# Supporting Information for "Eddy induced trapping and homogenization of freshwater in the Bay of Bengal"

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**Text S1. Attracting Lagrangian Coherent Structures (a-LCSs or Backward Finite Time Lyapunov Exponents (b-FTLEs)**: The mixing of freshwater is characterized from a Lagrangian perspective via so-called backward Finite Time Lyapunov Exponents (b-FTLEs) (Wiggins, 2005). These are ridges that represent attracting Lagrangian coherent structures in a flow (Haller, 2002). To compute the b-FTLEs, we first advect fluid parcel by integrating the following equations backward in time,

$$\frac{d\phi}{dt} = \frac{u(\phi, \lambda, t)}{R\cos(\lambda)}, \frac{d\lambda}{dt} = \frac{v(\phi, \lambda, t)}{R}. \tag{1}$$

Here, $\phi$, $\lambda$, $u$ and $v$ are the latitude, longitude, zonal and meridional velocity, respectively. $R$ is the radius of the earth. The time span is $t = t_0$ to $t = t_o - \tau$ and the numerical method employed is the 4th order Runge-Kutta scheme. The velocity data $(u, v)$ is given on a fixed grid and the flow has been interpolated by a bilinear interpolation scheme. We then compute the right Cauchy-Green Lagrange tensor $C_{t_0}^t$ associated with the flow map $F_{t_0}^t(\mathbf{x_0})$, which is defined as,

$$C_{t_0}^t(\boldsymbol{x_0}) = \left(\nabla F_{t_0}^t(\boldsymbol{x_0})\right)^T \nabla F_{t_0}^t(\boldsymbol{x_0}). \tag{2}$$

$F_{t_0}^t(\boldsymbol{x_0})$ denotes the position of a parcel at time $t$ backward in time, advected by the flow from an initial time and position $(t_0, \boldsymbol{x_0})$. $C_{t_0}^t(\boldsymbol{x_0})$ is symmetric and positive definite, its eigenvalues ($\lambda' s$) and eigenvectors ($\xi' s$) can be written as,

$$C_{t_0}^t(\boldsymbol{x_0}) = \lambda_i \xi_i,\ 0 < \lambda_1 \leq \lambda_2, i = 1, 2. \tag{3}$$

The gradient of the flow map $\nabla F_{t_0}^t(\boldsymbol{x_0})$ is computed using an auxiliary grid about the reference point (Onu et al., 2015), and can be written as,

$$\nabla F_{t_0}^t(\boldsymbol{x_0}) \approx \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix}, \tag{4}$$

where,

$$\alpha_{i,j} \equiv \frac{x_i(t; t_0, x_0 + \delta x_j) - x_i(t; t_0, x_0 - \delta x_j)}{2|\delta x_j|}. \tag{5}$$

Finally, the largest b-FTLE (Haller, 2002; Haller & Sapsis, 2011; Mathur et al., 2019) associated with the trajectory $\boldsymbol{x}(t, t_0, \boldsymbol{x_0})$ over the time interval [$t_0$, $t$] is defined as,

$$\lambda_\tau(\boldsymbol{x_0}) = -\frac{1}{|t-t_0|}\log\left(\sqrt{\lambda_{\max}[C_{t_0}^t(\boldsymbol{x_0})]}\right). \tag{6}$$

The backward integration time $|\tau| = |t - t_0|$ has been taken as 20 days and computed on a finer grid resolution $0.01^\circ \times 0.01^\circ$.

## References


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