On the Performance of a Real-Time Electron Radiation Belt Specification Model

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Key Points

- Real-time data-assimilative hindcasts of the outer electron radiation belt were accurate to within a factor of 1.5.Data assimilation substantially improved the error and bias of radiation belt specification but strongly influenced hindcast error and bias.
- Improved physics-based modeling and continuous real-time observations through the outer radiation belt are needed for accurate hindcasts.

Abstract

Maintaining accurate real-time hindcast and forecast specification of the radiation environment is essential for operators to monitor and mitigate the effects of hazardous radiation on satellite components. The Radiation Belt Forecasting Model and Framework (RBFMF) provides real-time forecasts and hindcasts of the electron radiation belt environment, which are used as inputs for the Satellite Charging Assessment Tool (SatCAT). We evaluated the long-term statistical error and bias of the RBFMF by comparing the 10-hour hindcast of electron phase space densities (PSD) to a multi-mission dataset of PSD observations. We found that, between the years 2016-2018, the RBFMF reproduced the radiation belt environment to within a factor of 1.5. While the error and bias of assimilated observations were found to influence the error and bias of the hindcast, data assimilation resulted in more accurate specification of the radiation belt state than real-time Van Allen Probe observations alone. Furthermore, when real time Van Allen Probe observations were no longer available, the hindcast errors increased by an order of magnitude. This highlights two needs; (i) the development of physics-based modelling incorporated into this framework, and (ii) the need for real-time observations which span the entire outer radiation belt.

Plain Language Summary

It is important to accurately predict and monitor the radiation levels in space to safeguard satellites from potential damage. This paper introduces a model called the Radiation Belt Forecasting Model and Framework (RBFMF), which provides real-time forecasts and historical data (hindcasts) of radiation levels. To test the model's accuracy, we compared its predictions to actual observations from satellites between 2016 and 2018. We found that generally, the model's predictions of the outer radiation belt were within 1.5 times of the actual measurements. Additionally, we discovered that incorporating both real-time satellite observations and physics-based simulation improved prediction accuracy compared to relying solely on either method. However, we noticed a significant increase in prediction errors when real-time observations through the heart of the Van Allen radiation belt were unavailable. This underscores the importance of enhancing the model with physics-based modeling and ensuring continuous real-time observations of radiation levels in space.

1 Introduction

The variability of the near-Earth radiation environment poses a serious hazard to telecommunications, navigational, and defense satellites which orbit through these regions. A key effect of spacecraft exposure to radiation is internal charging, where high energy electrons penetrate the surface shielding of a satellite and deposit charge into dielectric materials such as circuit boards or cable insulators, and on ungrounded material such as spot shields or connector contacts (e.g., Fennell et al., 2001; Lohmeyer et al., 2015). The accumulated charging eventually results in electrostatic discharges, which is one of the known causes of "satellite anomalies". Anomalies range in effect from more frequent temporary errors in non-critical systems to rare but catastrophic hardware damage and complete mission failure (Galvan et al., 2014) and can be instigated by a number of issues, such as command errors, manufacture flaws, and environmental exposures. Of all space environment effects on satellites, electrostatic discharges from internal charging have been reported to cause the most anomalies (Green et al., 2017; Koons et al., 1999).

Improved on-orbit anomaly detection tools have been cited by satellite operators as an industrial need for operators to easily and quickly attribute anomalies to space weather effects (Green et al., 2017). To attribute satellite anomalies to internal charging of equipment requires forensic reconstruction of the radiation environment and modelling of charge accumulation on component hardware, such as satellite shielding materials, for defined orbits (LEO, MEO, GEO). The Satellite Charging Assessment Tool (SatCAT) is one such tool which allows users to monitor the real time and long-term effects of internal charging due to fluence of radiation belt electrons. This paper will evaluate the Radiation Belt Forecasting Model and Framework (RBFMF) which SatCAT uses to specify the radiation environment both retrospectively and in real-time.

Several models have been developed to model the near-Earth radiation environment based upon the Van Allen radiation belt response to solar wind parameters and/or geomagnetic indices. These models usually fall within three categories; empirically based analytical descriptions (e.g., X. Li, 2004; Nagai, 1988; Roeder et al., 2005; Turner & Li, 2011), physics-based models (e.g., Horne et al., 2013; Subbotin & Shprits, 2009), and machine learning models (e.g., Boyd et al., 2023; Chen et al., 2019; Wei et al., 2018). Statistical approaches to modelling the radiation belt have shown success at identifying the key variables which influence radiation belt flux on different timescales. However, statistical approaches do not capture some extreme events, and could be flawed if they cannot reflect the non-linear relationship between input variables are output electron fluxes. While physics-based modelling allows the study of relative contributions of complex acceleration and loss mechanisms in the outer radiation belt, the computational power required to accurately model the interconnected magnetospheric system on an hourly basis provides a significant challenge. Machine learning models have predicted ultra-relativistic electron fluxes with high efficacy but show less accuracy at predicting lower energy fluxes (Camporeale, 2019; Shin et al., 2016) but, similarly to statistical modelling, could have limited capabilities for extreme event which are out of sample to training data.

By combining physics-based modelling with data assimilative techniques, RBFMF provides a lightweight and robust tool for real time radiation belt forecasting. This framework uses 1D Kalman filtering technique which has previously been shown through event-based studies to be a successful tool for reproducing the radiation belts (e.g., Coleman et al., 2018; Daae et al., 2011; Kellerman et al., 2014). This work will investigate the performance of this forecasting methodology by completing a detailed statistical evaluation of RBFMF over archived multi-year dataset of hindcasts. We aim to assess how data assimilation affects the error and bias of radiation belt forecasts, and the influence of variable geomagnetic conditions. In this way, we will identify how the RBFMF may be adapted in the future for improved hindcast reliability.

2 Radiation Belt Forecasting Model and Framework

The radiation belt forecast framework (RBFMF) is designed to combine physics-based modelling with data assimilative techniques to provide forecasts, nowcasts, and hindcasts of the radiation environment in real-time. This framework is based upon an existing data assimilative models developed by Kellerman et al. (2014) and Shprits et al. (2013), and has been further developed to provide a more robust forecasting infrastructure which provides data products which are integrated into the SatCAT tool. The implementation of this framework is summarized in Figure 1.

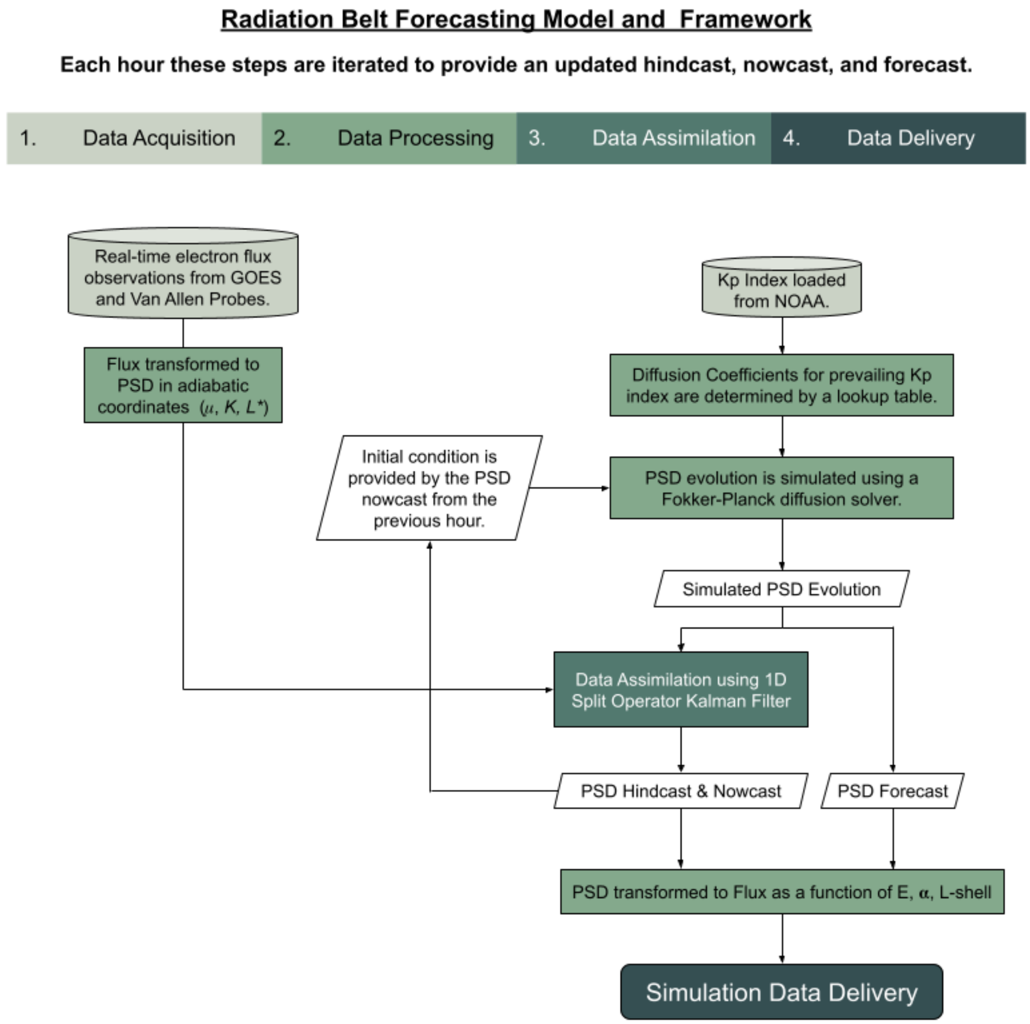

Figure 1 Flow chart summarizing the steps completed each hour in the RBFMF.

Diffusive Modelling

Radiation belt diffusion is modelled through implementation of the 3-D Fokker-Plank diffusion equation for radiation, Equation 1, (Roederer, 1970; Schulz & Lanzerotti, 1974; Walt, 1994).

$$\frac{\partial f}{\partial t} = \sum_{i,j} \frac{\partial}{\partial J_i}\left(D_{J_iJ_j}\frac{\partial f}{\partial J_j}\right) - Losses \qquad \text{Equation 1}$$

Where f is the phase averaged electron phase space density (PSD), J_i, J_j, represent the first, second, and third adiabatic invariants of adiabatic motion (μ, J, Φ respectively), and diffusion coefficients $D_{J_iJ_j} = \langle \Delta J_i \Delta J_j \rangle / (2\Delta t)$ which denote the scattering rates ($D_{\mu\mu}, D_{JJ}, D_{\Phi\Phi}$). In this paper we will use K ($\propto J$ assuming μ is conserved) as the second adiabatic invariant and L^* ($\propto 1/\Phi$) as the third adiabatic invariant (noting that $L^* = L$ in a dipolar magnetic field). The Versatile Electron Radiation Belt (VERB) code (Subbotin & Shprits, 2009) is used to implement a solution to this equation using precomputed diffusion coefficients.

Diffusion Coefficients

By precomputing diffusion matrices, the diffusion equation can be solved quickly at each time step by selecting diffusion coefficients for the prevailing *Kp* level. Mixed local diffusion terms are excluded, which enables larger grid steps. Three types of waves were used to derive the diffusion coefficient matrices; ULF waves (Brautigam & Albert, 2000), lower-band chorus (W. Li et al., 2007; Shprits et al., 2007) and plasmaspheric hiss (Spasojevic et al., 2015). The plasma density was obtained from (Sheeley et al., 2001) for diffusion coefficient computation. The diffusion coefficients were computed using the Full Diffusion Code at UCLA.

Initial and Boundary Conditions

Diffusion is solved for a dipolar magnetic field with a grid covering L-shells 1 to 7, energies 10 keV to 10 MeV, and pitch angles 0.3° - 89.7°. The upper boundary condition in energy is a Dirichlet boundary with constant $f = 0$, the lower-boundary condition is also Dirichlet for each time step, although updated by assimilation. The lower boundary condition in pitch angle is a Neumann boundary $\partial f / \partial \alpha = 0$, to allow for both weak and strong diffusion effects to be simulated. The upper boundary condition in L utilizes both Dirichlet and Neumann boundaries, depending on the last closed drift shell (LCDS) derived from Tsyganenko (1989) with a centered dipole (see Kellerman, 2018). The lower boundary in L is a Dirichlet boundary, with $f = 0$ at L_{min}, representing loss to the atmosphere.

The model was initialized through the average PSD observed by spacecraft from the previous month. For each additional forecast going forward through several years, the initial condition of each time step is set to the nowcast simulated for the previous hour.

Data Assimilation

Data assimilation uses filtering algorithms to estimate the state of a system using joint probability distributions from a simulated system and sparsely observed data. The Kalman Filter (Kalman, 1960) is a widely used data assimilation algorithm which estimates the system state by minimizing the mean square errors of the simulated state variables and observed state variables. Readers are referred to past works for detailed descriptions the Kalman filter to radiation belt analysis (e.g., Kondrashov et al., 2007; Naehr & Toffoletto, 2005; Ni et al., 2009; Shprits et al., 2007). Because the computational requirements of a 3D Kalman filter are very large, making it impractical for real time applications, the RBFMF instead employs a one-dimensional split-operator Kalman filter. This methodology has been validated in a synthetic-forecast analysis over multiple years (Kellerman, 2018). Due to several unknown errors in the model and real-time observational datasets, the errors were set equal for data and model.

2.1 Observations

Figure 2 shows how the modelled radiation belt PSD compares to the multi-mission PSD observations (described further in Section 3). The nowcast time is indicated in Figure 2 by the white line, with the preceding eight days showing hindcast PSD, and the following two days show the PSD forecast. By comparing the modelled PSD in Figure 2a to the measured PSD in Figure 2b, we can see that the RBFMF is highly successful at reproducing the structure and dynamical evolution of the radiation belt in this example. The magnitude of the radiation belt flux is well captured overall, although when the observed PSD became enhanced up to ~ 15 x 10^{-5} $(c/cm/MeV)^3$ at $5 < L^* < 6$ between 6-9th August, the hindcast PSD appeared to be slightly underestimated. Figure 2c shows that the percentage error was < 100% for most of the interval (i.e., PSD was estimated within a factor of 1 during the interval), but became largest (> 200%) near the outer boundary ($L^* > 6$) as electrons were lost on 3rd August, and along the inner edge of the radiation belt ($4 < L^* < 5$) during the enhancement between 4th to 7th August.

While the interval in Figure 2 appears to show that the RBFMF is a good representation of the outer radiation belt, we must quantitively evaluate the performance of the model to determine if it is accurate over long time periods, and identify when the simulation is inaccurate so that it can be improved. We will test the RBFMF against the observed state of the electron radiation belt for two time periods between 2016 - 2018 and between 2019 - 2020. The key difference between these years is than between 2016-2018 both GOES and Van Allen Probe data were assimilated into the hindcast in real time, whereas from 2019 onwards, only GOES data was assimilated in real time.

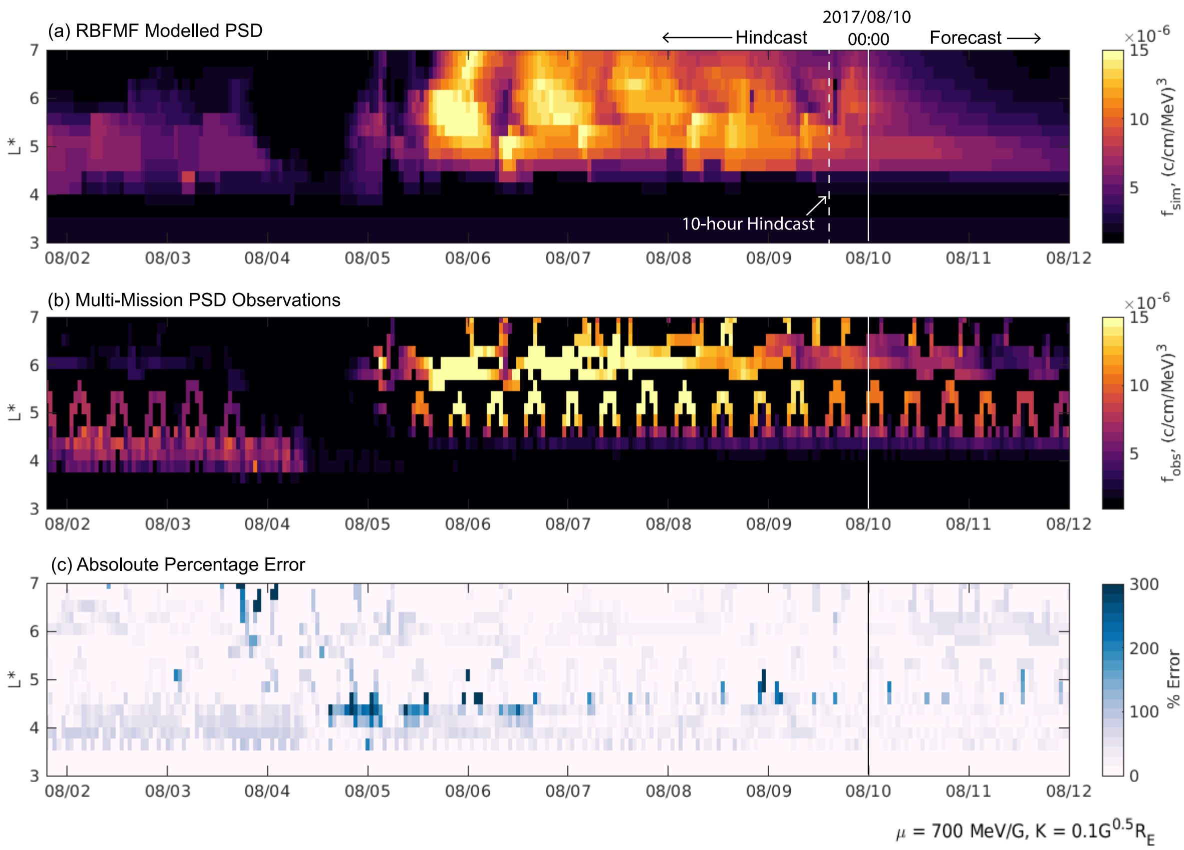

Figure 2 The simulated state of the radiation belt, f_{sim}, at 00 UT on 10th August 2017 is displayed in panel (a) as a function of L^* over time, for μ = 700 MeV/G and K = 0.1 G$^{0.5}$R$_E$. Panel (b) shows corresponding PSD observations, f_{obs}, taken by multiple missions. Panel (c) shows the absolute percentage error of the simulated PSD:$|f_{obs} - f_{sim}/f_{sim}| \times 100$.

3 Validation Method

In this analysis, the RBFMF will be validated against a multi-mission dataset of radiation belt observations (described in Section 3.1), which serves as the 'true' state of the radiation belt. The error and bias of the radiation belt hindcast of electron phase space density (PSD) is analyzed in adiabatic coordinates. To do this, the observed PSD (described in Sections 3.1) is converted onto the same resolution coordinate grid as the simulated PSD by interpolating across the first and second adiabatic invariants, μ and K, then averaging PSD into L^* bins on an hourly basis. The quotient ($Q_i = f_{sim}/f_{obs}$) of each simulation data point can then be determined if there is a corresponding PSD observation.

The statistical error and bias of the 10-hour hindcast is calculated between January 2016 – October 2019 and between March 2019 – December 2020. Special focus is given to the 10-hour hindcast (dashed line, Figure 2a) because this is the most frequently used by satellite operators for anomaly attribution. Error and bias are quantified using symmetric metrics described by Morley et al. (2018), which account for variable electron PSD magnitudes. The median symmetric accuracy (MSA) is described in Equation 2 and the symmetric signed percentage bias (SSPB) is described in Equation 3, where $\mathrm{M}()$ symbolizes the median calculation. In this scheme, the MSA is small if simulation errors are low, SSPB < 0 if the hindcast is biased towards underprediction of PSD, and SSPB > 0 if the hindcast is biased towards overprediction of PSD.

$$MSA = 100\,(\exp\,(\,\mathrm{M}(|\log_e(Q_i)\,|) - 1\,) \qquad \text{Equation 2}$$

$$SSPB = 100\,\mathrm{sgn}(M(\log_e(Q_i)))(\exp\,(\,\mathrm{M}(|\log_e(Q_i)\,|) - 1\,) \qquad \text{Equation 3}$$

3.1 Multi-Mission PSD Observations

PSD observations are taken from 32 individual satellites which are part of 5 different scientific missions and hosted payloads (see Staples et al., 2022; Staples et al., 2023 for usage of this dataset):

- Van Allen Probe Magnetic Electron Ion Spectrometer (MagEIS) and Relativistic Electron-Proton Telescope (REPT) instruments (Baker et al., 2014; Blake et al., 2014).

- GOES 13 and15 (Geostationary Operational Environmental Satellite) Magnetospheric Electron Detector (MAGED) Energetic Proton, Electron, and Alpha Detector (EPEAD) (Rodriguez, 2014a, 2014b; Sillanpää et al., 2017).
- GPS (Global Positioning System) Navstar Combined X-ray dosimeter (CXD) (Morley et al., 2017; Tuszewski et al., 2004).
- THEMIS (Time History of Events and Macroscale Interactions during Substorms) Electrostatic Analyzer (ESA) and Solid-State Telescope (SST) (Angelopoulos, 2008; Angelopoulos et al., 2008; McFadden et al., 2008).
- MMS (Magnetospheric Multiscale) Fly's Eye Electron Proton Spectrometer (FEEPS) (Blake et al., 2016; Burch et al., 2016).

All spacecraft data is calibrated to Van Allen Probe B and bias-corrected GOES 15 data, which are chosen as the "gold standards" for the calibration. The GPS pitch angle distributions are assumed using the Zhao et al. (2018) model. For each spacecraft instrument, the adiabatic invariants μ, K, and L^* are computed using a model magnetospheric field, represented by the International Geomagnetic Reference Field model (IGRF; Thébault et al., 2015) and Tsyganenko (1989) external magnetic field model (T89). The final PSD dataset is interpolated across dimensions of μ and K, but not across L^* or time.

4 Validation Results

4.1 Original Forecast Framework (2016-2018)

Figure 3 shows the MSA, SSPB, and number of samples, N, calculated for the 10-hour hindcast of radiation belt PSD as a function of the first and second adiabatic invariants, μ and K at three sampled L^*. The L^* values shown in Figure 3 are selected to represent trends in error and bias observed in the slot region (~ L^* = 2.44), the core of the outer radiation belt, and the outer radiation belt near medium earth orbit (~ L^* = 4.12) and at geostationary orbit (~ L^* = 6.04).

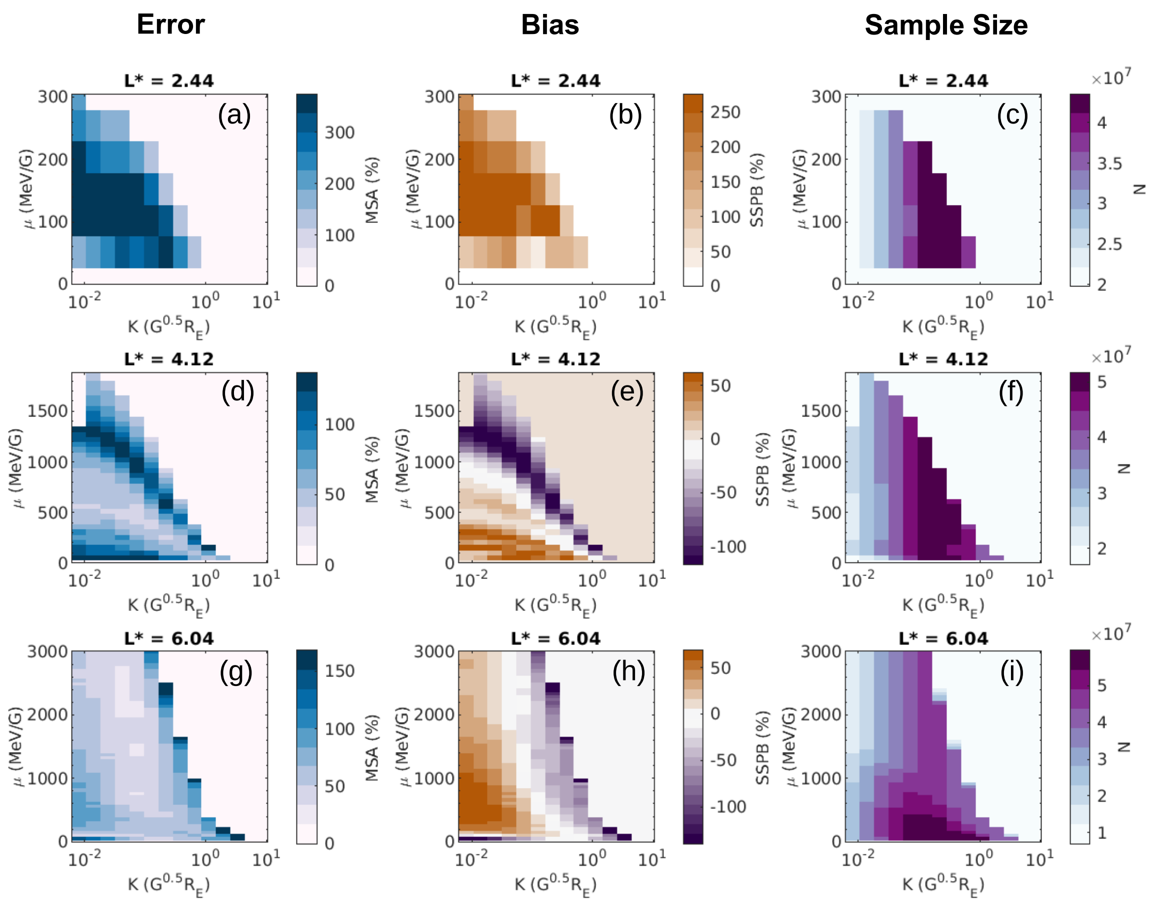

Figure 3 Left column shows the statistical error (MSA) of the 10-hour hindcast between 2016-2018, shown by color as a function of μ and K for a sample of simulated L^* = 2.44, 4.12, and 6.04. The middle column shows statistical bias (SSPB) in the same format, and right column shows the number of samples N used in the computation of MSA and SSPB.

We observe a large variance in the MSA and SSBP across the different L^* regions. At L^* = 2.44 the maximum percentage error was 350%, and the maximum bias was 250%. At L^* = 2.44, electrons are generally located inside the overlapping plasmasphere and slot region, so the large positive bias indicates that the model systematically overestimated the PSD in the slot region. It is because electron PSD is generally very low at L^* < 4, that comparatively small absolute differences in PSD create large errors and bias relative to the measured PSD.

In the outer radiation belt region (L* > 4) the error and bias were much smaller than the slot region, reaching a maximum of 150% percentage error and -150% bias. This indicates that statistically the hindcast accurately predicted the outer radiation belt PSD to within a factor of 1.5. Moreover, the hindcast error and bias was highly dependent upon μ and K.

At L^* = 4.12 the greatest errors were found to reach 150% corresponding to the highest measured μ values depending on K (i.e., highest energies, ~ multi-MeV) and was correspondingly biased toward underestimation of PSD by ~120%. At lower μ values (which correspond to the bulk of the radiation belt plasma population) the hindcast showed a statistical overestimation of PSD by 70% or less, showing that the hindcast predicted the core outer belt PSD to within a factor of 0.7. The hindcast error and bias showed similar trends at L^* = 6.04: The highest errors were observed at the highest measured μ (highest energies) with a negative bias > -100%, and at lower μ the hindcast overestimated PSD by up to 70%. In addition to these error and bias relationships, at L^* = 6.04 there was a strong error of 150% and bias of -150% observed at μ < 100 MeV/G. This population corresponds to lower energy < 300 keV electron populations. To ensure that error and bias is not dependent upon hindcast hour, the same analysis was completed for different hindcast hour, and no appreciable differences were observed.

The following analysis discusses the source of the statistical hindcast error and bias and evaluates the accuracy of the hindcast under different geomagnetic conditions.

4.1.1 Assimilated Data Bias

One potential source of error in the model hindcast is assimilated data bias. Between 2016-2018 the assimilated datasets were real time GOES data, and the Van Allen Probe Beacon data. Because the assimilated GOES data is similar to the final science data product, it is not informative to evaluate the bias of this data. The Van Allen Probe beacon data was provided in real time over the Van Allen Probe mission without the same corrections and post-processing as the long-term science data archive, and so is possibly more prone to errors. To assess if the error and bias of assimilated PSD observations influenced the hindcast PSD, we will evaluate the error and bias of the Van Allen Probe Beacon data (Figure 4) and compare the hindcast error and bias (Figure 3). The error and bias of Van Allen Probe beacon data is quantified by calculating the MSA and SSBP respectively using Equation 2 and Equation 3, where the quotient is calculated as $Q = PSD_{Beacon}/PSD_{Final}$, and PSD_{Final} is the science data product from the same Van Allen Probe. This analysis effectively shows the differences between the real time data product and the more highly processed science data as a function of *μ, K,* and *L**.

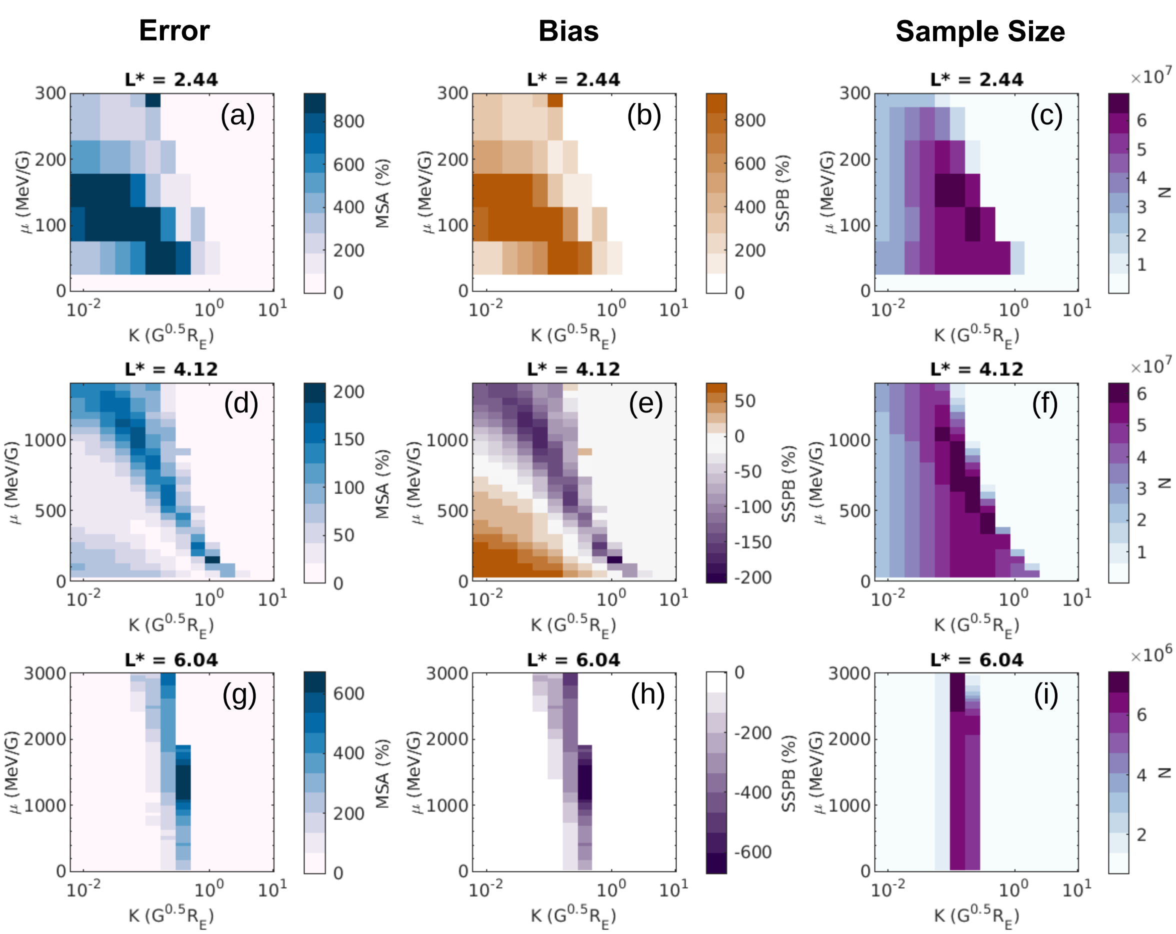

Figure 4 Left column shows the statistical error (MSA) and middle column shows the statistical bias (SSPB) of the Van Allen Probe B beacon data between 2016-2018, compared to the final Van Allen Probe science data. Right column shows the number of data samples N used in the computation of MSA and SSPB. Error, bias, and sample size are all shown by color as a function of *μ* and *K* at sampled bins of *L = 2.44; 4.12; 6.04 which were chosen to match the hindcast model grid.**

Figure 4 shows that universally (i.e., all *L**, *μ*, and *K*), the error and bias for Van Allen Probe beacon data was greater than the hindcast error and bias (Figure 3). For example, at *L** = 2.44, both the error and bias of beacon data reached > 800%, whereas the maximum hindcast error and bias were ~ 350% and ~250% respectively. The magnitude difference in error and bias was less significant at L^* = 4.12, which reached a maximum of 200% error and -200 % respectively.

Despite differences in magnitude, we highlight that the error and bias observed for Van Allen Probe beacon data (Figure 4) was nearly identically distributed across μ, K, and L^* compared to the 10-hour hindcast error and bias. For example, at L^* = 4.12, the maximum errors were at the highest μ values across all K, and were negatively biased, transitioning to a positive bias at low μ values. This observation shows that the assimilated data significantly influenced the error and bias of the hindcast. Though, because the magnitude of the hindcast error and bias was less than the assimilated data, the simulated hindcast gave a significantly improved estimation of radiation belt PSD than if Van Allen Probe beacon data was used alone.

4.1.2 Storm-time Error Analysis

Knowing how well the model captures the radiation belt evolution during different geomagnetic conditions is of particular interest because most impacts to satellites are observed during geomagnetically active periods. To analyze how the statistical error and bias varies with geomagnetic conditions, we extract geomagnetic storm intervals between 2016-2018 based upon Sym-H index evolution. A storm is identified by time periods where Sym-H decreased below -40 nT. The main storm phase is classified from intervals when Sym-H decreased from a value above 15 nT, to a Sym-H minimum below -40 nT. The recovery storm phase is classified from when the Sym-H increased after the main storm phase, until it reached a value above -15 nT. We further select times during geomagnetic storms when the AE index was in the upper 80% percentile of data. High AE index periods are known to be associated with substorm injections of lower energy source electrons from the magnetotail into the inner magnetosphere, and AE index values exceeding 150 nT have previously been used as a proxy for substorm injections (e.g., Meredith et al., 2000). As before, we calculate the statistical error and bias of each storm phase using Equation 2 and Equation 3. Figure 5 and Figure 6 show how the computed error and bias varied under different geomagnetic conditions at L^* = 4.12 and L^* = 6.04, respectively. To easily compare between geomagnetic conditions, the color bars have been saturated to $\pm$ 200%. We do not show the equivalent figure for L^* = 2.44 as no substantial differences between storm phases were observed.

Firstly, we observe from Figure 5 that the statistical error and bias under geomagnetically quiet conditions were effectively the same as for all data between 2016-2019 shown in Figure 3. This indicates that the overall statistical error and bias for the hindcast at L^* = 4.12 was not influenced by increased error or bias during geomagnetic variations. Figure 5 g-h show that high μ values (highest energies ≥ MeV) appear least accurate and most biased during the recovery storm phase (and during substorm injections) as MSA increased to 200% at the highest μ values (panel g), and SSPB decreases to -200% (panel h). This indicates that, during the recovery phase, the hindcast underestimated the PSD of MeV electrons, which is understood to become enhanced during this phase (e.g., Jaynes et al., 2015; K. R. Murphy et al., 2018; Sorathia et al., 2018). Conversely, PSD of low μ electrons ($\lesssim$ 700 keV for) is less accurate during the main storm phase (panel d), and during substorm related injections (panel j-k). In both cases the error is observed to reach 200% and the hindcast was biased towards underestimation of PSD down to -200%. Figure 5 shows that the hindcast consistently underestimated PSD across all storm conditions at L^* = 4.12, for all μ and K.

Similarly, Figure 6 shows that at L^* = 6.04, quiet times exhibited the same statistical error and bias as the overall time period (Figure 3), and PSD of high μ (energies ≥ MeV) were the least accurate and most biased during the recovery phase of the storm (Figure 6 g-h). However, Figure 6d shows that, irrespective of μ, the largest overall errors were observed at low K (i.e., equatorial electrons) during the main storm phase. Given Figure 6e shows a bias towards overestimation of PSD at these K, and that L^* = 6.04 is near geostationary orbit, it is possible that loss to the outer boundary was not well captured by the hindcast. Figure 6j also shows large hindcast errors for μ <500 MeV/G, and bias towards underestimation of PSD by up to -200%. Since this feature was most prominent during storms with the highest AE index, we expect these errors were caused by substorm injections of lower-energy (<500 keV) electrons.

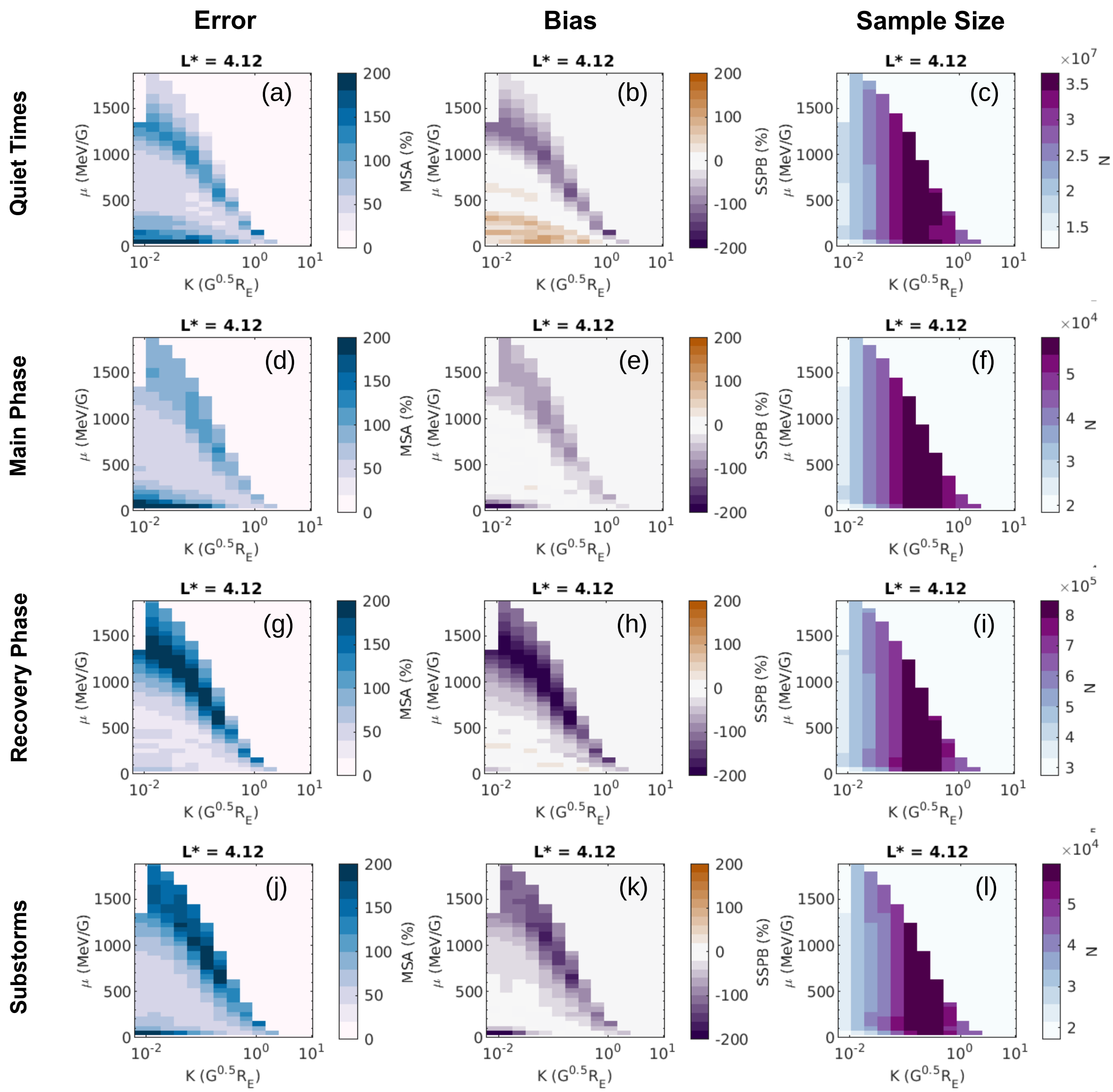

Figure 5 Left column shows the statistical error (MSA) and middle column shows the statistical bias (SSPB) of the 10-hour hindcast between 2016-2018, compared to the final multi-mission PSD observations. Right column shows the number of data samples N used in the computation of MSA and SSPB. Error, bias, and sample size are all shown by color as a function of μ and K at sampled bins of L^* = 4.12. Each row shows error and bias under different geomagnetic conditions a-c are quiet times, d-f are the main storm phase, g-I are the recovery storm phase, and j-l show storm intervals which contain substorm injections.

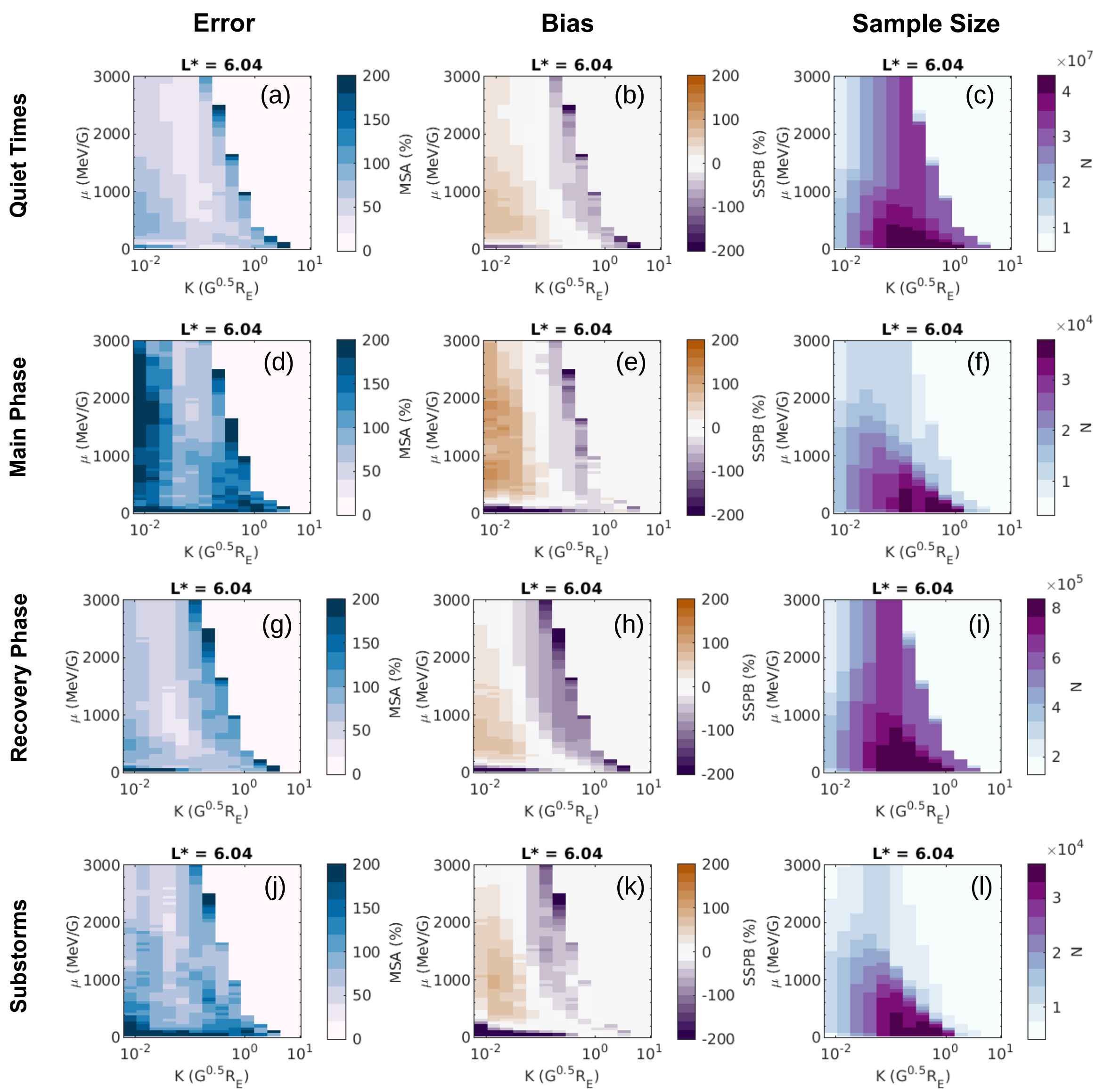

Figure 6 Statistical error and bias under different geomagnetic conditions, sampled at *L = 6.04 is shown in the same format as Figure 5.**

4.2 Updated Forecast Framework (2019-2020)

Since the end of the Van Allen Probe mission in early 2019, the RBFMF has been operating by assimilating only GOES observations in real time. To assess how this affected the accuracy of the 10 hour hindcast, we repeat the analysis presented in the previous section, comparing the 10 hour hindcast at L^* = 4.2 to PSD observations obtained from the GPS constellation between March 2019 – December 2020 (Figure 7). Since the real-time GOES data assimilated into the hindcast model is similar to the final science data product, it is not meaningful to complete an error analysis at geostationary orbit using this data.

Figure 7 shows that the hindcast error and bias at L^* = 4.12 were significantly increased compared to the Van Allen Probe era (Figure 3), reaching maxima of > 2000%, which is a factor of 10 greater than was observed between 2016-2018. The hindcast was strongly biased towards overestimation of PSD at all μ and K values by similar magnitudes to the error, which suggests that the model could be improved by the inclusion of a corrective factor. It is important to note that GPS satellites do not resolve electron flux by pitch angle, so an assumed pitch angle distribution is used to calculate PSD as a function of μ, K, and L^*. It is possible that error and bias determined at L^* = 4.12 were affected by the assumed pitch angle distribution used in GPS data processing, rather than actual errors in the hindcast. Another dataset of PSD observations which are pitch angle resolved is needed to test if this is the case (e.g., ARASE).

We emphasize that the diffusive simulation driving the hindcast provided a good first approximation of radiation belt dynamics, but is somewhat rudimentary as simplified diffusive modelling was employed. However, we chose not to modify the forecast model until a comprehensive analysis of hindcast performance was conducted. Since less observational data is now available to constrain the hindcast via data assimilation, the diffusion simulation should be improved by updating the precomputed diffusion coefficients using more modern methodologies of representing Chorus (e.g., Wong et al., 2024), Hiss (e.g., Agapitov et al., 2020; Watt et al., 2019), and ULF waves (e.g., Kyle R. Murphy et al., 2023). The diffusive effects of EMIC waves could also be incorporated (e.g., Ross et al., 2020) to improve the representation of electron loss in the inner magnetosphere.

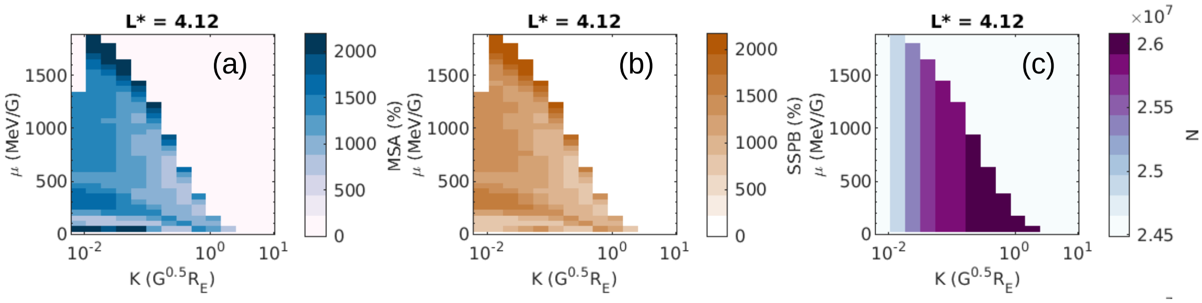

Figure 7 Statistical error (MSA, panel a) and bias (SSPB, panel b) are of the 10-hour hindcast in 2019-2020 are shown as a function of μ and K at L^* = 4.12. The number of data samples, N, is shown in the panel c.

5 Summary

We have conducted a comprehensive assessment of the accuracy and bias of the Radiation Belt Forecasting Model and Framework, which is used to specify the real-time radiation environment in the Satellite Charging Assessment Tool. Historical hindcasts were compared to observations of the radiation belt by computing the statistical errors and bias between the years January 2016 – October 2018 while the Van Allen Probes remained in operation, and between March 2019 – December 2020 following the end of the Van Allen Probe mission.

The hindcast was found to be accurate to within a factor of 1.5 in the outer radiation belt ($4 < L^* < 7$) during the years when the Van Allen Probe data was assimilated into the model (Figure 3d,g). We identified that the statistical hindcast bias was predominantly introduced by the assimilated Van Allen Probe data, which displayed the same dependence of bias upon μ and K (Figure 4). Analysis of geomagnetic storms between 2016-2018 also revealed increased hindcast error and bias compared to quiet times at $L^* > 4$. The most energetic electrons (> MeV) were more likely to be underestimated by the hindcast during storm recovery phase (Figure 5), error increased for equatorial electrons at $L^* \sim 6$ during the main storm phase (Figure 6d), and the hindcast underestimated lower energy electrons (< 500 keV) related to substorm injections (Figure 6j).

We have shown that the hindcast was much more accurate at predicting PSD than if the Van Allen Probe beacon data was used alone (Figure 4). Moreover, we found that the Van Allen Probe beacon data played a crucial role in constraining hindcast simulation between 2016-2019 as the hindcast error and bias increased tenfold when the Van Allen Probe data was no longer available (2018-2020). This highlights that combining coarsely processed data with physics-based modelling through data assimilation can improve the accuracy of radiation environment specification than either method used alone.

6 Future Work

Our analysis has emphasized the importance of real-time observations at multiple locations through the outer radiation belt. Even though the beacon Van Allen Probe data contained significant error and bias compared to the final processed data, assimilation of these observations into the RBFMF considerably improved the simulation compared to times where it was not assimilated. Furthermore, we showed that data assimilative techniques displayed reduced error and bias compared to the real time observations which were coarsely processed compared to final processed science data. Since the end of the Van Allen Probe mission, there are no similar observations available as the currently operational observatories (e.g., GPS, ARASE) do not provide publicly available data in real time. We emphasize that any provision of real time observations from existing or new missions enhance the operational impact of data, even if it is sub-optimally processed compared to science quality data. Furthermore, analysis of real-time data errors, analogous to our analysis of beacon Van Allen Probe data, can be used to inform the observational uncertainties used during data assimilation simulations.

In lieu of real-time observations to constrain this stimulation through the heart of the radiation belt, our analysis has highlighted key areas in which the physics simulation could be improved. Overestimation of PSD in the plasmasphere could be addressed by evaluating more recent diffusion coefficients computed for Plasmaspheric Hiss (e.g., Agapitov et al., 2020; Watt et al., 2019). Improved representation of electron loss at geostationary orbit during the storm main phase could be incorporated by using a dynamic outer boundary of the simulation (Bloch et al., 2021; Staples et al., 2020) and evaluating new radial diffusion coefficients (Kyle R. Murphy et al., 2023). Updating the radial diffusion coefficients could also improve hindcast with sparse real-time data by accurately propagating the effects of assimilated data across L^*. Underestimation of ultra-relativistic electrons during the storm recovery phase could be improved by updating energy diffusion through new parameterizations of Chorus waves (e.g., Wong et al., 2024). Furthermore, substorm injections of lower energy electrons are necessary, and could be incorporated through updates to the simulation boundaries. Continued development of the physics-based simulation is ongoing so that new versions of the RBFMF can be provided with improved hindcast accuracy.

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Open Research

Spacecraft data from GOES and the Van Allen Probes are publicly available via the NASA/GSFC CDAWeb service (https://cdaweb.gsfc.nasa.gov/index.html/). Solar Wind data and geomagnetic indices are publicly available through the NASA/GSFC Space Physics Data Facility OMNIWeb service (https://omniweb.gsfc.nasa.gov/). Due to the size of the specification model output and the electron PSD reference dataset, these cannot be hosted on an online repository platform, but will be made available upon request.

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