New Analytical 3D Magnetohydrostatic Equilibria for Modelling Solar Magnetic Fields

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I. Introduction

- The magnetic field in the solar corona is difficult to measure.
- Measurements at photospheric level are often used as boundary conditions for magnetic field models based on extrapolation methods.
- These methods are usually assuming that the magnetic field is force-free ($\mathbf{j} \parallel \mathbf{B}$).
- However, the photosphere and chromosphere are not force-free. Should this be taken into account?
- Some authors (e.g. Wiegelmann et al., 2015, 2017) have used analytical magnetohydrostatic (MHS) equilibria (Low, 1991) as basis for magnetic field extrapolation.

II. This Poster

- In this poster a new family of MHS equilibria is presented, albeit still based on the general method of Low (1991) – see section III. for some details.
- This family of magnetic fields can be "tuned" to include pressure gradients and gravity in the lower regions (MHS) and the magnetic fields can be potential, linear force-free or approximately linear force-free higher up.
- This is a compromise between simplifying assumptions and ease of implementation; it might alleviate some of the difficulties with the application of previous solutions (e.g. unrealistic plasma β).

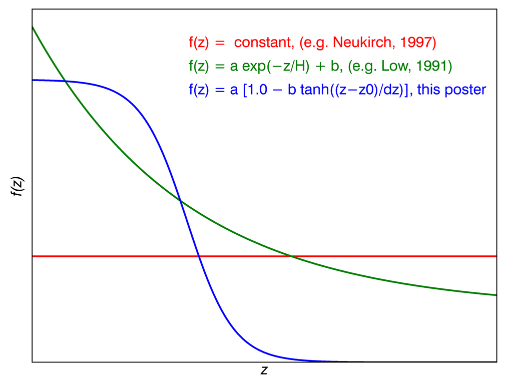

Figure 1: Examples of functions f(z) from the literature compared with the function type used for this poster.

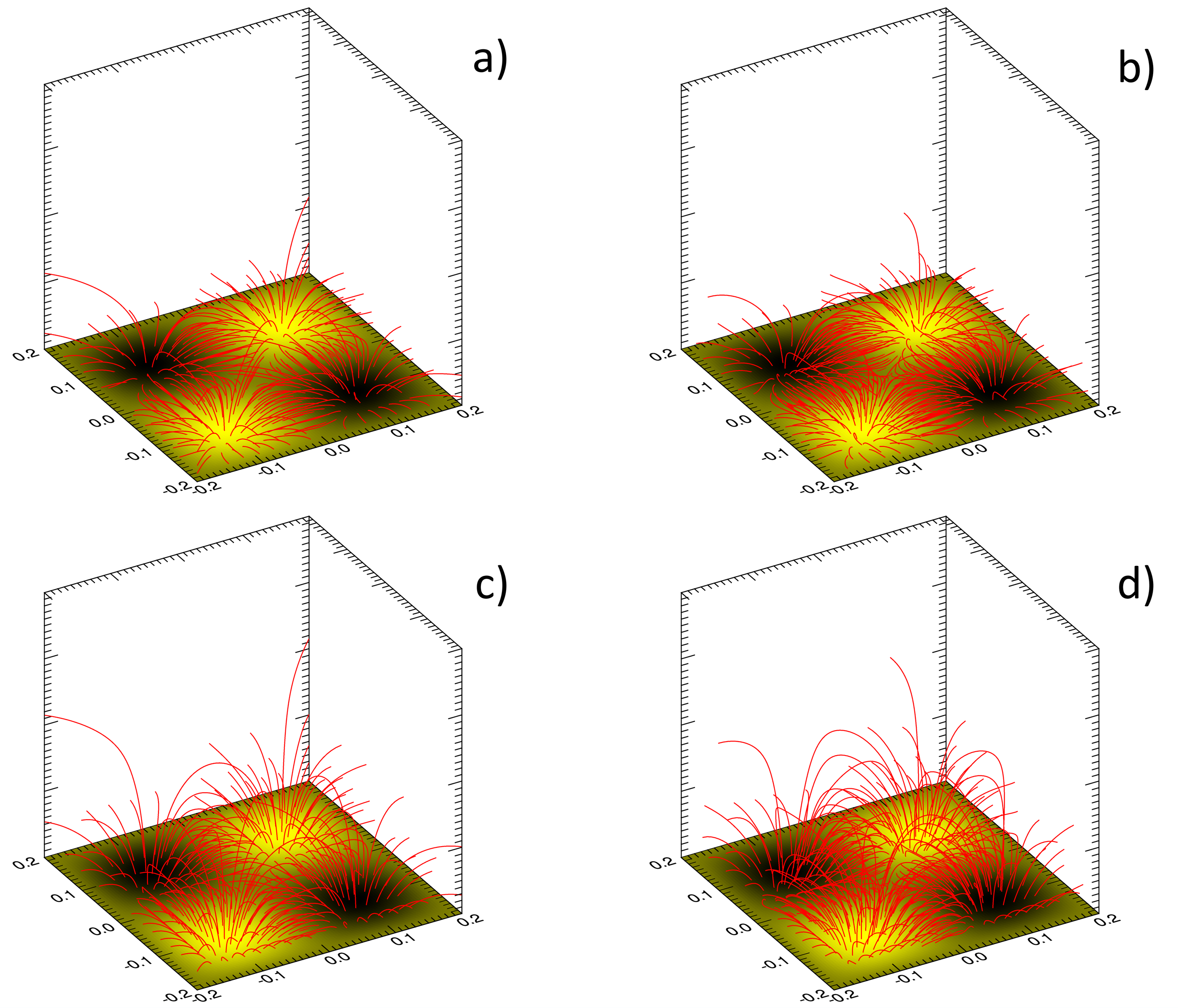

Figure 2: Field line plots for a simple test case; a) potential field; b) linear force-free field (α=5.0); c) MHS solution (a=0.4, b=1.0, z_0=0.2, Δz=0.02, α=0.0); d) same as c), but α=5.0.

- Different parameter values for the new solution family can generate different magnetic field models (see **Figure 2**).
- The simple examples presented above already show significant differences between e.g. a potential field (2a) and an MHS field becoming potential above a certain height (2c) (similarly for a linear force-free field (2b) and corresponding MHS field (2d)).

III. Theory

- Assume $\mathbf{j} = \alpha\mathbf{B} + f(z)\nabla B_z \times \mathbf{e}_z$
  $\alpha = \ \text{constant}, \ f(z) \ \text{a free function}$
- f(z) determines the "shape" of the current density (see **Figure 1** for examples).
- New family of solutions uses

$$f(z) = a\left[1 - b\tanh\left(\frac{z - z_0}{\Delta z}\right)\right]$$

- For details of how to calculate **B** see Low (1991) or Neukirch and Rastätter (1999).
- For the f(z) above the z-dependence of **B** is found in terms of hypergeometric functions.
- Pressure: $p = p_0(z) - f(z)\dfrac{B_z^2}{2\mu_0}$

  $p_0(z): \ \text{stratified background pressure}$
- Density: $\rho = \dfrac{1}{g}\left[-\dfrac{dp_0}{dz} + \dfrac{df}{dz}\dfrac{B_z^2}{2\mu_0} + \dfrac{f}{\mu_0}\mathbf{B}\cdot\nabla B_z\right]$

References:
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