^*(m))(\tilde{A}\breve{\psi} + \tilde{D}\breve{\phi})$, $\varepsilon = 1.8$. The matrices $A(z_q(m,t)), D(z_q(m,t)), G = (G_{qp})_{6\times6}, H = (H_{qp})_{6\times6}$ are chosen, respectively:

$$a_{11}(z_{q1}(m,t)) = \begin{cases} -0.33, & |z_{q1}(m,t)| \leqslant 1.5, \\ 0.15, & |z_{q1}(m,t)| > 1.5, \end{cases} \quad a_{12}(z_{q1}(m,t)) = \begin{cases} -0.18, & |z_{q1}(m,t)| \leqslant 1.5, \\ 0.43, & |z_{q1}(m,t)| > 1.5, \end{cases}$$

$$a_{13}(z_{q1}(m,t)) = \begin{cases} 0.12, & |z_{q1}(m,t)| \leqslant 1.5, \\ 0.23, & |z_{q1}(m,t)| > 1.5, \end{cases} \quad a_{21}(z_{q2}(m,t)) = \begin{cases} -0.35, & |z_{q2}(m,t)| \leqslant 1.5, \\ 0.16, & |z_{q2}(m,t)| > 1.5, \end{cases}$$

$$a_{22}(z_{q2}(m,t)) = \begin{cases} -0.24, & |z_{q2}(m,t)| \leqslant 1.5, \\ 0.44, & |z_{q2}(m,t)| > 1.5, \end{cases} \quad a_{23}(z_{q2}(m,t)) = \begin{cases} -0.52, & |z_{q2}(m,t)| \leqslant 1.5, \\ 0.36, & |z_{q2}(m,t)| > 1.5, \end{cases}$$

$$a_{31}(z_{q3}(m,t)) = \begin{cases} 0.45, & |z_{q3}(m,t)| \leqslant 1.5, \\ -0.28, & |z_{q3}(m,t)| > 1.5, \end{cases} \quad a_{32}(z_{q3}(m,t)) = \begin{cases} 0.35, & |z_{q3}(m,t)| \leqslant 1.5, \\ -0.27, & |z_{q3}(m,t)| > 1.5, \end{cases}$$

$$a_{33}(z_{q3}(m,t)) = \begin{cases} 0.11, & |z_{q3}(m,t)| \leqslant 1.5, \\ -0.26, & |z_{q3}(m,t)| > 1.5, \end{cases} \quad d_{11}(z_{q1}(m,t)) = \begin{cases} 0.49, & |z_{q1}(m,t)| \leqslant 1.5, \\ -0.35, & |z_{q1}(m,t)| > 1.5, \end{cases}$$

$$d_{12}(z_{q1}(m,t)) = \begin{cases} -0.54, & |z_{q1}(m,t)| \leqslant 1.5, \\ 0.13, & |z_{q1}(m,t)| > 1.5, \end{cases} \quad d_{13}(z_{q1}(m,t)) = \begin{cases} -0.36, & |z_{q1}(m,t)| \leqslant 1.5, \\ 0.25, & |z_{q1}(m,t)| > 1.5, \end{cases}$$

$$d_{21}(z_{q2}(m,t)) = \begin{cases} 0.33, & |z_{q2}(m,t)| \leqslant 1.5, \\ -0.45, & |z_{q2}(m,t)| > 1.5, \end{cases} \quad d_{22}(z_{q2}(m,t)) = \begin{cases} 0.47, & |z_{q2}(m,t)| \leqslant 1.5, \\ -0.37, & |z_{q2}(m,t)| > 1.5, \end{cases}$$

$$d_{23}(z_{q2}(m,t)) = \begin{cases} 0.34, & |z_{q2}(m,t)| \leqslant 1.5, \\ -0.47, & |z_{q2}(m,t)| > 1.5, \end{cases} \quad d_{31}(z_{q3}(m,t)) = \begin{cases} -0.29, & |z_{q3}(m,t)| \leqslant 1.5, \\ 0.35, & |z_{q3}(m,t)| > 1.5, \end{cases}$$

$$d_{32}(z_{q3}(m,t)) = \begin{cases} 0.12, & |z_{q3}(m,t)| \leqslant 1.5, \\ -0.35, & |z_{q3}(m,t)| > 1.5, \end{cases} \quad d_{33}(z_{q3}(m,t)) = \begin{cases} 0.34, & |z_{q3}(m,t)| \leqslant 1.5, \\ -0.41, & |z_{q3}(m,t)| > 1.5, \end{cases}$$

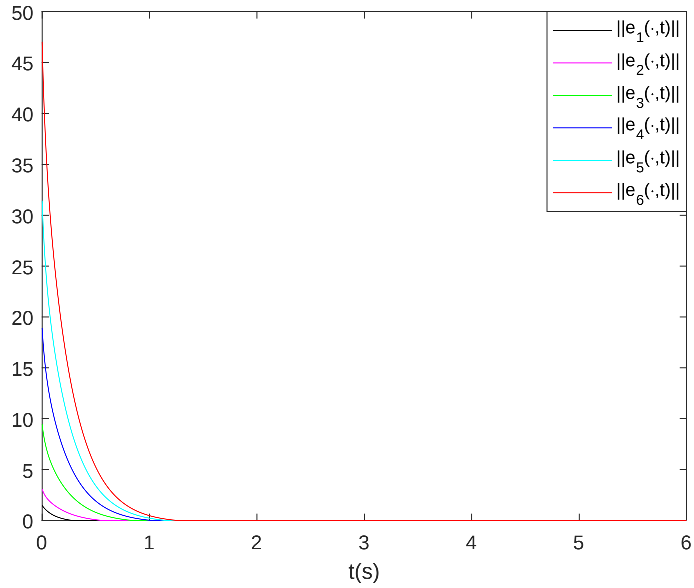

Figure 4: The evolutions of $e_q(m,t)$ in (40).

$$H=\begin{pmatrix} 0 & 0.002 & 0.002 & 0.003 & 0.04 & 0.005 \\ 0.003 & 0 & 0.005 & 0.002 & 0.003 & 0.004 \\ 0.001 & 0.06 & 0 & 0.01 & 0.02 & 0.03 \\ 0.003 & 0.007 & 0.002 & 0 & 0.003 & 0.004 \\ 0.004 & 0.002 & 0.003 & 0.004 & 0 & 0.001 \\ 0.01 & 0.02 & 0.003 & 0.002 & 0.005 & 0 \end{pmatrix}, G=\begin{pmatrix} -0.8 & 0.2 & 0.3 & 0.1 & 0.1 & 0.1 \\ 0.2 & -0.8 & 0.2 & 0.1 & 0.2 & 0.1 \\ 0.3 & 0.2 & -0.7 & 0 & 0 & 0.2 \\ 0.1 & 0.1 & 0 & -0.4 & 0.2 & 0 \\ 0.1 & 0.2 & 0 & 0.2 & -0.7 & 0.2 \\ 0.1 & 0.1 & 0.2 & 0 & 0.2 & -0.6 \end{pmatrix}.$$

Hence,

$$\tilde{A}=\begin{pmatrix} 0.48 & 0.61 & 0.11 \\ 0.51 & 0.68 & 0.88 \\ 0.73 & 0.62 & 0.37 \end{pmatrix}, \tilde{D}=\begin{pmatrix} 0.84 & 0.67 & 0.61 \\ 0.78 & 0.84 & 0.81 \\ 0.64 & 0.47 & 0.75 \end{pmatrix}.$$

Case 1: Evidently, $z^*(m)=(0,0,0)^T$, $\psi_j=\breve{\psi}_j=0.5$ and $\phi_j=\breve{\phi}_j=0.25$. Take $\theta=0.45$ in the condition (4), it is easy to calculate the following matrix P satisfying (30) and (31):

$$P=\begin{pmatrix} 0.6535 & 0 & 0 \\ 0 & 0.4427 & 0 \\ 0 & 0 & 0.3226 \end{pmatrix}.$$

On the basis of Theorem 4.1, the network (40) achieves event-triggered synchronization. Fig. 4 displays the evolutions of the error when network (40) is synchronized.

Case 2: $\psi_j, \breve{\psi}_j, \phi_j, \breve{\phi}_j, z^*(m), \theta$ are similar as those in *Case 1*. $\omega_q(m,t)=(1.2q\sqrt{t}\cos(\pi m), 1.8q\sqrt{t}\cos(\pi m), 1.6q\sqrt{t}\cos(\pi m))^T$. By means of employing MATLAB, the matrix P satisfying (36) and (37) with $\delta=3.6$ can be computed as follows:

$$P=\begin{pmatrix} 0.4068 & 0 & 0 \\ 0 & 0.2703 & 0 \\ 0 & 0 & 0.1994 \end{pmatrix}.$$

From Theorem 4.2, the network (40) with the disturbance attenuation level $\delta=3.6$ realizes event-triggered $\mathcal{H}_\infty$ synchronization. Fig. 5 displays the evolution of $e_q(m,t), \omega_q(m,t)$ when the network (40) is $\mathcal{H}_\infty$ synchronized, and the corresponding instants of triggering event are shown in Fig. 6.

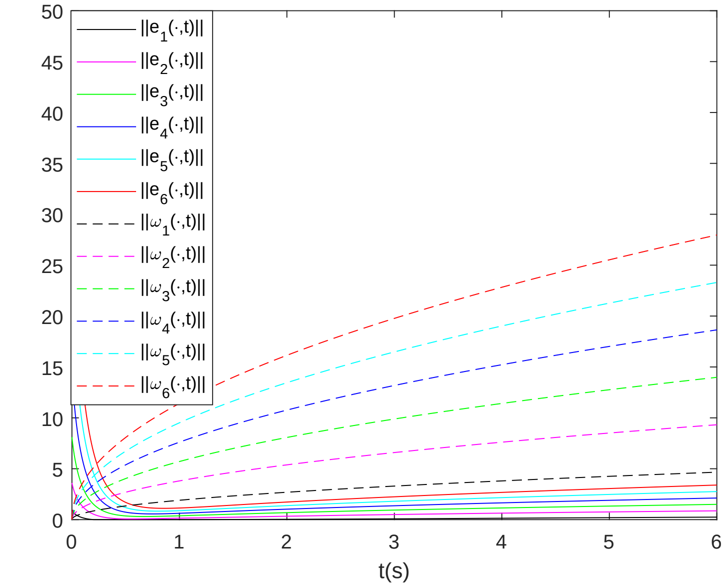

Figure 5: The evolutions of $e_q(m,t)$ in (40).

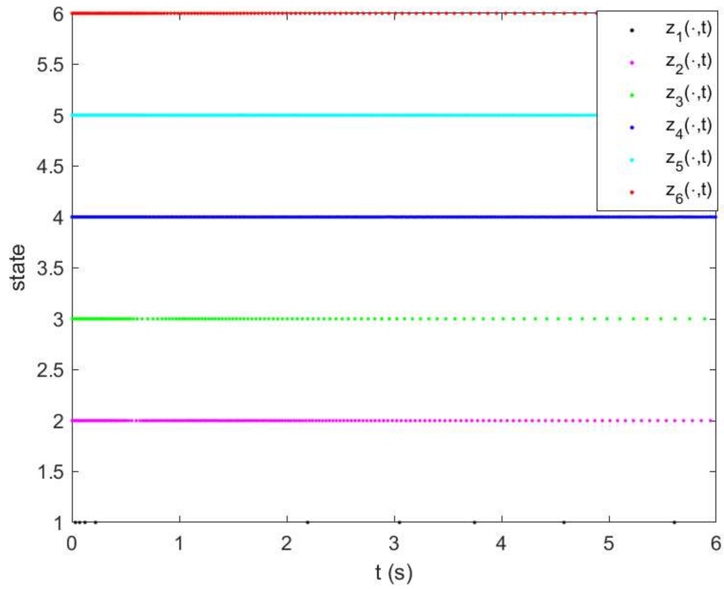

Figure 6: The change instants of triggering event of $z_q(m,t)$.

6. Conclusion

This paper has studied event-triggered synchronization and $\mathcal{H}_\infty$ synchronization of CDRDMNNs with state coupling and spatial diffusion coupling by means of exploiting the event-triggered controller. On the one hand, event-triggered synchronization and $\mathcal{H}_\infty$ synchronization criteria are derived for CDRDMNNs with state coupling on the basis of Lyapunov functional strategy. On the other hand, we also address the event-triggered synchronization and $\mathcal{H}_\infty$ synchronization problems for CDRDMNNNs with spatial diffusion coupling. In addition, two numerical examples have been given to confirm the effectiveness of the proposed event-triggered synchronization and $\mathcal{H}_\infty$ synchronization results.

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Data availability statement

Research data are not shared.

Declaration of interest

The authors declare that they have no conflict of interest.

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