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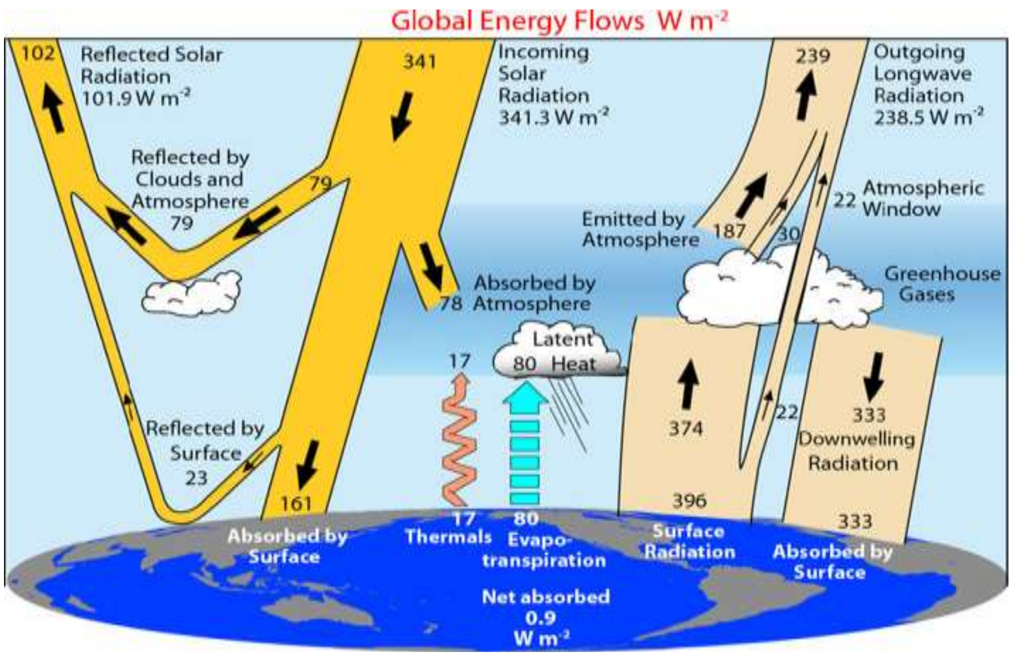
The Greenhouse Effect – Re-examination of the Impact of an Increase in Carbon Dioxide in the Atmosphere†

† Extract from Underwood, T. G. 2017. Solution of the Greenhouse Effect equations shows no increase in Earth's surface temperature from further increase in carbon dioxide. Unpublished.

Examination of the radiation budget at the surface of the Earth shows that there are five primary factors affecting the surface temperature; the amount of solar radiation absorbed by the atmosphere and by the surface respectively, the amount of heat emitted from the surface in the form of thermals and evaporation, and the proportion of infrared radiation emitted from the surface directly into space. The Greenhouse Effect equations are solved by calculating the downwelling flux from the atmosphere and substituting this in the equation for the radiative balance at the Earth's surface.



The global annual mean earth's energy budget for 2000–2005 (W m-2). The broad arrows indicate the schematic flow of energy in proportion to their importance. Adapted from Trenberth et al. (2009) (Fig. 1, Trenberth and Fasullo 2012).

Solution of the Greenhouse Effect equations

The Greenhouse Effect equations can be solved for the total infrared flux emitted by the Earth's surface, E_T, and consequently for the temperature at the surface, by calculating the downwelling flux from the atmosphere, E_D, and substituting this in the equation for the radiative balance at the

Earth's surface, $E_S = E_{TH} + E_{EV} + E_T – E_D$, where E_S is the solar energy absorbed by the surface, E_{TH} is the loss of energy from the surface through thermals, and E_{EV} is the loss of energy from the surface through evaporation.

The total downwelling infrared flux from the atmosphere $E_D = R_a + R_a f/2 + R_a f^2/4 + \ldots\ldots + R_a f^n/2^n = R_a + R_a r$, or $E_D = (1 + r)R_a$, where $R_a = (E_{TH} + E_{EV} + E_A + fE_S)/2$ is the initial downward infrared flux from the atmosphere, f is the proportion of terrestrial infrared radiation absorbed by the atmosphere, E_A is the solar energy absorbed by the atmosphere, and $r = f/2 + f^2/4 + \ldots\ldots + f^n/2^n$.

This assumes that the heat resulting from thermals and evaporation is absorbed by the atmosphere and, as with the solar radiation and terrestrial infrared radiation, changes the quantum state of the greenhouse gas molecules. On re-emission, half is radiated downward towards the Earth's surface, to be absorbed and re-emitted, and half radiated upward into the atmosphere. This is independent of the model assumed for the atmosphere, whether single or multi-layered.

$r = f/2 + f^2/4 + \ldots\ldots + f^n/2^n$ is a self-similar geometric series that can be summed by multiplying by the common ratio and subtracting. Multiplying by $f/2$ gives $rf/2 = f^2/4 + \ldots\ldots + f^n/2^n$, and subtracting from $r = f/2 + f^2/4 + \ldots\ldots + f^n/2^n$, gives $r – rf/2 = f/2$, or $r = f/(2 – f)$. Recognizing that $E_E = (1 – f)E_T$, where E_E is the total infrared radiation emitted from the Earth's surface that escapes directly into space, $E_E = E_T – f E_T$, so $f E_T = E_T – E_E$, or $f = (E_T – E_E)/ E_T$. Substituting values from the corrected Trenberth and Fasullo (2012) data in $f = (E_T – E_E)/ E_T$, gives $f = (396 – 22)/396 = 0.9444$. With $f = 0.9444$, $r = f/(2 – f) = 0.9444/1.0556 = 0.89$, which is effectively reached after seven iterations: $r = 0.9444/2 + 0.9444^2/4 + 0.9444^3/8 + 0.9444^4/16 + 0.9444^5/32 + \ldots\ldots + f^n/2^n$. Substituting $r = f/(2 – f)$ in $E_D = (1 + r)R_a$, gives $E_D = [1 + f/(2 – f)]R_a = [(2 – f + f)/(2 – f)]R_a$, so $E_D = 2R_a/(2 – f)$, and substituting $R_a = (E_{TH} + E_{EV} + E_A + fE_S)/2$ in $E_D = 2R_a/(2 – f)$, gives

$$\mathbf{E_D = (E_{TH} + E_{EV} + E_A + \mathit{f}E_S)/(2 – \mathit{f}).}$$

Substituting values from the corrected Trenberth and Fasullo (2012) data, gives $E_D = (17 + 80 + 78.2 + 0.9444 \text{ x } 160.2)/(2 – 0.9444)$, or $E_D = (17 + 80 + 78.2 + 151.3)/1.0556$, so $E_D = 326.5/1.0556 = 309.3\ W/m^2$, compared with Trenberth and Fasullo (2012)'s estimated residual of $333\ W/m^2$ and Wild et al. (2015)'s estimation of $342\ W/m^2$. This equation shows that the amount of the downwelling radiation, and consequently the infrared radiation from the surface, is highly dependent on the absorption of solar radiation by the atmosphere, and the existence of thermals and evaporation, in addition to the absorption of solar radiation by the Earth's surface.

Substituting $E_D = (E_{TH} + E_{EV} + E_A + fE_S)/(2 - f)$ into the equation for the radiative balance at Earth's surface, $E_S = E_{TH} + E_{EV} + E_T - E_D$, and expressing in terms of E_T, gives $E_T = E_S - E_{TH} - E_{EV} + E_D = E_S - E_{TH} - E_{EV} + (E_{TH} + E_{EV} + E_A + fE_S)/(2 - f)$, or $E_T(2 - f) = E_S(2 - f) - E_{TH}(2 - f) - E_{EV}(2 - f) + E_{TH} + E_{EV} + E_A + fE_S$, or $E_T(2 - f) = E_S(2 - f + f) - E_{TH}(1 - f) - E_{EV}(1 - f) + E_A$, which provides a solution of the Greenhouse Effect equations in terms of the surface temperature. At thermal equilibrium at the Earth's surface,

$$\mathbf{E_T = [2E_S - (1 - f)E_{TH} - (1 - f)E_{EV} + E_A]/(2 - f).}$$

Substituting values from the corrected Trenberth and Fasullo (2012) data,
$E_T = [(2 \text{ x } 160.2 - (1 - 0.9444) \text{ x } 17 - (1 - 0.9444) \text{ x } 80 + 78.2)/(2 - 0.9444)]$, or $E_T = (320.4 - 0.95 - 4.45 + 78.2)/1.0556 = 393.2/1.0556 = 372.5 \text{ W/m}^2$, compared with Trenberth and Fasullo (2012)'s value of 396 W/m^2. Under these assumptions and at this level of absorption, thermals and evaporation make only small contributions to the surface temperature, and the average global infrared radiation emitted by the Earth's surface at radiative equilibrium is approximately equal to twice the absorption of incoming solar radiation by the surface plus the incoming solar radiation absorbed by the atmosphere, divided by $(2 - f)$.

Substituting $E_T = 372.5 \text{ W/m}^2$ in Planck's formulation based on quantum mechanics of the Stefan-Boltzmann Law, $E = \sigma T^4$, where $\sigma = 2\pi^5 k^4/15c^2h^3 = 5.670373 \text{ x } 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$, results in $T^4 = E/\sigma = 372.5 \text{ x } 10^8/5.67 = 65.696649 \text{ x } 10^8$, and the average global surface temperature of the Earth, T = 284.70°K = 284.70 – 273.15 = 11.55°C, compared with 15.94°C based on Trenberth and Fasullo (2012)'s estimate for E_T of 396 W/m^2.

It should be remembered that this is an estimate of the Greenhouse Effect, the effect of absorption of greenhouse gases in the atmosphere on the surface temperature, not a complete estimate of the surface temperature. In the absence of an atmosphere, but allowing for an average reflection (albedo) from the Earth's surface of 6.8% (= 23.1/341.3), the temperature of the surface of the Earth would be given by $T^4 = E/\sigma = 340.3 \text{ x } (1 - 0.068) \text{ x } 10^8/5.67 = 55.936507 \text{ x } 10^8$, so T = 273.48°K = 273.48 – 273.15 = 0.3°C.

If $f = 1$, $E_T = [2E_S - (1 - f)E_{TH} - (1 - f)E_{EV} + E_A]/(2 - f)$ becomes $E_T = 2E_S + E_A$, independent of E_{TH} and E_{EV}. This states that with complete absorption by the atmosphere, the infrared flux emitted by the Earth's surface equals twice the solar radiation absorbed by the surface plus the solar radiation absorbed by the atmosphere, showing that the cycle of emissions and re-radiations effectively doubles the downward flux from the atmosphere. Substituting values from the corrected Trenberth and Fasullo (2012) data in $E_T = 2E_S + E_A$, gives $E_T = 2 \times 160.2 + 78.2 = 398.6$ W/m^2, corresponding to a maximum surface temperature under these assumptions of 16.4°C.

Substituting formulae for the fluxes in the Greenhouse Effect equation, $E_T = [2E_S – (1 – f)E_{TH} – (1 – f)E_{EV} + E_A]/(2 – f)$, gives $\sigma T^4 = [2(1 – \alpha_p – \alpha_a)S_0/4 – (1 – f)(E_{TH} + E_{EV}) + \alpha_a S_0/4]/(2 – f)$, or

$$\boldsymbol{\sigma T^4 = [(2 – 2\alpha_p – \alpha_a)S_0/4 – (1 – f)(E_{TH} + E_{EV})]/(2 – f)},$$

where α_p is the proportion of incoming solar radiation reflected back into space by the clouds and Earth's surface (the albedo), α_a is the proportion of incoming solar radiation absorbed by the atmosphere, and S_0 is the solar flux at the top of the atmosphere. From this it can be seen that the surface temperature is increased by a reduction in solar reflexivity or a reduction in solar atmospheric absorptivity as well as by an increase in the atmospheric infrared absorption coefficient; and, of course, is also increased by an increase in solar irradiance.

An increase in the infrared absorption coefficient, *f*, involves increasing the internal energy of the greenhouse gas molecule by changing its quantum state, which can only occur at particular wavelengths known as absorption bands. However, as seen above, *f* is currently around 0.9444, which suggests that at the current surface temperature all of the radiation within the emission bands is fully absorbed, and that the remaining 5.56 percent of the infrared emission represents radiation with wavelengths within the atmospheric window. If this is true, there can be no further increase in *f*, and no increase in the surface temperature with an increase in carbon dioxide, and any increase in surface temperature will depend on an increase in solar irradiance. After losing a critical instrument when NASA's Glory satellite failed to achieve orbit in 2011, it will now be another 20 years before we have enough data to be sure.