

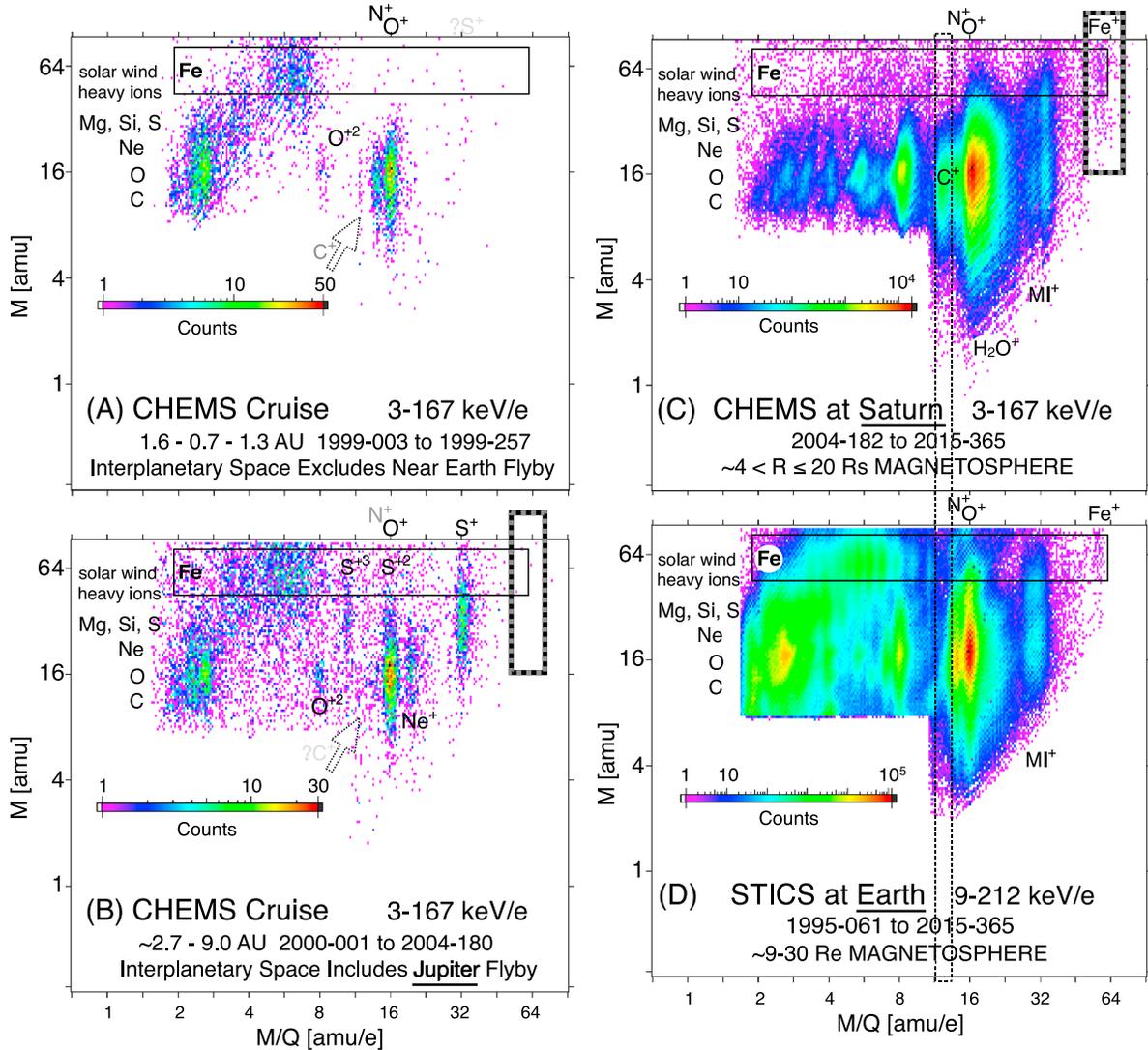
Energetic Fe Ions In And Near The Magnetospheres Of Earth, Jupiter, And Saturn

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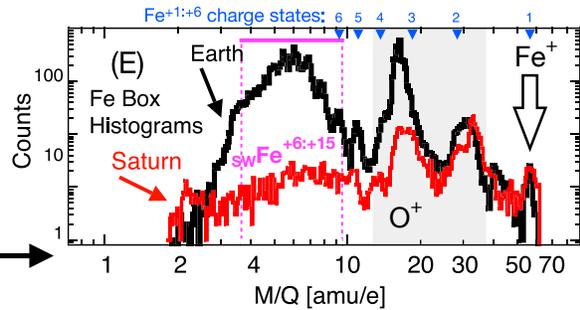
We examine long-term energetic heavy ion measurements including three planets' magnetospheres, focusing on Fe ions (specifically, but not exclusively, Fe⁺) in and near Earth's magnetosphere. We compare Fe data to that of other energetic ion species with masses greater than C (carbon) and consider the relationship(s) of energetic Fe ion measurements at the three planets to internal (ionospheres, exospheres, moons, rings, and trapped radiation) and external (solar wind and interplanetary dust) source candidates. Fe⁺ has been observed at Earth and Saturn, but not yet at Jupiter, as our observations there were brief. The measurements are from two functionally identical charge-energy-mass ion spectrometers: one on Geotail (~87-212 keV/e), orbiting Earth at ~9-30 Re; and the other on Cassini (~83-167 keV/e), in interplanetary space, during Jupiter flyby, and at ~4-20 Rs on its constantly varying orbits around Saturn. These ion spectrometers efficiently separate energetic light and heavy ions by mass, as well as lower charge state ions from higher charge state ions by mass-per-charge. Energetic low charge state ions often derive from magnetospheric sources, while energetic high charge state ions most often derive from the solar wind. We also enlist heavy ion measurements closer to the Earth from AMPTE/CCE which are used for C and Fe radiation-belt-modeling content, consideration, and estimation.

Fe⁺ is clearly observed at Earth and Saturn, but has not yet been detected at Jupiter

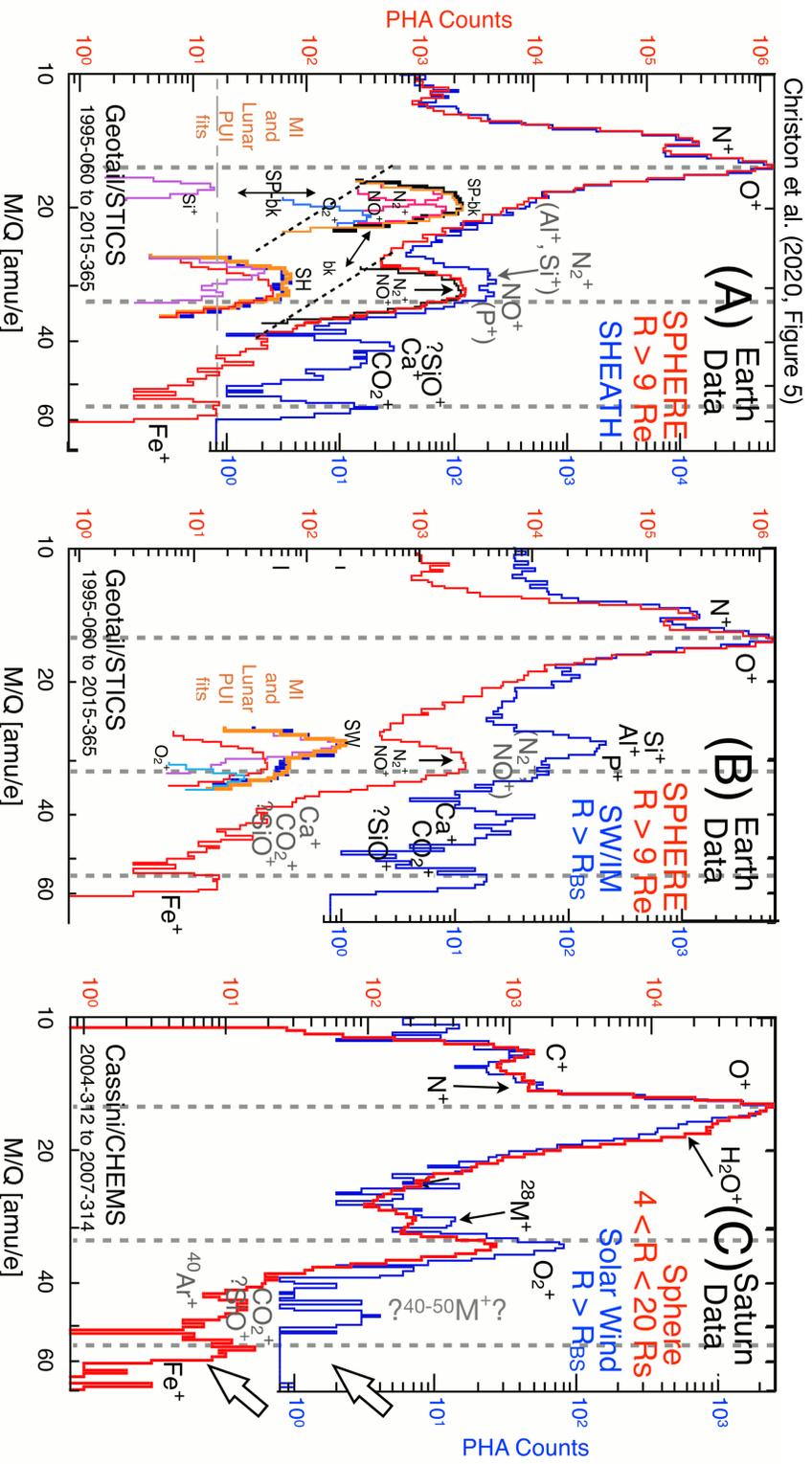


Christon et al. (2017, Figure 7)

- Any possible Fe⁺ near Jupiter is consistent with background
- The Fe⁺/swFe^{+6:+15} ratio is much higher at Saturn than at Earth



Although clearly observed inside Saturn's magnetosphere, Fe⁺ was not detected outside it

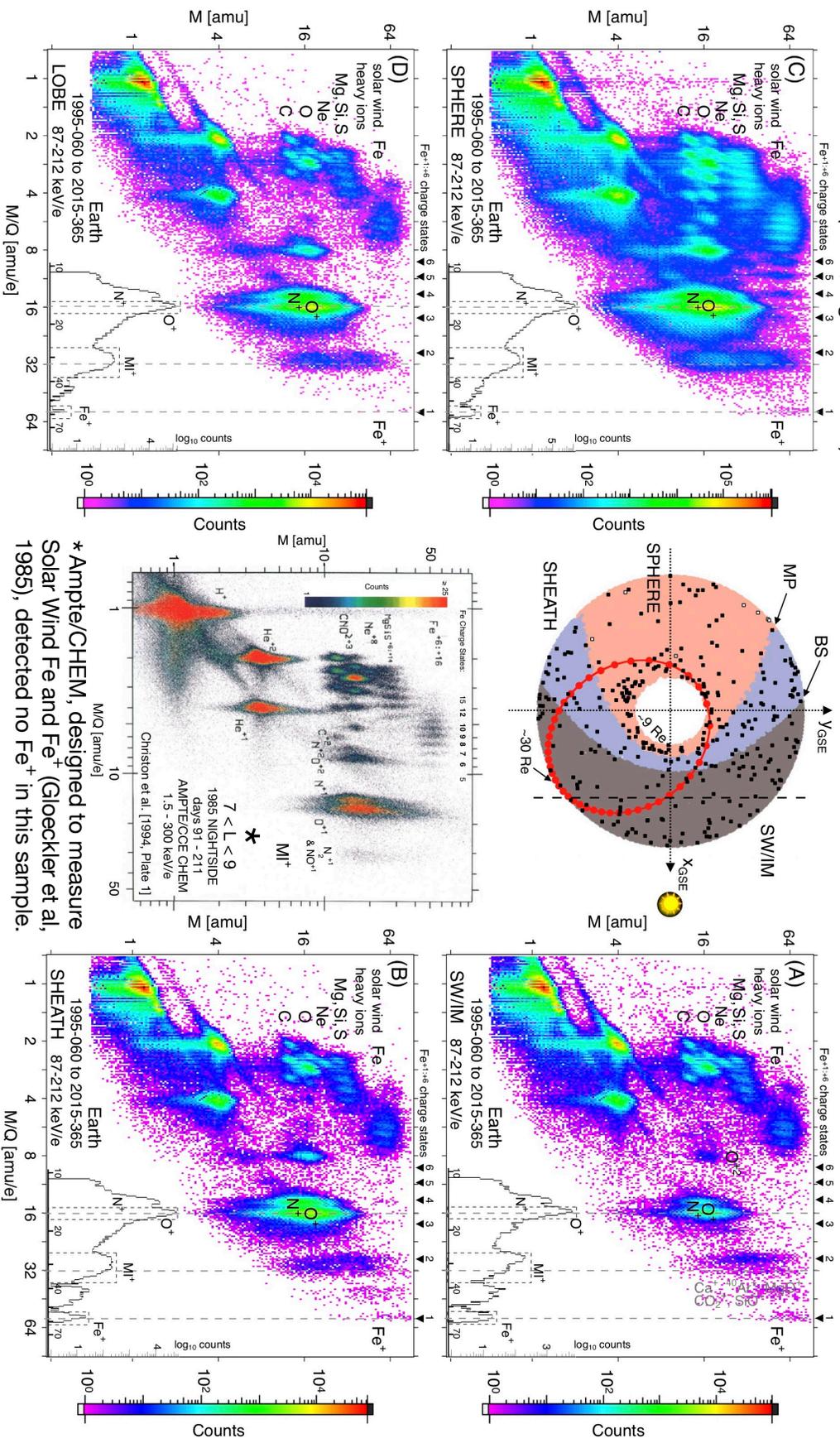


- At Earth (~21 years) and Saturn (~3 years), continuous, successively-sampled plasma regime intervals of Solar Wind (Sheath) and Sphere are compared
- No Fe⁺ was measured outside Saturn's magnetosphere during these intervals (right panel)
- A lack of Fe⁺ escape from Saturn's magnetosphere might result from internal dynamics or its size. In contrast, other unique internal heavy-ion species escape (i.e., ²⁸M⁺, O₂⁺, ^{?40-50}M⁺?)

Fe⁺ In And Near Planetary Magnetospheres • AGU Fall Meeting • Christon et al. (2020)

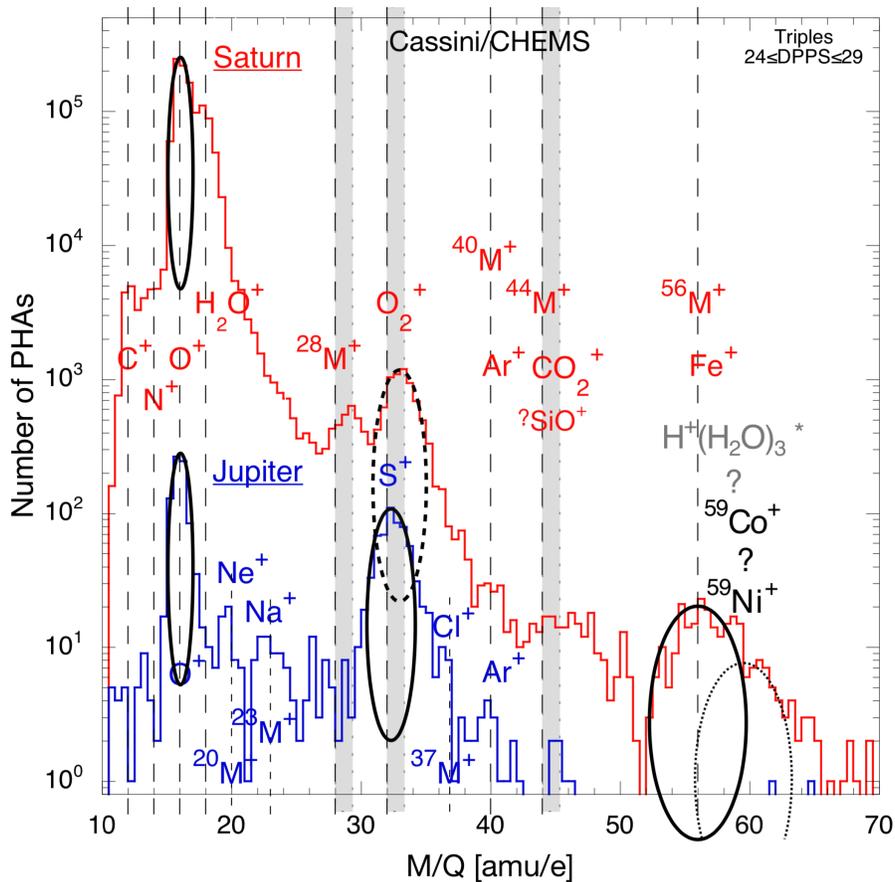
Although rare, Fe⁺ is observed in all near-Earth (~9-35 RE) plasma regimes

Christon et al. (2017, Figures 1 and 2)



* Ampte/CHEM, designed to measure Solar Wind Fe and Fe⁺ (Gloeckler et al, 1985), detected no Fe⁺ in this sample.

Q: Is Fe⁺ The Only Ion Observed At M/Q > 50 amu/e?



PHA M/Q histograms (with eyeball-fit ellipses) compare ion data at:

Saturn : red, R < 20 Rs.....2004-181 (SOI) to 2013-365;
and

Jupiter : blue, bow shock-to-sheath.....2000-363 to 2001-091.

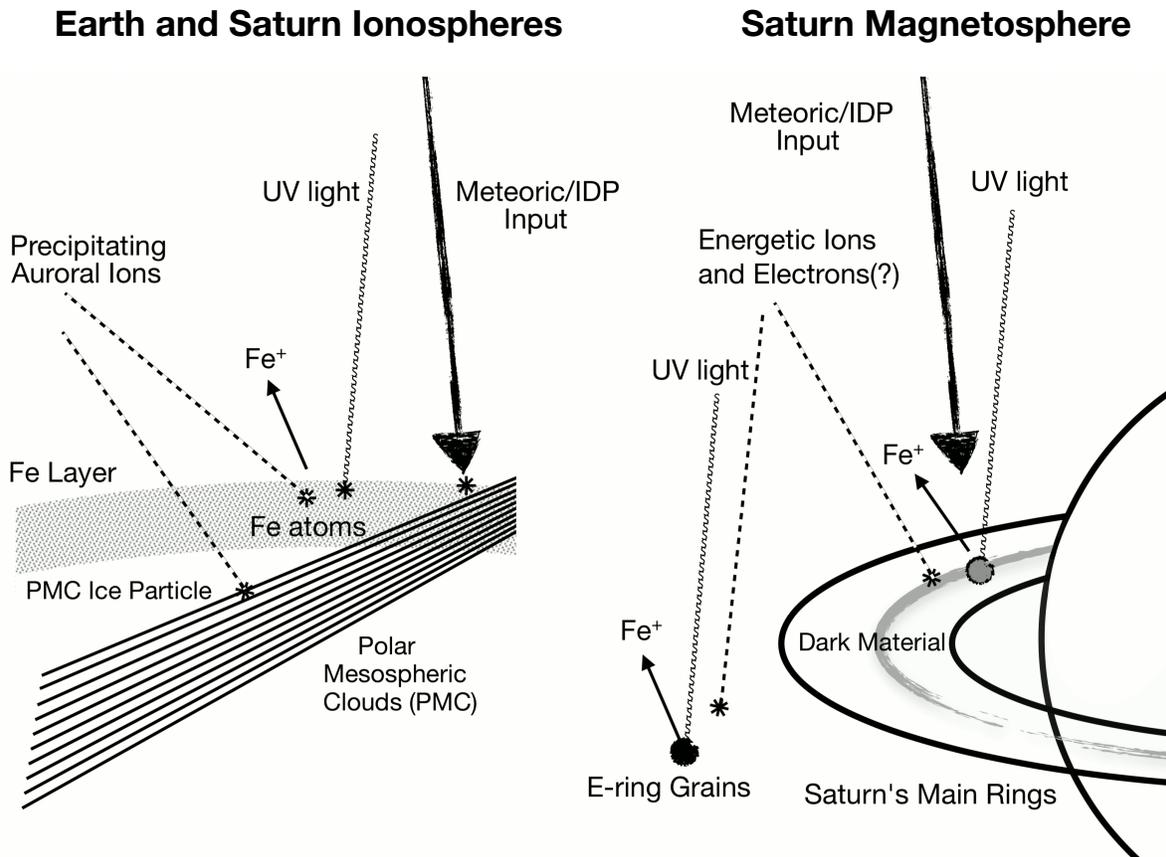
For the molecular ions ²⁸M⁺, O₂⁺, and CO₂⁺ (?or SiO⁺), nearly equal-mass atoms, gray bars extend from the ion's true M/Q to its peak's M/Q centroid location which is found at higher M/Q - as the molecular ion's energy losses are higher than those of its independent atomic ions, resulting in a lower time-of-flight.

⁵⁹Co⁺ and ⁵⁹Ni⁺, likely IDP products, may be present at M/Q ~ 60 amu/e

*or possibly H⁺(H₂O)₃, as suggested by Cassini/CDA (Postberg et al., 2018)

A: Probably Not At Saturn!

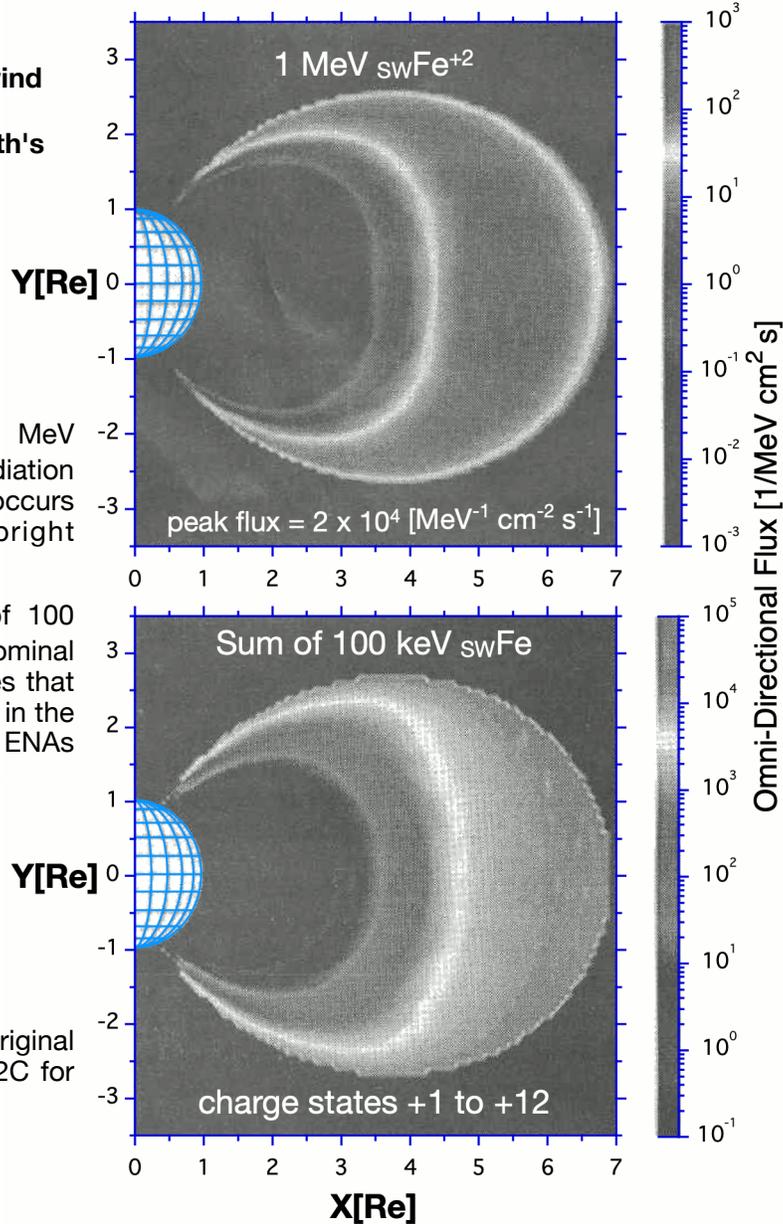
Likely Source and Production Scenarios for Fe⁺ at Earth, Saturn (and Jupiter)



Meteoric particles and interplanetary-dust (IDP) bombard and ablate in planets' magnetospheres and thermospheres. These processes produce Fe atoms, Fe-containing icy particles, and compounds from which singly-ionized Fe, Fe⁺¹ (\equiv Fe⁺), can result when impacted by precipitating auroral particles or irradiated by solar UV. That Fe⁺¹ often becomes an integral part of their ionospheres (Plane, 2012; Frankland & Plane, 2015; Christon et al., 2015; 2017, and references therein). The resulting Fe⁺¹ can then participate in the outward transport processes from the upper ionosphere into the magnetosphere. The same overall processes involving precipitating energetic particle impact and meteoric bombardment/ablation likely occur in all planets' thermospheres, rings, and ring atmospheres. To our knowledge though, no set of observations has yet provided detailed measurement and identification of the specific acceleration mechanisms involved in these processes for Fe⁺¹ in any magnetosphere.

The Solar Wind is Another Possible Source of Fe⁺ at Earth
Solar wind origin swFe^(+6:+15) processed near-Earth may contribute to the energetic Fe⁺¹ observed in interplanetary space near Earth.

Location of solar wind origin swFe ion distributions in Earth's Radiation Belts



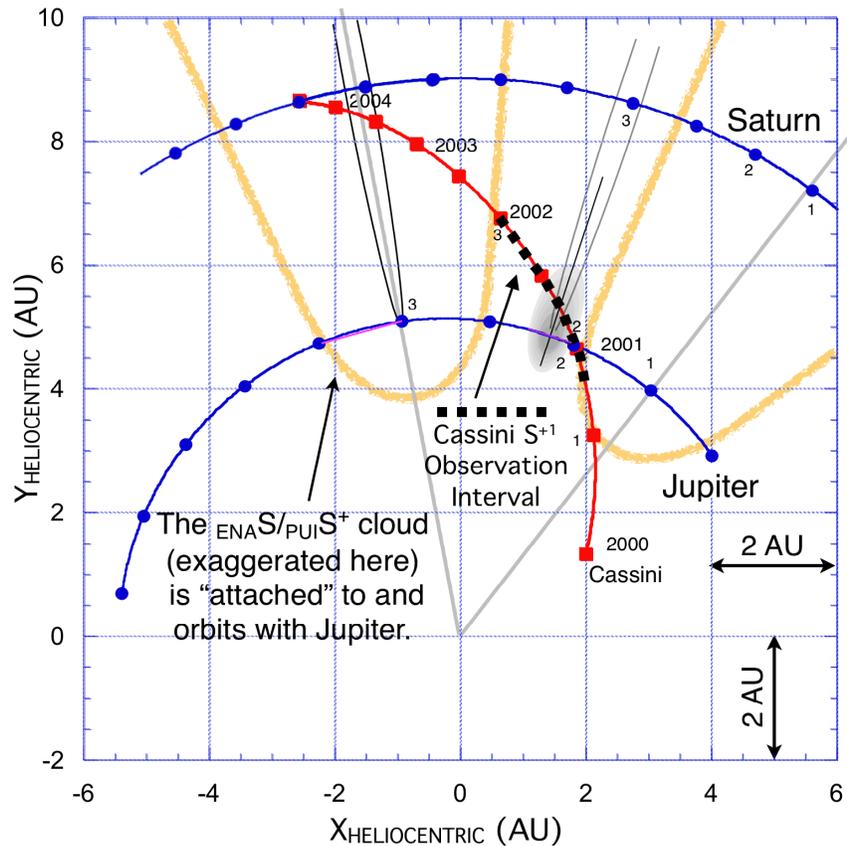
(Top) The locus of ~1 MeV swFe⁺² in Earth's Radiation Belts. The flux peak occurs between the two bright curves.

(Bottom) The locus of 100 keV swFe⁺¹ ions at a nominal flux level, demonstrates that 100 keV Fe⁺ is present in the regions from which ENAs are known to emanate.

(Grayscale copies of original color Figures 1A and 2C for Spjeldvik et al., 2002)

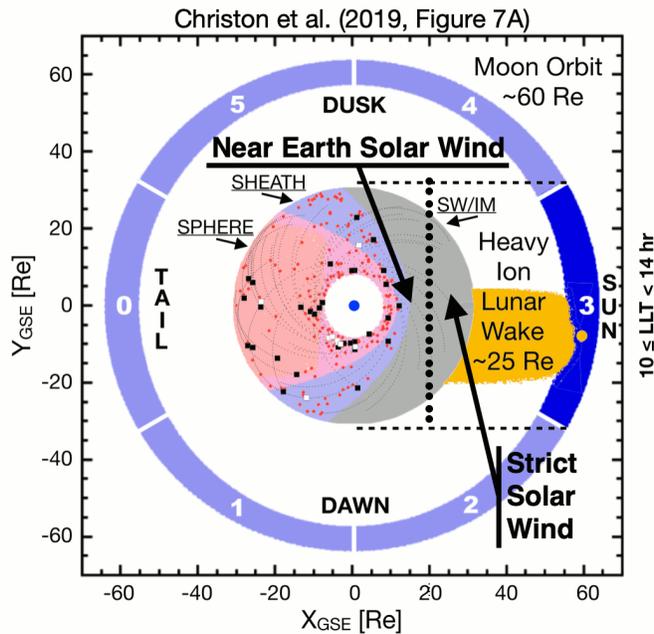
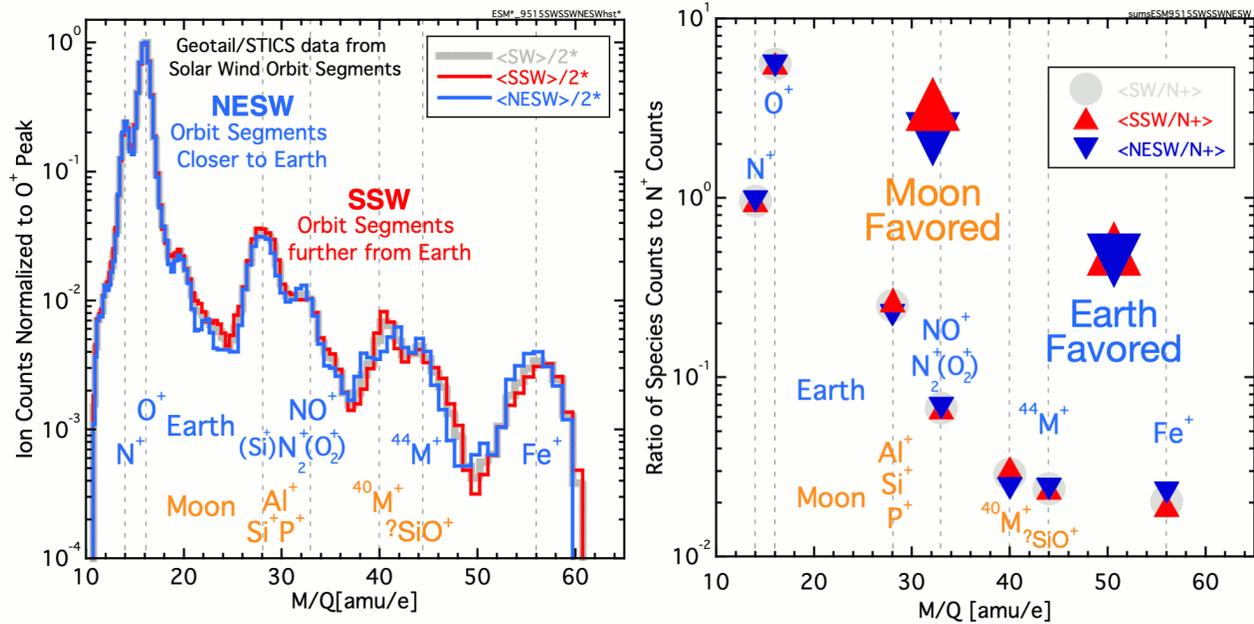
Solar Wind origin Fe, swFe, ion fluxes in Earth's radiation belts were calculated using a data-based swFe⁺¹² input spectrum (Spjeldvik et al., 2002; Spjeldvik, 1996). (bottom) These distributions include swFe^{+1:+5}. ~100 keV Energetic Neutral Atoms, ENAs, are produced locally at < 7 R_E from charge exchange (Brandt et al., 2002) - energies comparable to those of Fe⁺¹ observed outside Earth's magnetosphere (Christon et al., 2017). ENAs may be lost from the magnetosphere continually, or, at minimum, during disturbed magnetospheric intervals.

The S ENA - S⁺ PUI Component at Jupiter

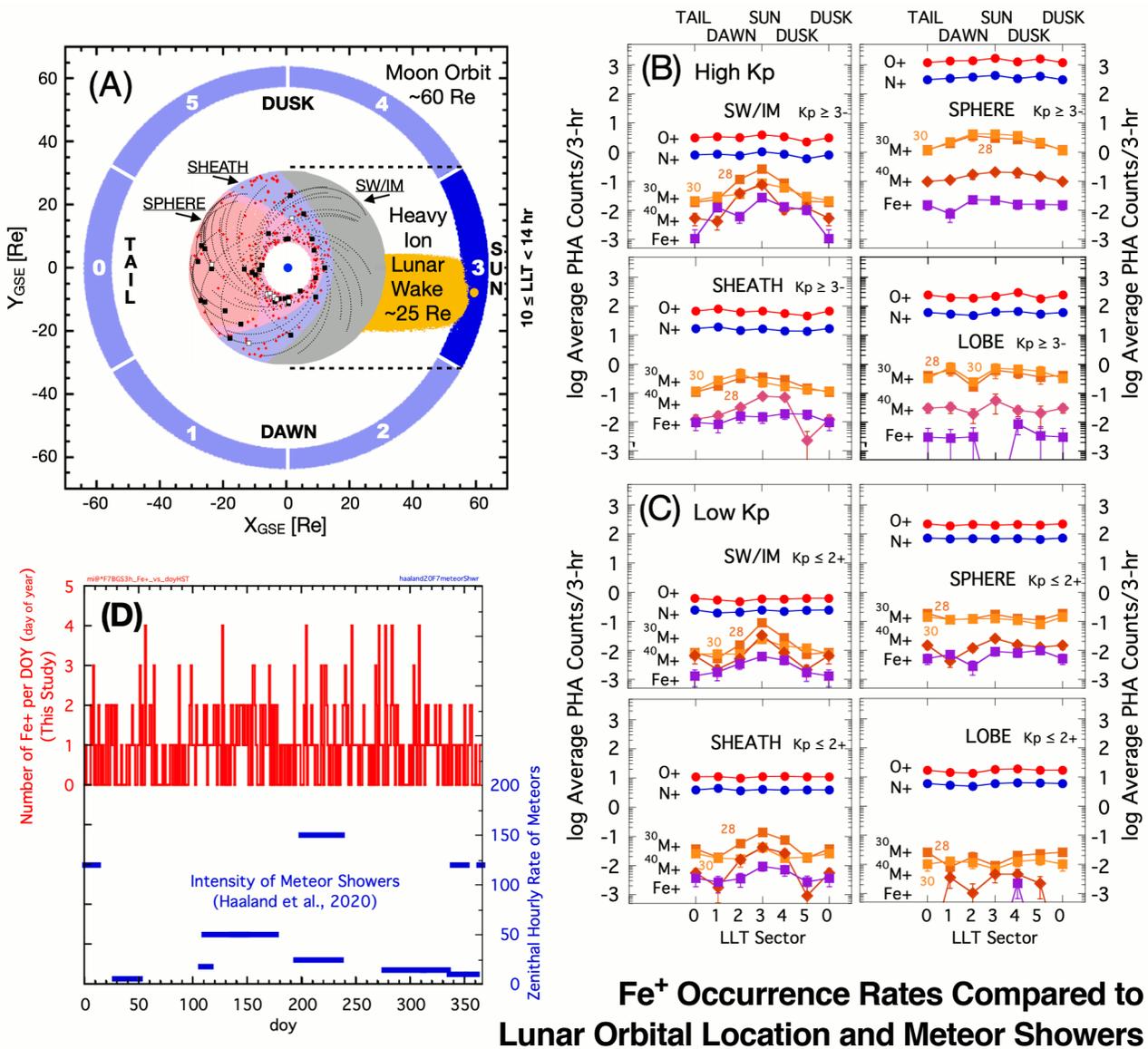


A sketch of Cassini's trajectory and Jupiter and Saturn orbits in heliocentric solar ecliptic coordinates. Shown at the start of each year, blue circles identify planets and red squares show Cassini spacecraft locations from 2000 to 2004. Cassini colocates with Saturn after mid-2004. Three common times are numbered along the trace and at the planets for reference. Jupiter's H, O, and S ENA/PUI cloud (e.g., Krimigis et al., 2002) is drawn both (1) exaggerated, as golden hyperbolae (radius of curvature ~ 1.4 AU), and (2) minimal, using a scaled image of Earth's neutral H cloud, width ~ 0.75 AU. Jovian origin S⁺, detected along the heavy, black-dashed trace (Christon et al., 2020), are likely pickup ions, PUI S^+ , expected from Jupiter's energetic neutral atoms, ENAS (Gruntman, 1997; Luhmann, 2003). As some of the S⁺ can travel along the IMF to the point of observation from the cloud, the cloud's nominal size is probably somewhere between these estimates.

The Main Source of Fe⁺ is Most Likely Earth, Not the Moon



(top panels) Geotail/STICS solar wind, SW, data are separated into near-Earth solar wind, **NESW**, and strict solar wind, **SSW**, groups (left), ordered by M/Q and summed into relevant groups. In the NESW, at X_{GSE} < 20 Re, Earth origin ion species counts are slightly higher than those of lunar origin ion species and vice-versa, lunar origin ion species counts are slightly higher than terrestrial origin ion species in the SSW at X_{GSE} ≥ 20 Re.



Fe⁺ Occurrence Rates Compared to Lunar Orbital Location and Meteor Showers

(A, B, C) Panels from Figure 7 of Christon et al. (2020) show heavy ion occurrence rates collected in six equal Lunar Local Time (LLT) sectors in the ~80-200 keV/e range. When measured in the solar wind/interplanetary medium, SW/IM, the heavier ions shown are expected to exhibit higher SW convection-peaking centered in LLT = 3. Lunar ions (³⁰M⁺ and ⁴⁰M⁺) exhibit pronounced peaks centered on LLT = 3. Fe⁺ exhibits a broad LLT = 3 centered enhancement in the SW/IM during high and low Kp intervals, not necessarily consistent with a lunar source. **(D)**. The Fe⁺ DOY-occurrence rate shows little relation to that of meteor showers listed by Haaland et al. (2020). Neither comparison supports an argument that Fe⁺ occurrence depends on these factors.

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- Fe⁺ is clearly observed at Earth and Saturn, but has not yet been detected at Jupiter
- Although clearly observed inside Saturn's magnetosphere, Fe⁺ was not detected outside it
- Although rare, Fe⁺ is observed in all near-Earth (~9-35 R_E) plasma regimes
- Fe⁺ occurrence times show little relation to lunar orbital location timing and/or to meteor shower occurrence
- Fe⁺ production likely results from UV irradiation, auroral precipitating particles and meteoric/IDP bombardment of and ablation in the thermospheres at Earth, Saturn, and Jupiter - as well as in other outer planet's thermospheres, rings, and ring atmospheres
- High-charge-state (+6:+15) solar wind origin iron, $_{sw}Fe$, processed in Earth's radiation belts may result in Fe ENAs which are then ionized and picked-up by the solar wind, becoming or contributing to the energetic Fe⁺ observed in interplanetary space near Earth. Such a Fe/Fe⁺ ENA/PUI cloud would be smaller than the H/H⁺ ENA/PUI cloud
- Fe⁺ is likely not the only ion observed at M/Q > 50 amu/e at Saturn
- At Earth, our data show that the main source of Fe⁺ is most likely Earth, not the Moon
- Data: • Cassini/MIMI/CHEMS data are at <http://pds.nasa.gov>.
• Geotail/EPIC/STICS data are at http://spdf.gsfc.nasa.gov/pub/data/geotail/epic/stics_pha_ascii_gzip and the JHU/APL Space Department.

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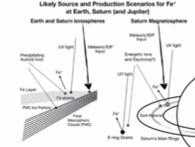
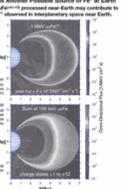
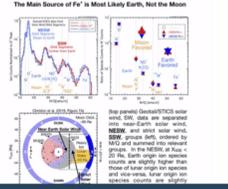
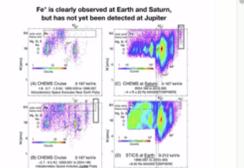


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(1) Focused Analysis and Research, Charleston, SC, USA, (2) University of Maryland, Department of Physics, College Park, Maryland, USA, (3) School of Chemistry, University of Leeds, Leeds, U.K., (4) Johns Hopkins University Applied Physics Laboratory, Laurel, Maryland, USA, (5) Weber State University, Department of Physics, Ogden, Utah, USA, (6) University of Surry, Guilford, U.K.



<p>Abstract</p> <p>We examine long-term energetic heavy ion measurements including three planets' magnetospheres, focusing on Fe ions (specifically, but not exclusively, Fe⁺) in and near Earth's magnetosphere. We compare Fe data to that of other energetic ion species with masses greater than C (carbon) and consider the relationship(s) of energetic Fe ion measurements at the three planets to internal (ionospheres, exospheres, moons, rings, and trapped radiation) and external (solar wind and interplanetary dust) source candidates. Fe⁺ has been observed at Earth and Saturn, but not yet at Jupiter, as our observations were brief. The measurements are from two functionally identical charge-energy-mass ion spectrometers: one on Geotail (~87-212 keV/e), orbiting Earth at ~9-30 R_E, and the other on Cassini (~83-167 keV/e), in interplanetary space, during Jupiter flyby, and at ~4-20 R_S on its constantly varying orbit around Saturn. These ion spectrometers efficiently separate energetic light and heavy ions by mass, as well as lower charge state ions from higher charge state ions by mass-per-charge. Energetic low charge state ions often derive from</p>	<p>Possible Sources - Most Likely</p> <p>Fe⁺ in And Near Planetary Magnetospheres • AGU Fall Meeting • Christon et al. (2020)</p> <p>Likely Source and Production Scenarios for Fe⁺ at Earth, Saturn (and Jupiter)</p>  <p>Meteoroid particles and interplanetary dust (IPD) bombard and ablate in planetary ionospheres and thermospheres. These processes which singly ionize Fe, Fe⁺ ions result when ionized by precipitating auroral electrons or ionized by solar UV. That Fe⁺ often becomes an integral part of their ionospheres (Shue, 2010; Frankland & Plane, 2010; Christon et al., 2010, 2011) and subsequent transport processes from the upper ionosphere into the magnetosphere. The same overall processes involving precipitating energetic particles and meteoroid bombardment/ablation being active at all planetary magnetospheres, rings, and ring atmospheres. In our research through, we aim at observations that will provide detailed measurement and identification of the specific acceleration mechanisms involved in these processes for Fe⁺ in any magnetosphere.</p>	<p>Possible Sources - Next Most Likely</p> <p>Fe⁺ in And Near Planetary Magnetospheres • AGU Fall Meeting • Christon et al. (2020)</p> <p>The Solar Wind is Another Possible Source of Fe⁺ at Earth Solar wind origin of Fe⁺ is proposed and Earth may contribute to the energetic Fe⁺ observed in interplanetary space near Earth.</p>  <p>Solar Wind origin Fe⁺ ions, ion fluxes in Earth's radiation belts were calculated using the Geotail ion spectrometer data from 1995-2000. Spjeldvik, 1996. Further, these calculations include Fe⁺ ions from charge exchange (Shue et al., 2010) energies comparable to those of Fe⁺ observed outside Earth's magnetosphere (Christon et al., 2017). Fe⁺ may be lost from the magnetosphere continually, or at intervals, during disturbed magnetospheric intervals.</p>	<p>Unlikely Source Candidates</p> <p>Fe⁺ in And Near Planetary Magnetospheres • AGU Fall Meeting • Christon et al. (2020)</p> <p>The Main Source of Fe⁺ is Most Likely Earth, Not the Moon</p>  <p>The poorly characterized solar wind Fe⁺ ions are expected to be much less abundant than Earth's solar wind. MEWS and other solar wind Fe⁺ ions are expected to be much less abundant than Earth's solar wind. The Earth's solar wind Fe⁺ ions are expected to be much less abundant than Earth's solar wind. The Earth's solar wind Fe⁺ ions are expected to be much less abundant than Earth's solar wind.</p>
<p>Selected Observational Aspects</p> <p>Fe⁺ in And Near Planetary Magnetospheres • AGU Fall Meeting • Christon et al. (2020)</p> <p>Fe⁺ is clearly observed at Earth and Saturn, but has not yet been detected at Jupiter</p> 	<p>Summary and References</p> <p>Summary</p> <p>Fe⁺ is clearly observed at Earth and Saturn, but has not yet been detected at Jupiter. Although clearly observed inside Saturn's magnetosphere, Fe⁺ was not detected outside it. Although rare, Fe⁺ is observed in all near-Earth (~9-35 R_E) plasma regimes. Fe⁺ occurrence times show little relation to lunar orbital location/timing and/or to meteor shower occurrence. Fe⁺ production likely results from UV irradiation, auroral precipitating particles, and meteoroid/IPD bombardment of and ablation in the thermospheres at Earth, Saturn, and Jupiter - as well as in other outer planet's thermospheres, rings, and ring atmospheres. High-charge-state (>E=15) solar wind origin ions, SWIFs, processed in Earth's radiation belts may result in Fe ENAs.</p>		

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• SM042-0010 • References •

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To support our reasons for referencing Brandt et al. (2002), we include the color copies of their inverted ion distributions derived from spacecraft ENA images, Figures 8-11, which appear on the article's webpage (whereas the downloadable pdf has only B/W low-resolution versions). Please see: <https://doi.org/10.1029/2002JA009307>

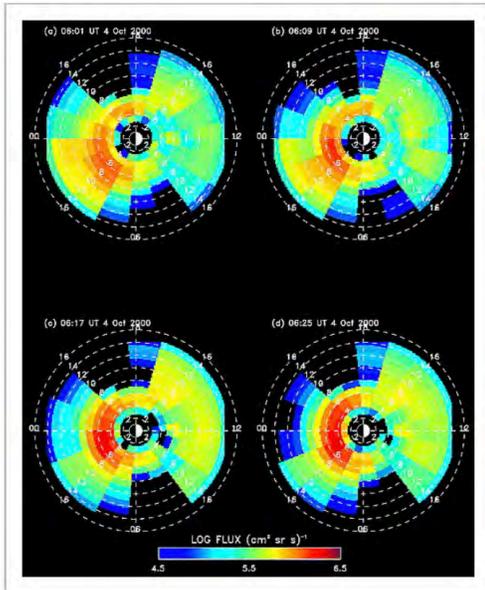


Figure 8

(a-d) The inverted ion distributions in the 10–60 keV range (6 min integration) for the 06:11 UT substorm using the symmetric *Rairden et al. [1986]* model exosphere (see equations (8) and (9)).

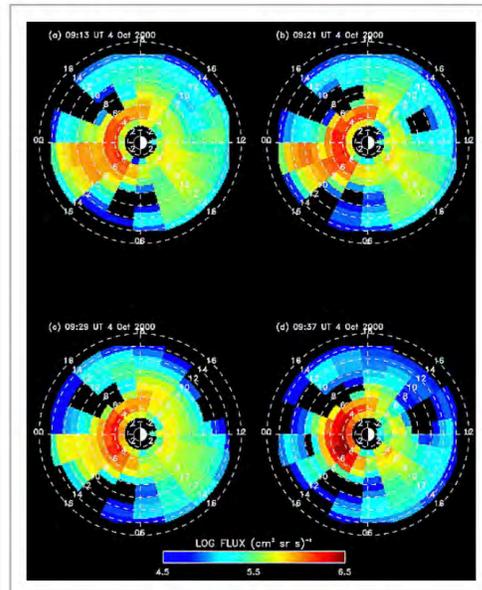


Figure 10

(a-d) The inverted ion distributions in the 10–60 keV range (6 min integration) for the 09:22 UT substorm using the *Rairden et al. [1986]* model exosphere (see equations (8) and (9)).

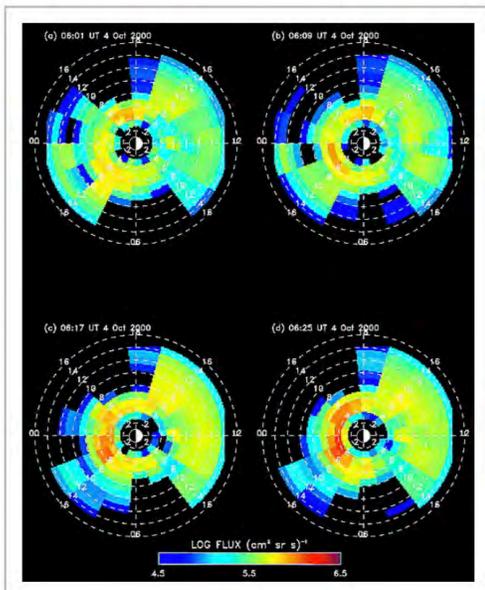


Figure 9

(a-d) The inverted ion distributions in the 10–60 keV range (6 min integration) for the 06:11 UT substorm using the asymmetric *Rairden et al. [1986]* model exosphere with $k = 0.3$ from equations (8) and (9).

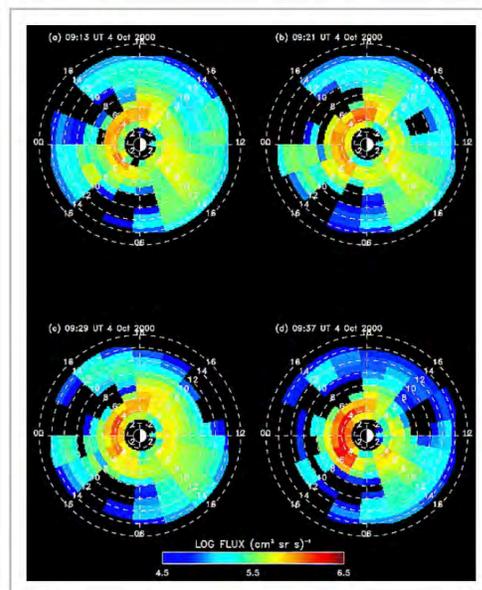


Figure 11

(a-d) The inverted ion distributions in the 10–60 keV range (6 min integration) for the 09:22 UT substorm using the asymmetric *Rairden et al. [1986]* model exosphere with $k = 0.3$ from equations (8) and (9).